EEG Dynamic Source Localization using Constrained Particle Filtering

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***Abstract*—We consider the dynamic EEG source localization problem with additional constraints on the expected value of the state. In dynamic EEG source localization, the source brains, also called dipoles, are not stationary but vary over time. Moreover, given our specific EEG experiment, we expect the dipoles to be located within a certain area of the brain (here, the visual cortex). We formulate this constrained dynamic source localization problem as a constrained non-linear state-estimation problem. Particle filters (PFs) are nowadays the state-of-the-art in optimal non-linear and non-Gaussian state estimation. However, PFs cannot handle additional constraints on the state that cannot be incorporated within the system model. In this case, the additional constraint is on the mean of the state, which means that realizations of the state, also called particles within the PF framework, may or may not satisfy the constraint. However, the state must satisfy the constraint on average. This is indeed the case when tracking brain dipoles from EEG experiments that try to target a specific cortex of the brain. Such constraints on the mean of the state are hard to deal with because they reflect global constraints on the posterior density of the state. The popular solution of constraining every particle in the PF may lead either to a stronger condition or to a different (unrelated) condition; both of which result in incorrect estimation of the state. We propose the Iterative Mean Density Truncation (IMeDeT) algorithm, which inductively samples particles that are guaranteed to satisfy the constraint on the mean. Application of IMeDeT on synthetic and real EEG data shows that incorporating a priori constraints on the state improves the tracking accuracy as well as the convergence rate of the tracker.**

I. Introduction

Electroencephalography (EEG) is an electrophysiological monitoring technique that records the electrical activity of the brain at the scalp by placing multiple electrodes at the surface of the head. It is non-invasive, cheap, portable and has high temporal resolution. These advantages led to the widespread application of EEG for research and diagnosis to understand brain function, disorders, strokes as well as build brain-computer interfaces [1], [2].

However, the advantages of the EEG and particularly its high temporal resolution are counterbalanced by i) a low signal to noise ratio and ii) the non-linearity of the EEG signal with respect to the brain source generating these surface signals.It is postulated that the EEG signals are generated by few source brains [3]. Localizing these source brains helps us understand the different areas in the brain. Brain source localization from EEG signals has been the subject of intense research [4], [5]. However, until recently, the source brains were assumed to be stationary. That is, the locations of these sources in the 3D brain does not change with time. Our recent work was one of the pioneers of the idea of tracking (rather than stationary estimation) of source brains over time [5]. The problem is formulated as a state-space model and a beamformer-particle filter is used to simultaneously track the positions of the sources and estimate their moments [5]. In our framework, the moments and positions of the source brains or neural generators are the components of the unknown or hidden state while the EEG measurements are the system observations model.

In a Bayesian context, optimal estimation of the hidden state is based on the posterior density function (pdf) of the state given the observations. In a linear and Gaussian system, optimal estimation is given by the Kalman filter [6]. However, the EEG state-space model is non-linear and may be non-Gaussian. Particle filters (PFs) solve the optimal estimation problem in non-linear and non-Gaussian state-spaces [7]. PFs approximate the posterior density of the state by a set of weighted samples, called *particles*. The particles are sampled from any accessible *proposal distribution* whose support contains the support of the posterior density, and then appropriately weighted to make up the difference between the proposal and posterior densities.The PF solution converges, in the mean-square error, to the optimal state [7]. However, a major weakness of the PF resides in the difficulty of handling additional constraints within the state space model.

We seek to improve the tracking performance of the EEG source localization problem by introducing additional a priori knowledge as constraints on the hidden state. In fact, in our particular EEG experiments, we expect the source brains to be in the visual cortex most of the time [4]. We subsequently add a constraint to the state-space model that imposes 3D coordinates within the visual cortex. In constrained particle filtering, the current research [8], [9] simply imposes the constraints on all particles of the PF. This approach actually constrains the support of the state posterior density. We term this approach of constraining all particles Pointwise Density Truncation abbreviated as the PoDeT method.

Ebinger *et al.* [10] presented a new approach named, Mean Density Truncation (MeDeT), that imposes the constraints on the conditional mean estimate of the state without further restricting the posterior distribution of the state. Specifically, MeDeT draws N unconstrained particles from the proposal distribution as in the unconstrained PF. If the weighted mean of these N particles, which corresponds to the conditional state estimate, satisfies the constraints, it is kept as the optimal constrained state estimate. Otherwise, an $(N+1)$st particle is sampled from a high likelihood region to enforce the constraints on the weighted mean. If one additional particle is not enough to ensure the constraints on the conditional mean, more particles are draw one at a time until the constraints are satisfied [10]. It is important to realize that PoDet and MeDet have two different views on the constrained problem. In PoDet, the constraint is assumed to be a *hard constraint*. Thus, all possible realizations of the state (all particles) must satisfy this constraint. In MeDet, the constraint is assumed to be a *soft constraint*, where the state satisfies the constraint on average or with high probability.

