

# ARMA COMPANDING SCHEME WITH IMPROVED SYMBOL ERROR RATE FOR PAPR REDUCTION IN OFDM SYSTEMS

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**Abstract**—This paper proposes a new nonlinear companding scheme with reduced symbol error rate (SER) that can be used to reduce peak-to-average power ratio (PAPR) in orthogonal frequency division multiplexing (OFDM) systems. The proposed system outperforms conventional companding systems using the same nonlinear companding functions with respect to SER without impairing PAPR reduction capability. The proposed system estimates a few autoregressive moving average (ARMA) model parameters of the difference signal between the companded and uncompanded OFDM envelopes and passes these parameters to the receiver. Upon receiving the ARMA model parameters, the receiver regenerates the difference signal and then adds it to the received companded OFDM envelope to recover the uncompanded OFDM signal. The traditional schemes employ decompanding process instead to reconstruct the uncompanded OFDM signal. Mathematical proofs and simulation results for three typical nonlinear companding functions show that, under mild sufficient conditions, the proposed scheme outperforms the conventional schemes with respect to SER performance without impairing PAPR reduction capability.

## I. INTRODUCTION

OFDM is a popular multi-carrier modulation technique that offers very high transmission rates and utilizes efficiently the available spectrum and network resources. It is a promising choice for future high speed data rate systems and is already incorporated in many applications and standards such as WLAN, Digital Audio Broadcasting (DAB), Digital Video Broadcasting (DVB), the European HIPERLAN/2, Worldwide Interoperability for Microwave Access (WiMAX) and Digital Subscriber Line (DSL) [1]. In OFDM modulation scheme, multiple data symbols are modulated simultaneously by multiple carriers by breaking the wide transmission band into narrower, multiple sub-bands. This process allows OFDM to effectively combat frequency-selective fading usually encountered in wireless channels. Despite the great advantages OFDM offers, it has a few drawbacks, the most serious of which is the non-constant signal envelope with high peaks. These high peaks produce signal excursions into nonlinear region of the high power amplifier at the transmitter, thereby leading to nonlinear distortion. For a binary data stream with rate  $R$  bps,  $N$  modulated symbols,  $a_k$ ,  $0 \leq k \leq N - 1$ , are stored for an interval of  $T_s = N/R$  using a serial-to-parallel converter. Subsequently, each one of the  $N$  symbols modulates one sub-carrier and then all modulated sub-carriers are transmitted simultaneously [2]. The OFDM signal  $x(t)$  can

be expressed as

$$\begin{aligned} x(t) &= \sum_{k=0}^{N-1} a_k \exp(j2\pi(f_c + k\Delta f)t) \\ &= \exp(j2\pi f_c t) \sum_{k=0}^{N-1} a_k \exp(j2\pi k\Delta f t) \\ &= \exp(j2\pi f_c t) a(t), \end{aligned} \quad (1)$$

where  $f_k = f_c + k\Delta f$ ,  $0 \leq k \leq N - 1$ , is the  $k^{th}$  sub-carrier, with  $f_c$  being the lowest sub-carrier frequency and  $\Delta f$  is the frequency spacing between adjacent sub-carriers, chosen to be  $1/T_s$  to ensure that the sub-carriers are orthogonal [2]. If  $a(t)$  is sampled at the rate of  $R$  samples per second, then  $a(t)$  is represented by the sampled function  $a[n]$  expressed as

$$a[n] = \sum_{k=0}^{N-1} a_k \exp(j2\pi kn/N). \quad (2)$$

This equation takes exactly the same form as the Inverse Discrete Fourier Transform (IDFT). Equations (1) and (2) demonstrate that OFDM can be generated by modulating the Inverse Fast Fourier Transform (IFFT) of the sequence  $\{a[n], 0 \leq n \leq N - 1\}$  by a single carrier of frequency  $f_c$  instead of by modulating  $N$  symbols by sub-carriers.

