EEG Dynamic Source Localization using Marginalized Particle Filtering

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***Abstract*—Localization of the brain neural generators that create Electroencephalographs (EEGs) has been an important problem in clinical, research and technological applications related to the brain. The active regions in the brain are modeled as equivalent current dipoles, and the positions and moments of these dipoles or brain sources are estimated. So far, the brain dipoles are assumed to be fixed or time-invariant. However, recent neurological studies are showing that brain sources are not static but vary (in terms of location and moment) depending on various internal and external stimuli. This paper presents a shift in the current paradigm of brain source localization by considering dynamic sources in the brain. We formulate the brain source estimation problem from EEG measurements as a (nonlinear) state-space model. We use the Particle Filter (PF), essentially a sequential Monte Carlo method, to track the trajectory of the moving dipoles in the brain. We further address the "curse of dimensionality," issue of the PF by taking advantage of the structure of the EEG state-space model, and marginalizing out the linearly evolving states. A Kalman Filter is used to optimally estimate the linear elements, whereas the PF is used to track only the non-linear components. This technique reduces the dimension of the problem; thus exponentially reducing the computational cost. Our simulation results show that, where the PF fails, the Marginalized PF is able to successfully track two dipoles in the brain with only 500 particles.**

***Keywords*—Bayesian estimation, EEG inverse problem, Spatial-temporal brain source localization, Particle filtering, Kalman Filtering.**

I. Introduction

We formulate the brain source localization problem as a (nonlinear) state-space model, where the positions and moments of the neural generators constitute the unknown or hidden state and the EEG measurements are the observations of the system. In a Bayesian context, inference of the hidden state given a realization of the observations relies upon the posterior density function (pdf) [1]. For systems with linear dynamics and Gaussian noise, the posterior distribution is Gaussian whose mean and covariance can be computed using the Kalman filter. For systems with non-linear dynamics, a Monte Carlo method, called the *Particle Filter* (PF) has emerged, which uses the concept of Sequential Importance Sampling (SIS) to estimate the posterior pdf using a finite number of weighted samples. In particular, the PF does not make any assumptions about the pdfs or the linearity of the system model. The power of the PF, however, comes at a computational cost. In particular, the number of particles needed for the estimation increases super-exponentially with the dimension of the state [2]. This problem is commonly known as the "curse of dimensionality", and makes it unreasonable to use the Particle filter for tracking problems in high dimensional spaces. In the context of EEG source localization, the dimension of the state space is six times the number of dipoles, causing the tracking of even two dipoles (12-dimensional problem) to be inaccurate unless a very large number of particles are used. To deal with the high-dimensionality issue, we propose to *marginalize* out the states in the system that are linear with respect to the measurements [3]. This allows the linear states in the state-space model to be estimated optimally using the Kalman Filter, whereas the non-linear states are estimated using the PF. By decreasing the dimensionality of the state, less particles can be used, allowing a decrease in computation time. Simulation results show that even a two dipole model cannot be localized using the traditional PF, but can be tracked accurately using the marginalized PF.

II. EEG Source Localization Model

Given M equivalent active dipoles in the brain, the measured multichannel EEG signal $\boldsymbol{z}_k$ from n_z sensors at time k can be modeled as follows:

$$\boldsymbol{z}_k = \sum_{m=1}^{M} \boldsymbol{L}_m(\boldsymbol{d}_k(m))\boldsymbol{s}_k(m) + \boldsymbol{e}_k, \tag{1}$$

where M is the total number of dipoles, $\boldsymbol{d}_k(m)$ is a 3×1 spatial position vector in the brain of dipole m at discrete time k. Each dipole m is defined as $\boldsymbol{d}_k(m) = [x_k(m), y_k(m), z_k(m)]^t$. $\boldsymbol{L}_m(\boldsymbol{d}_k(m))$ is the $n_z \times 3$-dimensional lead-field matrix for the m^{th} dipole. $\boldsymbol{s}_k(m)$ is a 3×1-dimensional moment of the m^{th} dipole at time k. $\boldsymbol{e}_k$ is a zero-mean white Gaussian noise with covariance $\boldsymbol{R}_k$. Most notably, the components of the leadfield matrix $\boldsymbol{L}_m$ are non-linear functions of the dipole locations, electrodes' positions and head position [4]. The EEG measurement equation described in (1) can be written concisely as

$$\boldsymbol{z}_k = \boldsymbol{L}_k(\boldsymbol{d}_k)\boldsymbol{s}_k + \boldsymbol{e}_k. \tag{2}$$

