CHAPTER 2
Sets, Functions, and Reasoning

Being able to reason is essential to understanding mathematics. By developing ideas, exploring phenomena, justifying results, and using mathematical conjectures in all content areas and—with different expectations of sophistication—at all grade levels, students should see and expect that mathematics makes sense. Building on the considerable reasoning skills that children bring to school, teachers can help students learn what mathematical reasoning entails.¹

Activity Set 2.1 SORTING AND CLASSIFYING WITH ATTRIBUTE PIECES

PURPOSE
To use attribute pieces in games and activities for sorting and classifying, reasoning logically, formulating and verifying hypotheses, and introducing set terminology and operations.

MATERIALS
Attribute Pieces from the Manipulative Kit or from the Virtual Manipulatives, Attribute Label Cards from Material Card 3, and scissors to cut them out, Attribute Game Grids from Material Card 4 and the Two-Circle and Three Circle Venn Diagrams on Material Cards 5 and 6, respectively.

INTRODUCTION
“Mathematics is reasoning. One cannot do mathematics without reasoning. The standard does not suggest, however, that formal reasoning strategies be taught in grades K–4. At this level, the mathematical reasoning should involve the kind of informal thinking, conjecturing, and validating that helps children to see that mathematics makes sense.

“Manipulatives and other physical models help children relate processes to their conceptual underpinnings and give them concrete objects to talk about in explaining and justifying their thinking. Observing children interact with objects in this way allows teachers to reinforce thinking processes and evaluate any possible misunderstandings.”²

Unit 3
Enrichment

Venn Diagrams
You can use data from a diagram to draw conclusions.

School Clubs

Solve.
1. Who attends only the Book Club?


3. How many children attend more than 1 club? _____

Look! Ken attends all 3 clubs.
In the late 1960s, sets of geometric figures called attribute pieces became a popular physical model for activities that promote logical thinking. These attribute pieces are also well suited for introducing ideas and terminology related to sets. The 24 attribute pieces used in this activity set vary in shape, color, and size. There are four shapes (triangle, square, hexagon, and circle), three colors (red, blue, and yellow), and two sizes (large and small). Each attribute piece differs from every other piece in at least one of the attributes of shape, color, or size. The letters on each piece refer to its three attributes. (For example, LRT means large, red, triangle.) Most of the games and activities in this activity set can be adapted for use with children.

1. Place your small blue square, small red square, and small yellow circle on the indicated squares of the grid below. In the first row of the grid, the small blue square differs from the small red square in exactly one attribute—color. In the third column, the small red square differs from the small yellow circle in exactly two attributes—color and shape. Use your remaining small attribute pieces to fill the grid so that adjacent pieces in rows differ in exactly one attribute and adjacent pieces in columns differ in exactly two attributes. Record your results on the grid. There are many possibilities.

2. Attribute Guessing Game (2 players): To play this game, display all 24 attribute pieces on a flat surface. One player thinks of a specific attribute piece. The other player tries to determine that piece by asking questions that can be answered yes or no. Pieces that are ruled out by questions may be physically separated from the rest. The score is the number of questions needed to identify the piece. Players alternate roles. Low score wins.

*a. If the player trying to guess the piece restricts all questions to the attributes of color, size, and shape, what is the minimum number of guesses necessary to ensure the identification of a randomly chosen piece? List the questions you would ask in order to identify a piece in the minimum number of guesses.
b. If the player guessing is allowed to use additional attributes of the pieces (like number of sides, size of angles, opposite sides parallel), explain how the minimum number of guesses can be reduced.

3. **Attribute-Grid Game** (2 or more players or teams, or solitaire): To begin this game, place any attribute piece on the center square of the grid (Material Card 4). The *large red hexagon* was used to start the game on the grid shown below. The players take turns placing an attribute piece on the grid according to the following conditions: the piece that is played must be placed on a square that is adjacent, by row, column, or diagonal, to a piece that has already been played; the adjacent attribute pieces in the rows must differ in one attribute (color, size, or shape); the adjacent pieces in the columns must differ in two attributes; and the adjacent pieces in the diagonals must differ in three attributes. A player’s score on each turn is the total number of attributes in which the piece played differs from *all* adjacent attribute pieces.

**Examples:** The first piece played on the grid shown below was a *small blue square*, which differs in three attributes from the large red hexagon. The first player scored 3 points. The second player played the *small red hexagon*. This piece differs from the small blue square in two attributes and from the large red hexagon in one attribute. This player also scored 3 points. The game ends when no more pieces can be played on the grid.