The PAPR for the continuous-time signal  $x(t)$  is the ratio of the maximum instantaneous power to the average power. For the discrete-time version  $x[n]$ , PAPR is expressed as

$$\max_{0 \leq n \leq N-1} \frac{|x[n]|^2}{E[|x[n]|^2]}, \quad (3)$$

where  $E[\cdot]$  denotes the expectation operator. Since IFFT generates the OFDM signal, the resulting discrete-time OFDM signal samples are obtained at the Nyquist rate. The peak values computed using these samples may not coincide with the peak value of the continuous-time OFDM signal [3]. It is found that the PAPR of the oversampled discrete-time signal offers an accurate approximation of the continuous-time signal one if the oversampling factor is at least 4 [4].

Many PAPR reduction techniques have been proposed in the literature [5], some examples of which include techniques such as clipping and filtering, signal companding, peak windowing,

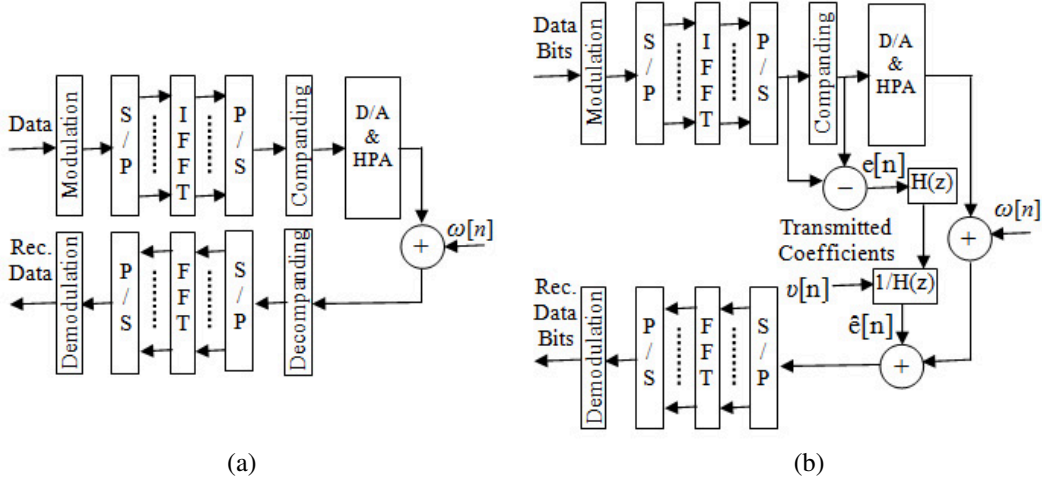


Fig. 1. OFDM Companding system: (a) Conventional system; (b) Proposed system.

selective mapping, partial transmit sequence, tone injection, tone reservation and linear block coding. An overview of these methods is provided in [5].

In signal companding scheme, the time-domain OFDM signal encounters a transformation that attenuates high peaks and enhances low amplitudes at the transmitter. Obviously this process will decrease the PAPR. At the receiver, the inverse transformation is applied to reconstruct the original uncompanded signal. Companding is an attractive technique to achieve PAPR reduction due to its relatively low implementation complexity regardless of the number of sub-carriers in the OFDM system. Moreover, companding does not require side information and does not cause bit rate loss. Figure 1(a) shows the conventional block diagram of an OFDM system with nonlinear companding for PAPR reduction. Observe that the companding process takes place before the digital-to-analog converter (D/A) and the high power amplifier (HPA). At the receiver, the decompanding is implemented before the FFT block.

One of the major drawbacks of current companding techniques is the tradeoff between PAPR reduction and SER performance. An increase in the PAPR reduction capability inherently leads to a degradation in the SER performance. This occurs because channel noise is decompanded at the receiver resulting in a higher number of errors in the recovered data symbols and hence a higher SER. To maintain high transmission rates, wireless communication systems sacrifice SER performance, especially under severe channel conditions.