The hidden state (to be estimated) is given by the dipole positions and moments: $\boldsymbol{x}_k = [\boldsymbol{d}_k^t, \boldsymbol{s}_k^t]^t$.

It is important to note that the measurements $\boldsymbol{z}_k$ are linear with respect to the dipole moments $\boldsymbol{s}_k$ and non-linear with respect to the dipole spatial positions $\boldsymbol{d}_k$. This model allows us to consider marginalization of the linear states to be estimated by the Kalman filter, thus reducing the dimensionality of the state estimated by the PF. We further assume a random walk model for the state transition dynamics. A random walk model does not assume any *a priori* knowledge about the source locations and moments. The EEG source localization state-space model is then given by

$$\begin{cases} \boldsymbol{x}_k = \boldsymbol{x}_{k-1} + \boldsymbol{v}_k, \\ \boldsymbol{z}_k = \boldsymbol{L}(\boldsymbol{d}_k)\boldsymbol{s}_k + \boldsymbol{e}_k, \end{cases} \tag{3}$$

where $\boldsymbol{v}_k$ is the state noise at time k, assumed to be zero-mean, white Gaussian process. The goal is to use the model in (3) to estimate, at every time instant, the dipole locations $\boldsymbol{d}_k$ and moments $\boldsymbol{s}_k$ given the EEG measurements $\boldsymbol{z}_k$.

III. The Particle Filter

Consider the following discrete-time state-space model defined by possibly nonlinear state and measurement equations

$$\begin{aligned} \boldsymbol{x}_k &= \boldsymbol{f}_k(\boldsymbol{x}_{k-1}) + \boldsymbol{v}_k, \\ \boldsymbol{y}_k &= \boldsymbol{h}_k(\boldsymbol{x}_k) + \boldsymbol{e}_k, \end{aligned} \tag{4}$$

where $\boldsymbol{x}_k \in \mathbb{R}^{n_x}$ and $\boldsymbol{y}_k \in \mathbb{R}^{n_y}$ represent, respectively, the hidden state and the measurement vectors. The functions $\boldsymbol{f}_k$ and $\boldsymbol{h}_k$ are known, possibly non-linear, mappings; and $\boldsymbol{v}_k$ and $\boldsymbol{e}_k$ are realizations of the zero-mean process and measurement noise with known probability density functions (PDFs) g_k and r_k, respectively. We wish to estimate the state of the system $\boldsymbol{x}_k$ at every time step k, given the history of measurements $\boldsymbol{Y}^k = \{\boldsymbol{y}_1, \boldsymbol{y}_2, ..., \boldsymbol{y}_k\}$.

In the Bayesian framework, the optimal state estimate is given by the mean of the posterior density $p(\boldsymbol{x}_k|\boldsymbol{Y}^k)$. Using Bayes rule, the posterior distribution, at time k, can be computed sequentially from the following two-step prediction-update formula:

$$p(\boldsymbol{x}_k|\boldsymbol{Y}^{k-1}) = \int g_k(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})p(\boldsymbol{x}_{k-1}|\boldsymbol{Y}^{k-1})d\boldsymbol{x}_{k-1} \tag{5}$$

$$p(\boldsymbol{x}_k|\boldsymbol{Y}^k) = \frac{r_k(\boldsymbol{y}_k|\boldsymbol{x}_k)p(\boldsymbol{x}_k|\boldsymbol{Y}^{k-1})}{\int r_k(\boldsymbol{y}_k|\boldsymbol{x}_k)p(\boldsymbol{x}_k|\boldsymbol{Y}^{k-1})d\boldsymbol{x}_k}. \tag{6}$$