In this paper, we propose a new contribution to the constrained particle filtering problem and apply it to the dynamic brain source localization problem based on EEG real data. In particular, we formulate the EEG source localization problem as a softly constrained state-space model because we expect the sources to be in a certain area of the brain with high probability. Our proposed constrained PF method is different from [10] in that, instead of drawing the additional particles one at a time until the constraints are satisfied, we propose a systematic and inductive procedure that guarantees that the constraints are satisfied with a draw of N particles.

The paper is organized as follows: In Section II, we review the unconstrained PF framework. In Section III, we present the PoDeT, MeDeT and IMeDeT approaches as the state-of-the-art in constrained particle filtering methods. Section IV formulates the brain source localization model as a constrained state-space model, and derives IMeDeT for the EEG dipole source localization problem. Simulation and comparison results of synthetic and real EEG data are presented in Section V. Finally, Section VI summarizes the main findings and concludes the paper.

II. Unconstrained Particle Filtering

Consider a discrete-time state-space model defined by a state transition model and a measurement model (or observation model). Both models may be non-Gaussian or nonlinear.

$$x_{n+1} = f_n(x_n) + u_n, \tag{1}$$

$$z_n = h_n(x_n) + v_n, \tag{2}$$

where x_n, z_n represent, respectively, the state and the measurement vectors at time n. f_n, h_n are, respectively, the state and measurement (non-linear) functions. u_n, v_n are, respectively, the transition and the measurement noise with known probability density functions, respectively, g and r. The main objective is to estimate the state of the system x_n at every time step n, using the history of measurements $Z^n = [z_1, ..., z_n]$ up to time n.

In a Bayesian context, the optimal state estimate x_n is given by the mean of the posterior distribution $p(x_n|Z^n)$. Using Bayes rule, this distribution can be computed in two steps: prediction and update as given by the following equations:

$$p(x_n|Z^{n-1}) = \int g(x_n|x_{n-1}) p(x_{n-1}|Z^{n-1})\,\mathrm{d}x_{n-1}, \tag{3}$$

$$p(x_n|Z^n) = \frac{r(z_n|x_n)p(x_n|Z^{n-1})}{\int r(z_n|x_n)p(x_n|Z^{n-1})\,\mathrm{d}x_n}, \tag{4}$$

Unfortunately, for the nonlinear case, the models given in Eqs. (1)-(2) are only a conceptual solution, due to the fact that the integrals are generally intractable.

The particle filter is a sequential Monte Carlo method designed to estimate the posterior density function of the state, at time n, using a set of random samples, named *particles*, and their associated weights : $\{x_n^{(i)}, w_n^{(i)}\}_{i=1}^N$. The posterior pdf is approximated as:

$$\hat{p}(x_n|Z^n) = \sum_{i=1}^N w_n^{(i)} \delta(x_n - x_n^{(i)}), \tag{5}$$

where δ is the dirac delta function.

In the ideal case, the particles are sampled from the true posterior,which is not always available. Thus, an *importance distribution* or a *proposal distribution* $q(x_n|x_{n-1}, z_n)$ is evoked. Theoretically, the only condition on the importance distribution is that its support includes the support of the posterior distribution. However, in practice, the number of particles is finite and thus, the importance distribution should be chosen to approximate the posterior distribution [7]. The importance weights are given by:

$$\tilde{w}_n^{(i)} = w_{n-1}^{(i)} \frac{r(z_n|x_n^{(i)})g(x_n^{(i)}|x_{n-1}^{(i)})}{q(x_n^{(i)}|x_{n-1}^{(i)}, z_n)}, \tag{6}$$

The weights are then normalized such that: $\sum\limits_{i=1}^N w_n^{(i)} = 1$.

The conditional mean estimate of the state at time n is then given by the weighted mean of the particles:

$$\hat{x}_n = E[x_n|Z^n] \approx \sum_{i=1}^N w_n^{(i)} x_n^{(i)}. \tag{7}$$

III. Constrained Particle Filtering

We consider the state-space model given in (1)-(2) with the following additional constraint on the state:

$$a_n \leq \phi_n(\hat{x}_n) \leq b_n. \tag{8}$$

where ϕ_n is the constraint function at time n and $\phi_n(\hat{x}_n) = \phi_n(E[x_n|Z^n]) \approx \phi_n\left(\sum\limits_{j=1}^N w_n^{(i)} x_n^{(i)}\right)$.

