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An improved interpretation of recoil compressive failure data for high-performance polymers

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Abstract

This paper describes an improved methodology for the accurate interpretation of recoil compressive failure data. This new procedure uses a wider portion of the data set in its interpretation than does the method of Allen and is far simpler and less data intensive than the Weibull model proposed by Hayes. Computer simulations using various statistical distributions for the intrinsic recoil compressive strength of a batch of filaments show that this method quickly and accurately converges to an accurate approximation of the mean for the sample set.

1. Introduction

Despite their exceptional tensile strength, high-performance polymers are limited in their applicability to structural applications by their comparatively low compressive strength [1]. For example, Kevlar (poly p-phenylene terephthalamide) fibre-reinforced composites typically possess compressive strengths more than five times less than their tensile strengths [2–4]. Similar behaviours have been shown for other high-performance polymers including PBT (poly p-phenylenebisthiazole) and PBO (poly p-phenylene benzobisoxazole) [5, 6]. The repeat unit of Kevlar is shown in figure 1. It is generally accepted that the compressive failure in high-performance fibre-reinforced composites results from the low compressive strengths of the fibres [5–7]. Therefore, the accurate and simple measurement of compressive failure data in fibres is essential.

Compressive testing of composites is well established by ASTM [8], but these tests involve many steps and large quantities of fibre. Therefore, it would be advantageous to directly measure the compressive strengths of the fibres themselves. At least four different test procedures exist to estimate the compressive strength of fibres. These tests include the bend test, the elastic loop test, testing based on a single filament imbedded in a matrix, and the recoil test. For polymeric fibres, the fibre compressive strengths estimated from composite compression tests agree most accurately with those obtained by the recoil test [9].

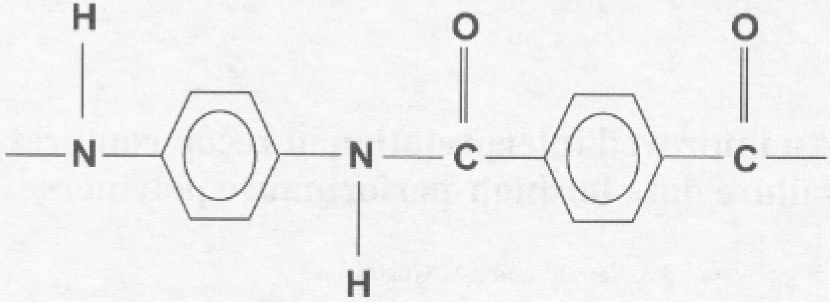

Figure 1. The repeat unit of Kevlar.

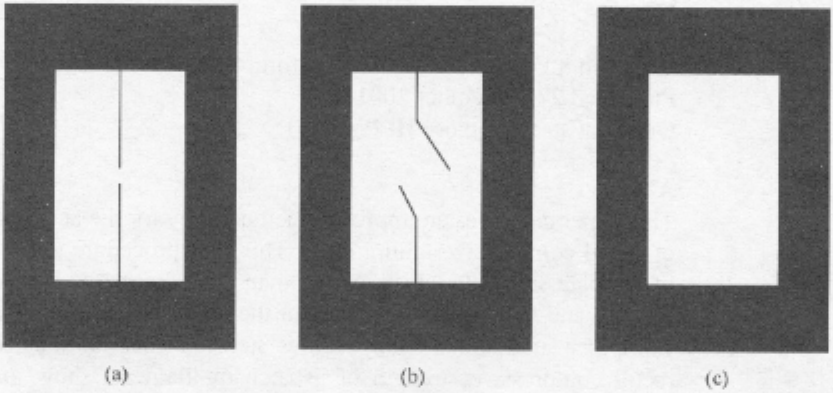

Figure 2. Visual depiction of fibre survival and failure after recoil compressive testing. (a) Survival of both fibres (statistically: 1,1); (b) failure of both fibres in the kinked mode (statistically: 0,0); (c) failure of both fibres by disappearance (statistically: 0,0).

The recoil test is conceptually simple. Fibres are placed under a static tensile load. Next, the fibre is ruptured using either an electric discharge or surgical scissors. The stored energy travels down both halves of the fibre in the form of a recoil compressive wave. The two fibre halves either survive this recoil wave or they experience a secondary failure. Figure 2 depicts recoil compressive failures and survivals. This analysis makes several assumptions [10] about the fibre. Specifically, the fibre obeys Hook's law for linearly elastic materials, is rigidly clamped at each end, has no initial velocity, and has a uniform initial stress along its length at failure. From these assumptions, the axial stress history following tensile failure can be represented by the solution of the Fourier series shown in equation (1),

$$\frac{\sigma(x,t)}{\sigma_0} = \sum_{m=0}^{\infty} \frac{4}{(2m+1)\pi} \sin\frac{(2m+1)\pi}{2} \cos\frac{(2m+1)\pi x}{2L} \cos\frac{(2m+1)(E/\rho)^{0.5}t}{2L} \qquad (1)$$

where x is the fibre coordinate from a clamped end, σ/σ_0 is the normalized stress, L is the length of the broken fibre end, m is an integer between zero and infinity, t is time, E is Young's modulus, and ρ is density.