Unfortunately, except for the linear case, these equations are only a conceptual solution, due to the intractability of the integrals defined. The PF is a Monte Carlo method that represents the posterior pdf, at time k, using a set of N *particles* $\{\boldsymbol{x}_k^{(i)}\}_{i=1}^N$ and their associated weights $\{w_k^{(i)}\}_{i=1}^N$:

$$p(\boldsymbol{x}_k|\boldsymbol{Y}^k) \approx \sum_{i=1}^N w_k^{(i)}\delta(\boldsymbol{x}_k - \boldsymbol{x}_k^{(i)}), \tag{7}$$

where $\delta(.)$ is the Dirac delta function and N is the number of particles. The conditional mean estimate at time k is then given by

$$\hat{\boldsymbol{x}}_k = E[\boldsymbol{x}_k|\boldsymbol{Y}^k] \approx \sum_{i=1}^N w_k^{(i)}\boldsymbol{x}_k^{(i)}. \tag{8}$$

A known pdf, called the *importance distribution* or *proposal distribution*, $q(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}, \boldsymbol{y}_k)$, is used to sample the particles: $\boldsymbol{x}_k^{(i)} \sim q(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}^{(i)}, \boldsymbol{y}_k)$. To make up for the difference between the importance distribution and the posterior density, the weight of each particle $\boldsymbol{x}_k^{(i)}$ is computed as

$$w_k^{(i)} = w_{k-1}^{(i)} \frac{r_k(\boldsymbol{y}_k|\boldsymbol{x}_k^{(i)})g_k(\boldsymbol{x}_k^{(i)}|\boldsymbol{x}_{k-1}^{(i)})}{q(\boldsymbol{x}_k^{(i)}|\boldsymbol{x}_{k-1}^{(i)}, \boldsymbol{y}_k)}. \tag{9}$$

The weights are then normalized. A common choice for the importance distribution is the prior, i.e., $q(\boldsymbol{x}_k|\boldsymbol{x}_{k-1}, \boldsymbol{y}_k) = g_k(\boldsymbol{x}_k|\boldsymbol{x}_{k-1})$. The price to be paid for the flexibility and numerical power of the PF is computational. This computational cost is especially prohibitive in higher dimensional state spaces, where the number of particles needed increases super-exponentially with the dimension of the state [2].

IV. The Marginalized Particle Filter

The main idea of the marginalized PF (MPF) is to partition the state vector as $\boldsymbol{x}_k = [(\boldsymbol{x}_k^l)^t, (\boldsymbol{x}_k^n)^t]^t$, where $\boldsymbol{x}_k^l$ denotes the state variable partition with conditionally linear dynamics and $\boldsymbol{x}_k^n$ denotes the state variable partition with non-linear dynamics. Let us consider the following marginalization model:

$$\begin{cases} \boldsymbol{x}_k^n &= \boldsymbol{f}_k(\boldsymbol{x}_{k-1}^n) + \boldsymbol{v}_k^n, \\ \boldsymbol{x}_k^l &= \boldsymbol{A}_k(\boldsymbol{x}_{k-1}^n)\boldsymbol{x}_{k-1}^l + \boldsymbol{v}_k^l, \\ \boldsymbol{y}_k &= \boldsymbol{h}_k(\boldsymbol{x}_k^n) + \boldsymbol{C}_k(\boldsymbol{x}_k^n)\boldsymbol{x}_k^l + \boldsymbol{e}_k, \end{cases} \tag{10}$$