Notice that the constraint is imposed on the conditional mean estimate. In particular, not all realizations of the state must satisfy this constraint but on average, we expect the state to satisfy this constraint. We refer to such a constraint as a *soft* constraint in contrast to a *hard* constraint, where all realizations of the state (with low and high probabilities) must satisfy the constraints. This soft constraint imposed on the mean is harder to solve in an optimal way because it imposes a global condition on the density. Thus, the constrained posterior density, if it exists, is not merely the projection of the unconstrained density onto the constraint set.

A. Point wise Density Truncation

One popular approach to dealing with constrained nonlinear state estimation is to enforce the state constraints on all particles [8], [9]. Enforcing the constraint on all particles results, in this case, in a stronger constraint and possibly a totally different or even irrelevant condition. In fact, constraining every particle is equivalent to constraining the support of the posterior distribution to the mentioned interval. This is a much stronger condition than constraining the mean of the distribution or any point estimate, to be inside the interval [10]. We refer to this approach as *Point wise Density Truncation* or *Particle Density Truncation* (PoDeT). We will show in our simulations that PoDeT leads to erroneous estimates of the density and state when soft constraints are imposed.

B. Mean Density Truncation

The mean density truncation (MeDeT) methods is constructed to satisfy the constraint on the conditional mean estimate rather than the posterior density itself [10]. The main idea of MeDeT is to first sample N unconstrained particles from the proposal distribution. If this N-order estimate the state satisfies the constraints, we keep it. Otherwise, we sample an $(N+1)^{th}$ particle from the high probability region (or high likelihood), to enforce the constraints on the mean. The sampling of the $(N+1)^{th}$ particle can be viewed as a perturbation of the unconstrained posterior distribution so that its mean shifts in the desired boundaries. If a one-particle perturbation does not suffice to shift the mean, we draw another particle in the high-likelihood region and recheck for the condition, and so on. We keep drawing particles until the desired condition is satisfied. Assume m is the number of additional particles required to shift the mean. Notice that when $m = N$, the N-particle perturbation is still very different from the PoDeT method: In the PoDeT method, the original constraint is enforced on all particles, whereas the N-particle MeDeT imposes the desirable condition only on the mean estimate.

C. Inductive Mean Density Truncation

In MeDeT, the minimum number of particles required to shift the conditional mean estimate to the desired boundaries depends on the state-space model at hand and especially on the choice of the proposal distribution from which the particles are draw. If the proposal distribution is chosen poorly, i.e., far from the posterior density of the state, it may take a large number of additional particles ($m \ll 1$) to shift the mean of the distribution to the desired boundaries. This iterative process of drawing 1 additional particle at a time may be time-consuming and not efficient. This is especially true for high-dimensional systems where the number of particles N must be large; thus leading to a large m as well. To address this computational inefficiency, we propose an inductive procedure where the particles are chosen inductively from $n = 1, \cdots, N$ such that every subset of n particles satisfies the constraint on the weighted mean state. Mathematically, we want the constraint to be satisfied for any number of particles $j = 1, \cdots, N$,

$$a_n \leq \phi_n(\sum_{i=1}^{j} w_n^{(i)} x_n^{(i)}) \leq b_n \; for\, all\, j = 1, ..., N. \tag{9}$$

For simplicity and without loss of generality, we assume that the proposal distribution is chosen to be the prior distribution and hence the weights are given by the likelihood. Separating the summation of the $(j-1)$ unconstrained particles from the j^{th} particle, the constraint expression (9) becomes:

$$a_n \leq \phi_n\Big(\frac{\sum_{i=1}^{j-1} p(z_n|x_n^{(i)}) x_n^{(i)} + p(z_n|x_n^{(j)}) x_n^{(j)}}{\sum_{i=1}^{j} p(z_n|x_n^{(i)})}\Big) \leq b_n. \tag{10}$$

If we further assume that Φ_n is given by the identity function, which corresponds to an interval type constraint, the above inequality can be expressed in terms of the j^{th} particle only as follows:

$$\Big\{\; q_1(x_n^{(j)}) \leq C_1(\{x_n^{(i)}\}_{i=1}^{j-1}), q_2(x_n^{(j)}) \geq C_2(\{x_n^{(i)}\}_{i=1}^{j-1}), \tag{11}$$

Where: C_1, C_2 are two constants that depend on the already sampled $(j-1)$ particles and q_1, q_2 are given by

$$\begin{cases} q_1(x_n^{(j)}) = p(z_n|x_n^{(j)})(a_n - x_n^{(j)}), \\ q_2(x_n^{(j)}) = p(z_n|x_n^{(j)})(x_n^{(j)} - b_n), \end{cases} \tag{12}$$

Finding a j^{th} particle that satisfies Eq. (11) could be done analytically or numerically. The set of solutions to Eq. (11) enforces the constraint on the conditional mean estimate estimate for any subset of particles j . The following algorithm details the steps of the inductive MeDeT approach.