During the recoil, each half of the cut fibre will experience the same relaxation and compressive stresses. Thus, each test will lead to two data points (the top and bottom halves of the fibre). However, the actual compressive strength of a tested filament cannot be determined directly using the recoil test. Because failure is artificially induced, the resulting recoil compressive wave may be significantly above or significantly below the intrinsic compressive strength of the fibre. If the fibre ends experience a secondary failure, there is no way to ascertain how much stress above the intrinsic compressive strength the recoil wave imparted on the fibre. Similarly, there is no way to know by how much the intrinsic compressive strength of the fibre exceeded the stress imposed by the recoil compressive wave that resulted in two survivals.

Several methods have been proposed to evaluate recoil test data. These methods fall into two distinct categories. The first method used by Allen [10], Wang *et al* [11], and Crasto and Kumar [12] arranges the data in order of ascending load levels. A stress range is identified over which the data change from all survivals to all failures. The mean of the two endpoints of this range is presented as the average recoil compressive strength of the fibre set. In essence, this approach disregards the majority of data points and characterizes the compressive properties of the entire fibre batch based on two points. Additionally, the next fibre tested could always radically change the estimated mean strength if it were to become the highest survivor or lowest failure.

The second approach, developed by Hayes [13], is to gather large quantities of recoil compression data and apply a Weibull model to characterize the underlying distribution. Hayes estimated the average recoil compressive (σ_{rc}) strength as

$$\sigma_{\mathrm{rc}} = \sigma_0 \Gamma\left(1 + \frac{1}{M}\right) \tag{2}$$

where σ_0 and m are the Weibull scale and shape parameters, respectively, and G is the statistical Gamma function. Although the Hayes method provided a reasonable fit of recoil failure data, the large amount of data required to accurately characterize a single sample makes this method impractical for widespread application.

2. Theoretical

This paper proposes a new methodology in which each fibre end is classified as a survival (1) or failure (0) depending on whether it has survived the recoil wave. Any individual filament may survive well beyond the mean recoil compressive strength of the sample, or fail well below it. First, a filament is tested in recoil compression at a random initial stress level ($\sigma_{\mathrm{rc},i}$). If the sum of the test results is two (both ends survive), then the next test is performed at a stress level approximately 0.020 GPa higher than the previous trial as shown in equation (3)

$$\sigma_{\mathrm{rc},i+1} = \sigma_{\mathrm{rc},i} + 0.02\ \mathrm{GPa}. \tag{3}$$

If the sum of the test results is zero (both ends fail), the next sample is test at approximately 0.020 GPa lower, as shown in equation (4).

$$\sigma_{\mathrm{rc},i+1} = \sigma_{\mathrm{rc},i} - 0.02\ \mathrm{GPa}. \tag{4}$$

Finally, if the sum of the test results is one (one end survives while the other fails), the next sample is tested at the same stress level, as shown in equation (5).

$$\sigma_{\mathrm{rc},i+1} = \sigma_{\mathrm{rc},i}. \tag{5}$$

Although the selection of the 0.02 GPa increment is empirical, it was selected because it represents approximately 5% of the typical recoil compressive strength of most high-performance polymers. The incremental increase/decrease process is repeated and the mean test level is recorded as the estimate of the mean recoil compressive strength of the fibre batch, as shown in equation (6).

$$\sigma_{\mathrm{rc}} = \left(\sum_{i=1}^{N} \sigma_i\right) \Big/ N \tag{6}$$

where N is the number of fibres tested. Clearly, the mean test level should eventually approach the mean recoil compressive strength. If the current test level is significantly below the mean, the filament is more likely to survive, thereby raising the estimate with the next test. Conversely, if the current test level is significantly above the mean recoil strength, the filament is more likely to fail, resulting in a lower estimate.

The results would be biased by the initial random guess. Therefore, only the last thirty filaments tested are used in the moving average as shown in equation (7)

$$\sigma_{\mathrm{rc}} = \left(\sum_{i=N-30}^{N} \sigma_i \right) \Big/ 30 \tag{7}$$

3. Experimental

A series of simulations were performed to validate the new moving-average methodology. A random number generator was used to generate simulated intrinsic recoil compressive strength data for filaments following normal, exponential, and Weibull distributions. The Weibull distribution was selected because it is the most likely to represent the true failure of fibres [13–17]. Once the data were obtained, comparisons between the new methodology and the method of Allen were performed.