where $\boldsymbol{f}_k$ and $\boldsymbol{h}_k$ are non-linear functions, whereas $\boldsymbol{A}_k(\boldsymbol{x}_{k-1}^n)$ is conditionally linear on $\boldsymbol{x}_{k-1}^l$ and $\boldsymbol{C}_k(\boldsymbol{x}_k^n)$ is conditionally linear on $\boldsymbol{x}_k^l$. The system and measurement noise are assumed to be white Gaussian processes distributed according to $\boldsymbol{e}_k \sim N(0, \boldsymbol{R}_k)$, $\boldsymbol{v}_k = \begin{bmatrix} \boldsymbol{v}_k^l \\ \boldsymbol{v}_k^n \end{bmatrix} \sim N(0, \boldsymbol{Q}_k), \boldsymbol{Q}_k = \begin{bmatrix} \boldsymbol{Q}_k^l & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{Q}_k^n \end{bmatrix}$.

The posterior pdf of the state $\boldsymbol{x}_k$ can then be found as $p(\boldsymbol{x}_k^l, \boldsymbol{X}^{n,k}|\boldsymbol{Y}^k)$, where $\boldsymbol{X}^{n,k} = \{\boldsymbol{x}_0^n, \boldsymbol{x}_1^n, ..., \boldsymbol{x}_k^n\}$ and the marginal of which is $p(\boldsymbol{x}_k^l, \boldsymbol{x}_k^n|\boldsymbol{Y}^k)$. By marginalizing out the conditionally linear states $\boldsymbol{x}_k^l$ using Bayes' theorem, we have

$$p(\boldsymbol{x}_k^l, \boldsymbol{X}^{n,k}|\boldsymbol{Y}^k) = p(\boldsymbol{x}_k^l|\boldsymbol{X}^{n,k}, \boldsymbol{Y}^k)p(\boldsymbol{X}^{n,k}|\boldsymbol{Y}^k). \tag{11}$$

The distribution $p(\boldsymbol{x}_k^l|\boldsymbol{X}^{n,k}, \boldsymbol{Y}^k)$ is analytically tractable because it is conditioned on the non-linear states $\boldsymbol{X}^{n,k}$ and, therefore, can be found optimally using the Kalman Filter [3]. The distribution $p(\boldsymbol{X}^{n,k}|\boldsymbol{Y}^k)$ depends only on the nonlinear states and can be estimated using the PF.

Since the pdf $p(\boldsymbol{x}_k^l|\boldsymbol{X}^{n,k}, \boldsymbol{Y}^k)$ is conditioned on the non-linear states $\boldsymbol{X}^{n,k}$, we have $\boldsymbol{A}_k(\boldsymbol{x}_{k-1}^n)$ and $\boldsymbol{C}_k(\boldsymbol{x}_k^n)$ as fixed constant matrices, thus allowing for a statistically optimal estimate of the linear state. The Kalman Filter algorithm to compute the optimal estimate of $\boldsymbol{x}_k^l$ given the non-linear states is outlined below.

Given the initial conditions $\boldsymbol{x}^l_{0|0}$ and $\boldsymbol{P}_{0|0}$, compute the prediction equations

$$\begin{array}{rcl} \boldsymbol{x}^l_{k|k-1} & = & \boldsymbol{A}_k(\boldsymbol{x}^n_{k-1})\boldsymbol{x}^l_{k-1} \\ \boldsymbol{P}_{k|k-1} & = & \boldsymbol{A}_k(\boldsymbol{x}^n_{k-1})\boldsymbol{P}_{k|k-1}\boldsymbol{A}_k(\boldsymbol{x}^n_{k-1})^T + \boldsymbol{Q}^l_k \end{array} \quad (12)$$