IV. BRAIN SOURCE TRACKING

A. The EEG state-space model

Brain electrical activity at the macroscopic level is generated by the neuronal source brains as equivalent electric dipoles. Normally, there are several active sources (or dipole) at the same time in different brain regions with different intensities that are projected above the scalp with positive or negative polarity depending on the orientation of the dipole relative to the position of the electrodes. We denote by M the number of active dipoles in the brain that are the source of the electrical activity measured by the multichannel EEG signal Z_n from n_z sensor at time n Let $s_n(m)$ be the moment signal generated at dipole m at time n. Let $d_n(m)$ denote the location of the m^{th} dipole at time n. The EEG signal is related to the

Algorithm 1 Inductive Mean Density Truncation (IMeDeT)

Denote by C_n the constraint region::$C_n = \{x_n : a_n \le \phi(\hat{x}_n) \le b_n\}$.

Unconstrained sampling

for $n = 1, 2, ..., T($ where T : time lengh) **do**

  for $j = 1, 2, ..., N($ where N is the number of particles) **do**

    Generate samples from an accessible proposal distribution $x_n^{(j)} \sim q_n(x_n)$.

    Calculate the weights $w_n^{(j)}$ of $x_n^{(j)}$ using Eq.(6); then , normalize the weights.

    IMeDeT

    for i=1,2,...,j **do**

      if $\sum_{i=1}^{j} w_n^{(i)} x_n^{(i)} \in C_n$ **then**

        Go to the next step.

      else

        Find a particle $x_n^{(j)}$ such that $\sum_{i=1}^{N} w_n^{(i)} x_n^{(i)} \in C_n$.

      end if

    end for

  end for

  $\hat{x}_n = \sum_{i=1}^{N} w_n^{(i)} x_n^{(i)}$.

end for

dipoles locations and moments through the following non-linear equation [2,10] :

$$z_n = \sum_{m=1}^{M} L_m(d_n(m)) \ s_n(m) + u_n, \tag{13}$$

where $L_m(d_n(m))$ is the $n_z \times 3$ lead field matrix at time n for dipole m, which depends on the dipoles location. u_n is a zero mean, white Gaussian noise with covariance C_u. The EEG observation equation in (13) can be written in vector form as:

$$z_n = L(d_n) \ s_n + u_n, \tag{14}$$

Where $d_n = [d_n(1), \cdots, d_n(M)]^t$ has the 3D location coordinates of all M dipoles, $L(d_n) = [L_1(d_n(1)), \cdots, L_M(d_n(M))]$ is the $n_z \times 3M$ lead field matrix of the M dipoles at time n, and $s_n = [s_n(1), \cdots, s_n(M)]^t$ is the vector of dipole moments for the M dipoles. The unknown state vector in the EEG problem is defined by the dipoles positions and source moments, i.e., $x_n = [d_n^t, s_n^t]^t$. The EEG state-space model is given by:

$$\begin{cases} x_n = x_{n-1} + u_n, \\ z_n = L_n(d_n) \ s_n + v_n, \end{cases} \tag{15}$$

Observe that we used a random walk for the state transition model. This random walk model reflects the fact that we have no specific a priori knowledge (flat prior) on the state dynamics. We would like to use the model in (15) to estimate, at every time step, the dipole locations d_n and moments s_n given the EEG measurement z_n. The likelihood of each measurement can be derived from the Gaussianity of the noise and Eq. (16) [5] as $\mathcal{L}(z_n|(x_n, S_n))$

$$\propto exp[-\frac{(z_n - L(x_n)s_n)^t C_v^{-1}(z_n - L(x_n)s_n)}{2}]. \tag{16}$$

B. Constrained EEG source tracking using IMeDeT

Our EEG experiments are focused on the visual cortex of the brain (the experiments are detailed in the sequel). Therefore, we expect the estimated dipoles to be in the visual cortex most of the time, which corresponds in our head model geometry to $dy_n < 0$, where dy is the estimated location of the dipole along the y-axis. We therefore add the constraint $E[dy_n] < 0$. We expect that adding this constraint will improve the tracking accuracy. The IMeDeT algorithm applied to the EEG dynamic source localization problem in (15) is presented below.