**Solitaire Version:** Place an attribute piece on the center square. Try to place as many pieces as possible on the grid according to the rules. Record your results. No one has been able to place all 24 pieces on the grid.

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Attribute-Grid Game
Rows, 1 difference (1 point)
Columns, 2 differences (2 points)
Diagonals, 3 differences (3 points)
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4. Label your two-circle Venn diagram (Material Card 5) with the attribute labels RED and TRIANGULAR (from Material Card 3), as shown in the diagram below. In this diagram, the large red circle is placed in the circle labeled RED, and the small blue triangle is placed in the circle labeled TRIANGULAR. The small red triangle possesses both the red attribute and the triangular attribute, so it is placed in the intersection of the two circles. The large yellow hexagon, having neither attribute, is placed outside both circles. Place the same four pieces on your diagram. Distribute your remaining attribute pieces on the correct regions of the Venn diagram on Material Card 5.

![Venn Diagram](image)

**a.** The attribute pieces that lie in the intersection of the two circles are described as being RED and TRIANGULAR. This set is called the **intersection** of the set of red pieces and the set of triangular pieces. Referring to your diagram, complete the following list of pieces in the intersection of the two sets.

Red and Triangular: SRT, __________

**b.** The attribute pieces that lie in the RED circle or the TRIANGULAR circle or both circles are described as being RED or TRIANGULAR. This set is called the **union** of the set of red pieces with the set of triangular pieces. Referring to your diagram, complete the following list of pieces in the union of the two sets.

Red or Triangular: LRC, SRT, SBT, __________

5. Label your two-circle Venn diagram on Material Card 5 as shown here. Distribute your attribute pieces in the appropriate regions.

![Venn Diagram](image)

**a.** Record the set of attribute pieces in the **intersection** of the following two sets.

Circular and Not Blue: __________
b. Record the set of attribute pieces in the union of the following two sets.

Circular or Not Blue:

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c. Using only word combinations found on the attribute label cards of Material Card 3, describe the pieces that are outside both circles in the Venn diagram on the previous page.

6. Mathematicians invent symbols for the purpose of abbreviating and representing ideas. In this activity the symbols NR, S, T, NY, and NB will represent the NOT RED, SMALL, TRIANGULAR, NOT YELLOW, and NOT BLUE sets of attribute pieces, respectively. The symbol $\cup$ will represent the union of sets, and the symbol $\cap$ will represent the intersection of sets. Use your labels and two-circle Venn diagram to determine and record the following sets of attribute pieces.

a. $\text{NR} \cap S$

*b. Using only words found on the attribute label cards, describe the pieces that are outside both circles in part a.

*c. $T \cup NY$

d. $\text{NB} \cup NY$

7. Label your three-circle Venn diagram (Material Card 6) with the attribute labels shown in the diagram. There are eight different regions within the rectangle. Seven of the regions are enclosed by one or more circles, and one region is outside all three circles. Distribute the attribute pieces on the correct regions of your diagram on Material Card 6.

a. Record your solution by writing the abbreviation for each piece on the correct region of the diagram above.
b. The region $H \cap L$ can be described as all of the large hexagons. For each of the following, describe the region in a complete sentence and shade the corresponding Venn diagram to show the indicated region.

1. $H \cap NY$

2. $(H \cap NY) \cap L$

3. $(NY \cap L) \cup H$

4. $(H \cap L) \cup H$
JUST FOR FUN

ATTRIBUTE IDENTITY GAME (2 teams or 2 players)

Draw three large circles and label them X, Y, and Z. Each circle represents a set of attribute pieces. The labeling team (or player) decides what the sets are to be, but does not tell the guessing team (or player). The descriptions of the sets should be written down. The guessing team selects an attribute piece, and the labeling team places that piece on the correct region of the diagram, either inside one or more of the circles or outside all three circles. The guessing team continues selecting pieces for placement until its members believe they have correctly identified sets X, Y, and Z. The labeling team answers yes or no to each guess and records the number of guesses. If part or all of a proposed identification is incorrect, the guessing team continues selecting pieces for placement until its members are able to guess all three sets.

After the guessing team correctly identifies the three sets, the teams reverse roles and play again. The winner is the team requiring the fewest total number of pieces (placed on the diagram) and guesses to make an identification of all three sets.

Example: The guessing team has selected 10 attribute pieces, and the labeling team has placed them in the three sets as shown here. As a member of the guessing team, what do you think is the identity of each of the three sets?