and the update equations

$$\begin{array}{rcl} \boldsymbol{S}_k & = & \boldsymbol{C}_k(\boldsymbol{x}^n_k)\boldsymbol{P}_{k|k-1}\boldsymbol{C}_k(\boldsymbol{x}^n_k)^T + \boldsymbol{R}_k \\ \boldsymbol{K}_k & = & \boldsymbol{P}_{k|k-1}\boldsymbol{C}_k(\boldsymbol{x}^n_k)^T\boldsymbol{S}_k^{-1} \\ \boldsymbol{x}^l_{k|k} & = & \boldsymbol{x}^l_{k|k-1} + \boldsymbol{K}_k\left(\boldsymbol{y}_k - \boldsymbol{h}_k(\boldsymbol{x}^n_k) - \boldsymbol{C}_k(\boldsymbol{x}^n_k)\boldsymbol{x}^l_{k|k-1}\right) \\ \boldsymbol{P}_{k|k} & = & \boldsymbol{P}_{k|k-1} - \boldsymbol{K}_k\boldsymbol{C}_k(\boldsymbol{x}^n_k)\boldsymbol{P}_{k|k-1} \end{array} \quad (13)$$

The optimal estimate of $\boldsymbol{x}^l_k$ is then given by $\boldsymbol{x}^l_{k|k}$.

The second conditional probability in Eq. (11), $p(\boldsymbol{X}^{n,k}|\boldsymbol{Y}^k)$, can be expressed as

$$p(\boldsymbol{X}^{n,k}|\boldsymbol{Y}^k) \propto p(\boldsymbol{y}_k|\boldsymbol{X}^{n,k},\boldsymbol{Y}^{k-1})p(\boldsymbol{x}^n_k|\boldsymbol{X}^{n,k-1},\boldsymbol{Y}^{k-1}) \\ p(\boldsymbol{X}^{n,k-1}|\boldsymbol{Y}^{k-1}) \quad (14)$$

We use the distribution $p(\boldsymbol{x}^n_k|\boldsymbol{X}^{n,k-1},\boldsymbol{Y}^{k-1})$ as the importance density. We have $p(\boldsymbol{x}^n_k|\boldsymbol{X}^{n,k-1},\boldsymbol{Y}^{k-1}) = \mathcal{N}(\boldsymbol{f}_k(\boldsymbol{x}^n_{k-1}),\boldsymbol{Q}^n_k)$, where $\mathcal{N}(\boldsymbol{x},\boldsymbol{C})$ denotes the normal distribution with mean $\boldsymbol{x}$ and covariance matrix $\boldsymbol{C}$. The weights of the particles are calculated as

$$\tilde{w}^{(i)}_k = w^{(i)}_{k-1}p(\boldsymbol{y}_k|\boldsymbol{X}^{n,k},\boldsymbol{Y}^{k-1}), \quad (15)$$

where

$$p(\boldsymbol{y}_k|\boldsymbol{X}^{n,k},\boldsymbol{Y}^{k-1}) = \mathcal{N}\left(\boldsymbol{h}_k(\boldsymbol{x}^n_k) + \boldsymbol{C}_k(\boldsymbol{x}^n_k)\boldsymbol{x}^l_{k|k-1},\boldsymbol{S}_k\right).$$

The optimal state estimate at time k is then given by

$$\hat{\boldsymbol{x}}_k = \left[\boldsymbol{x}^l_{k|k}, \sum_{i=1}^{N} w^{(i)}_k\boldsymbol{x}^{n,(i)}_k\right]^t. \quad (16)$$

The Marginalized PF algorithm is summarized below.