Algorithm 2 EEG dynamic source localization using IMeDeT

Initialization

The constraint region is given by :$C_n = \{x_n : E[dy_n] \le 0\}$.

For all points of the grid, compute the lead field matrix L by solving the Maxwell equations in [5].

for j=1,2,...,N **do**

  Sample $x_0^{(j)} \sim q_0(x_0^{(j)})$.

  Set initial weights $w_0^{(j)} = \frac{1}{N}$.

end for

Sampling

for n=1,2,...,T **do**

  for j=1,2,...,N **do**

    Generate samples from the state transition model: $x_n^{(j)} = x_{n-1}^{(j)} + u_n^{(j)}$.

    For the predicted dipole, find the lead field matrix $L(x_n^{(j)})$ from the calculation made at the initial step.

    Compute the weights by : $w_n^j = w_{n-1}^{(j)} \mathcal{L}(z_n|(x_n, L(x_n^{(j)})), s_n^{(j)})$.

    Normalize the weights so they sum up to unity.

    Compute the weighted mean $\hat{x}_n = \sum_{i=1}^{j} w_n^{(i)} x_n^{(i)}$.

    Enforcing the constraint

    for i=1,2,...,j **do**

      if $\hat{x_n} \in C_n$ **then**

        Go to the next step.

      else

        Find a particle $x_n^{(j)}$ such that $\sum_{i=1}^{j} w_n^{(i)} x_n^{(i)} \in C_n$.

      end if

    end for

  end for

  $\hat{x}_n = \sum_{i=1}^{N} w_n^{(i)} x_n^{(i)}$.

end for

V. SIMULATIONS RESULTS

A. Simulation results on synthetic data

We assume that the observed EEG measurements are generated by one moving dipole. The moments are supposed to

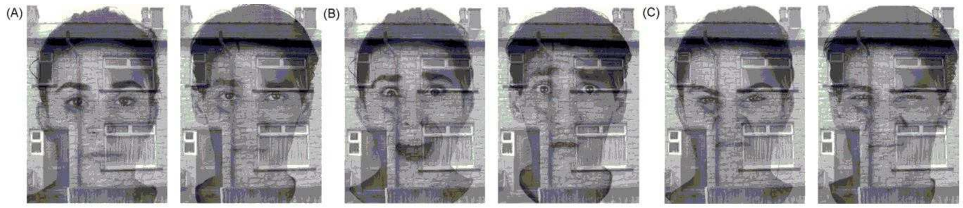

Fig. 1. A group of images shown to the subjects in the experimental framework.

be sinusoidal waveforms with varying frequencies and amplitudes. We apply the unconstrained PF and the constrained PF using PoDeT and inductive MeDeT algorithms on EEG dynamic source localization based on synthetic data that we generate using the model in (15). We use the mean-square error (MSE) to assess the three algorithms. We perform 10 Monte Carlo runs using 1000 particles for all methods.

Figures 2, 4 and 3 show the tracking of the dipole (position and moment) using, respectively, the unconstrained particle filter (Fig. 2), the inductive MeDeT (Fig. 4) and PoDeT (Fig. 3). The star lines represent the estimated positions and the continuous lines represent the true state for each dimension. Herein, x-position is in blue, y-position is in green and z-position is in red. Observe that adding the constraint (in PoDeT and IMeDeT) seems to increase the convergence and "locking-on rate" of the PF. While the unconstrained PF locks onto the dy location trajectory around time step 23 and locks onto the x-moment around time step 13, PoDeT and IMeDeT lock onto their respective trajectories (both location and moment) very early on. This result of higher convergence rate, though unexpected, is actually pretty intuitive because enforcing the constraint helps direct the tracker to the optimal state early on.

Figure 5 shows the MSE of IMeDeT, PoDeT and the unconstrained PF. IMeDeT is able to track the dipole better (with the minimum error) than PoDeT and the unconstrained PF. Moreover, the unconstrained PF seems to have the highest MSE both in the location and moment estimations.