For example, from a normally distributed fibre set with true intrinsic recoil compressive strength of 400 MPa, the first filament simulated may have an intrinsic strength of 350 MPa. A random first stress level is selected for the first test (say 300 MPa). In either the method of Allen or the moving-average methodology, both filament ends should survive. In the method of Allen, the next test level would be random (say 400 MPa). In the new methodology, the next test would occur at 20 MPa higher than the previous (in this case, 320 MPa). If the intrinsic recoil compressive strength of the second filament was 370 MPa, the method of Allen would have resulted in failure, while the moving-average method would have led to a second survival. This process was repeated for 200 fibres per distribution with estimated means reported after 10 filaments, 25 filaments, and then every 25 filaments until 200.

Next, actual fibre samples of Kevlar-29 were mounted on paper tensile testing tabs and secured with Epoxy 220. Fibre diameters were determined by laser diffraction. The tab was then secured in the locking grips of a modified Instron ultimate testing machine. After a static tensile load was applied to the fibre, the fibre was cut in the centre using sharpened surgical scissors. The filament pieces were examined under a magnifying lens for classification as survivals (1) or failures (0).

4. Results and discussion

Tables 1–3 compare the method of Allen to this new moving-average methodology when applied to the simulated normal, exponential, and Weibull distributions. Although the moving-average method bases its estimate on the previous thirty trials, data from simulations after 10 and 25 trials are included in the tables. These data points show the initial biasing of the estimate caused by the random first guess. Between trials 25 and 50, the effects of this biasing disappear.

These tables clearly indicate the advantages of the new methodology. In every case, the new methodology converged upon the true sample mean more quickly and reliably than the method of Allen. More importantly, the new method makes it far easier to determine when the mean has been adequately characterized. The simulations of the method of Allen demonstrated the susceptibility to single statistical outliers. In the case of the normal distribution, the

Table 1. Results of the method of Allen and the moving-average method applied to simulated normally distributed failure data ($\sigma_{rc} = 400$ MPa) (average of 100 simulations).

	Method of Allen	Moving-average method
Number of fibres	Number of times the estimated σ_{rc} fell within 5% of the true value	Number of times the estimated σ_{rc} fell within 5% of the true value
10	74	62
25	75	73
50	80	84
75	82	88
100	81	94
125	86	97
150	87	97
175	89	98
200	91	98

Table 2. Results of the method of Allen and the moving-average method applied to simulated exponentially distributed failure data ($\sigma_{rc} = 400$ MPa) (average of 100 simulations).

	Method of Allen	Moving-average method
Number of fibres	Number of times the estimated σ_{rc} fell within 5% of the true value	Number of times the estimated σ_{rc} fell within 5% of the true value
10	53	42
25	61	60
50	64	82
75	69	83
100	74	88
125	77	91
150	82	92
175	84	94
200	84	96

estimation of the method of Allen actually became worse between 75 and 100 tests because of an outlier.

The new methodology was then applied to as-received Kevlar-29 fibres. Table 4 shows the effectiveness of the methodology in quickly converging on a mean recoil compressive strength for the sample. Within 45 single filament trials, the testing had converged upon a mean recoil compressive strength of approximately 280 MPa. Although this value is somewhat lower than the typical range of 300–400 MPa reported in the literature for Kevlar-29 [9, 18, 19], it is reasonable for the specific fibre batch tested. More importantly, the data clearly indicate that running several more trials are unlikely to substantially change the estimate.

Table 3. Results of method of Allen and the moving-average method applied to simulated Weibull distributed failure data ($\sigma_{rc} = 400$ MPa) (average of 100 simulations).

	Method of Allen	Moving-average method
Number of fibres	Number of times the estimated σ_{rc} fell within 5% of the true value	Number of times the estimated σ_{rc} fell within 5% of the true value
10	52	44
25	57	50
50	66	78
75	67	84
100	72	88
125	74	90
150	77	92
175	79	94
200	80	94

Table 4. Moving-average method applied to Kevlar-29 fibres.

Number of samples	Estimated compressive strength (MPa)
10	240
25	268
35	274
40	278
45	280
50	281
55	280
60	281
65	280

5. Summary

The simulation runs indicate that the moving-average approach provided a better estimate of recoil compressive failure data than the method of Allen. More importantly, the moving-average technique is less affected by statistical outliers, utilizes more of the data set, and converges more conclusively.

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