Algorithm 1: Marginalized PF algorithm

for $i = 1,2,...,N$ **do**
  Initialize particles $\boldsymbol{x}^{n,(i)}_{0|-1} \sim p_{\boldsymbol{x}^n_0}(\boldsymbol{x}^n_0)$ and set $\{\boldsymbol{x}^{l,(i)}_{0|-1},\boldsymbol{P}^{(i)}_{0|-1}\} = \{\bar{\boldsymbol{x}}^l_0,\bar{\boldsymbol{P}}^l_0\}$
for $k = 1,2,...$ **do**
  for $i = 1,2,...,N$ **do**
    Evaluate the weights $\tilde{w}^{(i)}_k$ using Eq. (15).
  Normalize the weights $\boldsymbol{w}_k \leftarrow \frac{\boldsymbol{w}_k}{\sum_{i=1}^N \tilde{w}^{(i)}_k}$.
  for $i = 1,2,...,N$ **do**
    Update $\hat{\boldsymbol{x}}^l_{k|k}$ using Eq. (13).
    Update $\boldsymbol{P}_{k|k}$ using Eq. (13).
    Calculate the mean estimate $\hat{\boldsymbol{x}}_k$ using Eq. (16).
    Sample $\hat{\boldsymbol{x}}^{n,(i)}_k \sim p(\boldsymbol{x}^n_k|\boldsymbol{X}^{n,k-1},\boldsymbol{Y}^{k-1})$.
    Update $\hat{\boldsymbol{x}}^l_{k+1|k}$ using Equation (12).
    Update $\boldsymbol{P}_{k+1|k}$ using Equation (12).

A resampling step may be introduced after normalizing the weights to avoid degeneracy of the PF [1].

V. Results and Discussion

A. Simulation Results on Synthetic Data

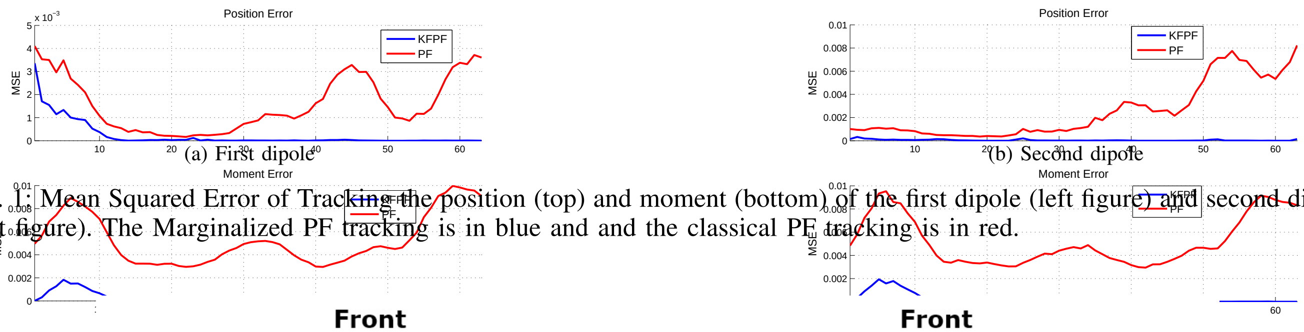
In our experiments, we considered two moving dipoles generating the observed EEG measurements. We compared the performance of the "traditional" Particle Filter algorithm, where both the linear and nonlinear components of the state are estimated using the PF, with the proposed Marginalized PF. The moments are assumed to be sinusoidal waveforms with varying amplitudes and frequencies. We performed 100 Monte Carlo simulations and computed the Mean Squared Error (MSE) of the true state $\boldsymbol{x}_k$ versus the estimated state.

The simulation was repeated for 100 Monte Carlo runs using 500 particles. The average MSE is shown in Figure 1. Not only does the PF produce a larger MSE, but in 12D it fails to track the state. The Marginalized PF is able to track the state because it is only using the PF to estimate the non-linear part and uses the Kalman filter to estimate the linear part. For the EEG localization model presented in this paper, half of the states are linear, allowing the Marginalized PF to calculate the same result using a reduced 6D state space model.