B. Application to real data

We apply the proposed IMeDeT algorithm to real EEG data recorded from twelve female subjects aged between 20 and 28 years old. The experimental application was elaborated by Santos et al. [11] for their study on subject perception and attention using the evoked potential signals (VEP). Santos and co-authors were interested in different facial expressions such as fearful, disgusted and neutral of the subjects. These different facial expressions were displayed as the result of exposing the subjects to a sequence of images superimposed on houses (see Fig.5). For each test, the participants task was to determine if the current face or house is the same as the one shown on the previous test. Each test remains 1600 ms (400 points of digitized EEG samples with sampling rate 250 Hz, comprising a pre-stimulus interval of 148 ms (37 samples) and post stimulus onset interval of 1452 ms [5]. The distribution of the electrodes whose responsible of recording the EEG signals emanate from the scalp around 16 electrodes (Fz, Cz, Pz, Oz , F7, F8,Fp1, Fp2, F3, F4, C3, C4, P3, P4, O1, O2,) and two Electrooculogram (EOG) electrodes (vertical and horizontal EOG) according to the 10/20 International system. In this experiment, the data corresponds to VEP signals and the brain activity is perception of visual stimulus. Therefore, we expect the dipoles to be located, on average, in the primary visual cortex. We assume the same constraint as in the synthetic data sets, namely $E[dy_n] \leq 0$ for all time steps n. We implemented IMeDeT using 1000 particles. We considered estimating one dipole source for each participant.

We show the source localization results on two subjects in Figs. 6 and 7. In each figure, the top row shows tracking of the dipole position and the bottom row shows the dipole moments over time. Observe that the constraint is satisfied, i.e., the position of the estimated dipole is located in the visual cortex $(d_y < 0)$. Another observation is that the x-position of the dipole location varies between positive and negative values, which correspond to the right and left lobes of the brain. There are no significant differences in the dipole locations for the two subjects. However, the moment signals are different for the two subjects. Finally, we postulate that in order to observe significant or abrupt changes in brain source locations, we need to design an experiment, where two or more areas of the brain (e.g., visual and motor) are invoked.

VI. CONCLUSION

We proposed the Iterative Mean Density Truncation (IMeDeT) to optimally track a state in a non-linear state-space model with additional constraints on the expected value of the state. In many dynamical state-space models, additional a priori knowledge on the state is available. The proposed approach extends the particle filter to handling constraints by inductively drawing particles that satisfy the desired constraints on the mean state. We applied IMeDeT to the EEG dynamic source localization problem, where the source brains are expected to be located in the visual cortex of the brain. Our results showed that, incorporating additional a priori knowledge on the state as constraints improves the estimation accuracy as well as increases the convergence and locking-on rate of the non-linear tracker. Moreover, we showed that IMeDeT has a superior performance compared to the main approach, termed PoDeT, widely used for constrained particle filtering. PoDeT relies on imposing the constraints on every realization of the state, which results in more stringent and may be completely unrelated conditions than the original conditions on the mean value of the state. In our future work, we will investigate the

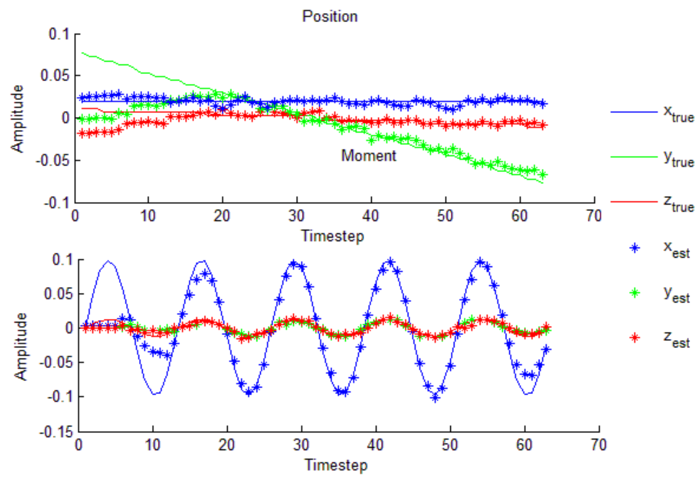

Fig. 2. Tracking of the dipole using unconstrained PF: Position in the top and moment signal in the bottom.

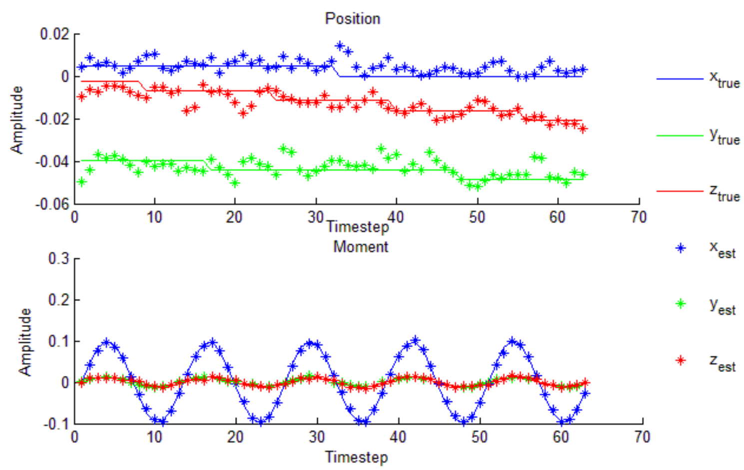

Fig. 3. Tracking of the dipole using Point wise Density Truncation (PoDeT): Position in the top and moment signal in the bottom.

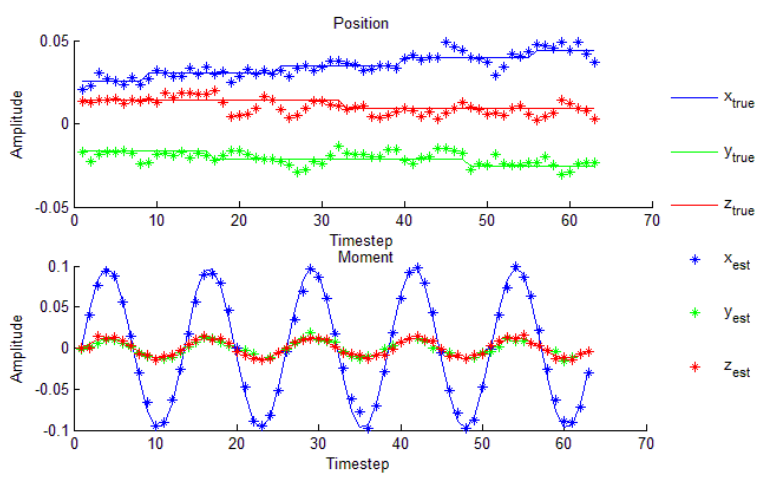

Fig. 4. Tracking of the dipole using Inductive Mean Density Truncation (IMeDeT): Position in the top and moment signal in the bottom.

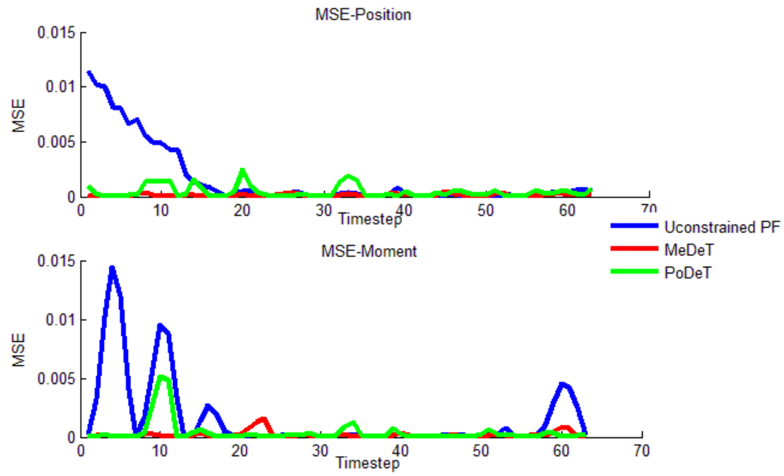

Fig. 5. The performance of tracking the dipole using MSE are illustrated by the location (top) and moment (bottom).

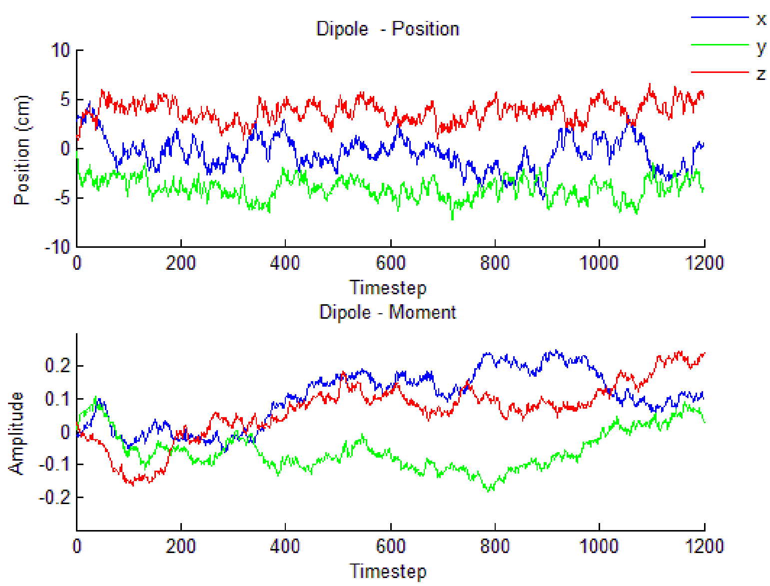

Fig. 6. Subject 1: Tracking the dipole position and moments using IMeDeT.

convergence properties and theoretical bounds of the proposed IMeDeT method for non-linear state estimation with additional constraints on the mean of the state.