B. Application to Real EEG Data

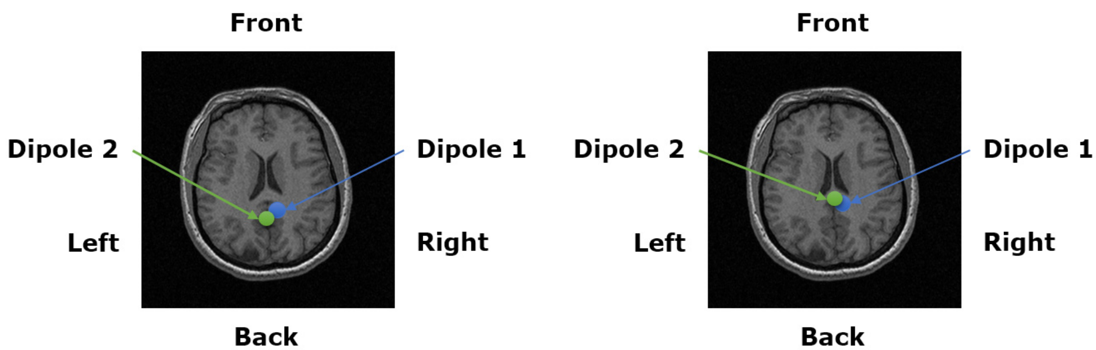
In this section, we apply the proposed Marginalized PF algorithm to real EEG data recorded from twelve female subjects (20-28 years old). The experimental setup was designed by Santos *et al*. [5] for their study on subject attention and perception. The subjects were exposed to a sequence of images of different facial expressions (neutral, fearful and disgusted) superimposed on houses. The participants task was to determine, on each trial, if the current house or face is the same as the one presented on the previous trial. Each trial lasts 1600 ms (400 samples with sampling rate 250 Hz) comprising a pre-stimulus interval of 148 ms (37 samples) and post-stimulus onset interval of 1452 ms. EEG signals were recorded from 16 channels (Fp1, Fp2, F3, F4, C3, C4, P3, P4, O1, O2; F7, F8, Fz, Cz, Pz, Oz) and two Electrooculogram (EOG) channels (horizontal and vertical EOG) located according to the 10/20 International system. Since the primary brain task in this experiment is perception of visual stimulus, the neural activity is supposed to happen in the visual cortex. Therefore, the Marginalized PF algorithm is expected to estimate the strongest dipoles that may have originated the registered VEPs in the occipital brain zone which corresponds to the visual cortex.

We considered the estimation of two sources for each patient. We used 1000 particles in the Marginalized PF for the real data. It is very interesting to observe that the dipole coordinates are located in the zone of the primary visual cortex as shown in Fig. 2. In this sense, the proposed approach seems to be coherent in tracking the brain sources over time. Another noteworthy observation is the fact that the 3D locations of the dipoles does not vary significantly over time or between trials. This is due to the fact that the EEG experimental setup was designed to study attention and perception. We postulate that in order to observe significant or abrupt changes in brain source locations, we need to design an experiment, where two or more areas of the brain (e.g., visual and motor) are invoked. We also observed that there is no significant variability between the



(a) First dipole (b) Second dipole

Fig. 1: Mean Squared Error of Tracking the position (top) and moment (bottom) of the first dipole (left figure) and second dipole (left figure). The Marginalized PF tracking is in blue and and the classical PF tracking is in red.



(a) Subject 1 (b) Subject 2

Fig. 2: Axial view of primary visual cortex zone. The arrows point the estimated source locations.

subjects in the locations of the brain dipoles. However, there was a notable variability in the moments between the subjects.

VI. Conclusion

This paper considered, for the first time, moving dipoles estimation, which would contribute to a physiologically more plausible brain technologies such as source-based BCI. Non-linear tracking algorithms, notably the Particle Filter (PF), are emerging as promising solutions in the localization of equivalent current dipole models from EEG measurements. However, the numerical nature of particle filters, which constitutes their strength for multidimensional numerical integration, becomes their major weakness in high-dimensional state-space models. In this paper, we proposed to handle the curse of dimensionality problem in the PF by taking advantage of the linear substructures in the EEG state space model. The moments were "Marginalized" out and computed optimally using the Kalman filter. The remaining non-linear positions were then estimated numerically using the classical Particle Filter. We showed that the Marginalized Particle Filter was able to successfully track two dipoles with no *a priori* knowledge of their positions or moments using only 500 particles. The classic PF failed in tracking this same system, due to the high-dimensionality of the problem and the small number of particles used.

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