Acknowledgment

This work was supported by the National Science Foundation under Grants NSF CCF-1527822 and NSF ACI-1429467. P. Georgieva acknowledges the sabbatical grant provided by the Portuguese Foundation of Science and Technology (FCT) for her visiting period in Rowan University.

References

[1] S. Sanei and J. Chambers, EEG *Signal Processing*. John Wiley & Sons, July 2007, p. 312.

[2] M. Grosse-Wentrup, C. Liefhold, K. Gramann, , and M. Buss, "Beamforming in non-invasive brain-computer interfaces," *IEEE Transactions on Biomedical Engineering*, vol. 56, no. 4, pp. 1209 – 1219, April 2009.

[3] M. Scherg and D. V. Cramon, "Two bilateral sources of the late AEP as identified by a spatiotemporal dipole model," *Electroencephalogr. Clin. Neurophysiol*, vol. 62, pp. 32 – 44, 1985.

[4] B. Ebinger, N. Bouaynaya, P. Georgieva, and L. Mihaylova, "Eeg dynamic source localizationusing marginalized particle filtering," in *International Conference on Bioinformatics and Biomedicine (BIBM)*, Washington, DC., November 2015.

[5] P. Georgieva, N. Bouaynaya, F. Silva, M. Lyudmila, and J. Lakhmi, "A beamformer-particle filter framework for localization of correlated EEG sources," *IEEE journal of biomedical and health informatics*.

[6] D. Simon, *Optimal State Estimation: Kalman, H Infinity, and Nonlinear Approaches*. Wiley, 2006.

[7] A. Doucet and A. M. Johansen, *Handbook of Nonlinear Filtering*, 2009, vol. 12, ch. A tutorial on particle filtering and smoothing: Fifteen years later, pp. 656–704.

[8] F. Papi, M. Podt, Y. Boers, and G. Battistello, "On constraints exploitation for particle filtering based target tracking," in *International Conference on Information Fusion*, Singapore, July 2012, pp. 455 – 462.

[9] X. Shao, B. Huang, and J. M. Lee, "Constrained bayesian state esti-

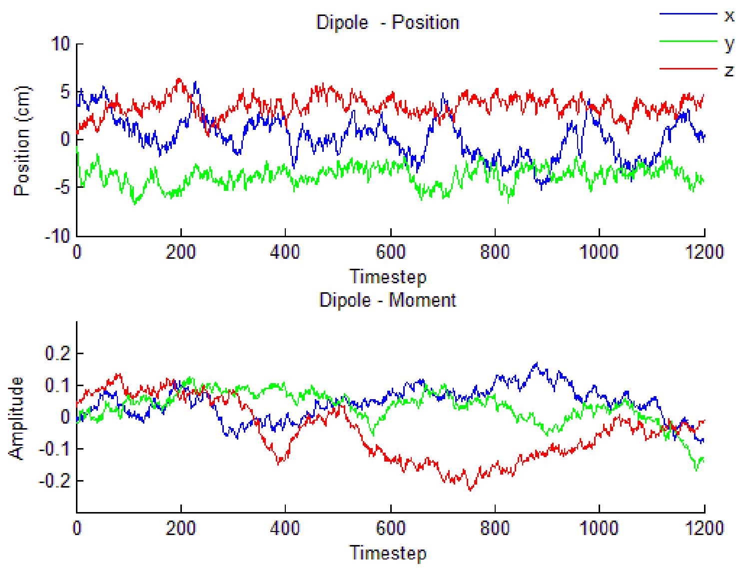

Fig. 7. Subject 2: Tracking the dipole position and moments using IMeDeT

mation: A comparative study and a new particle filter based approach," *Journal of Process Control*, vol. 20, no. 2, pp. 143–157, 2010.

[10] B. Ebinger, N. Bouaynaya, R. Polikar, and R. Shterenberg, "Constrained state estimation in particle filters," in *International Conference on Acoustics Speech and Signal Processing (ICASSP)*, South Brisbane, QLD., April 2015.

[11] I. Santos, J. Iglesias, E. I. Olivares, and A. Young, "Differential effects of object-based attention on evoked potentials to fearful and disgusted faces," *Neuropsychologia*, vol. 46, no. 5, pp. 1468–1479, 2008.