In this paper, we propose a new companding scheme that improves the SER performance compared to the conventional companding schemes. The proposed scheme relies on an ARMA representation of the difference signal between the uncompanded and companded OFDM envelopes. We show that, under a mild condition on the first derivative of the companding function, the ARMA-based companding scheme achieves a lower SER rate than the conventional companding schemes, while maintaining the same PAPR capability.

Alternatively, keeping SER the same for both the proposed and conventional schemes, the proposed scheme can achieve a higher level of companding and hence, a better PAPR reduction capability. This performance comes at the cost of transmitting side information corresponding to the ARMA model parameters. However, we show that, in practice, the overhead is negligible with respect to the number of sub-carriers.

This paper is organized as follows: In section II, we describe the proposed ARMA-based companding scheme for a general nonlinear companding function. We subsequently derive a sufficient condition that ensures the superiority of the proposed scheme compared to the conventional companding methods, in terms of SER performance. In section III, we apply the proposed scheme to three widely used nonlinear companding functions and show that the proposed scheme outperforms the three companding schemes. Section IV presents simulation results that confirm our theoretical analysis. Finally, section V provides some conclusions.

## II. DESCRIPTION OF THE PROPOSED SYSTEM

In this section, we propose a new nonlinear companding system structure that improves the SER performance of conventional companding OFDM systems without diminishing the PAPR reduction capability. At the transmitter of the proposed system, we model the difference signal between the uncompanded and companded OFDM envelopes as an Auto-Regressive Moving Average (ARMA) process given by

$$\sum_{i=0}^p a_i e[n-i] = \sum_{i=0}^q b_i v[n-i], \quad (4)$$

where  $e[n]$  is the discrete difference signal between the uncompanded and companded OFDM envelopes, and  $v[n]$  is a white noise sequence. Without loss of generality, we set  $a_0 = 1$ . The ARMA model is then given by

$$H(z) = \frac{\sum_{i=0}^q b_i z^{-i}}{1 + \sum_{i=1}^p a_i z^{-i}}. \quad (5)$$

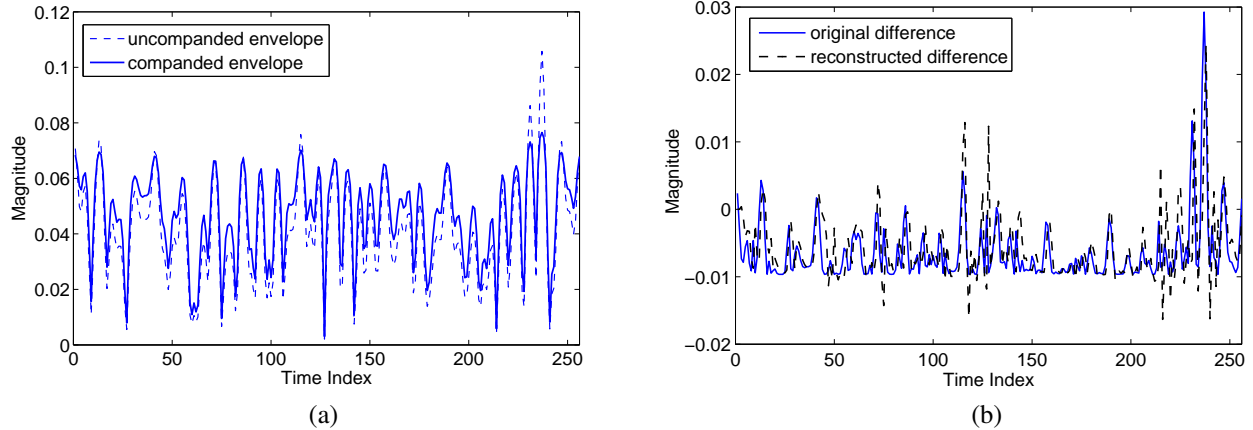


Fig. 2. (a) uncompanded and companded envelopes of an OFDM signal of 64 sub-carriers with an oversampling factor of 4 and QPSK modulation. Companding is performed using the error function (erf); (b) difference and ARMA(2,2) reconstructed difference signals.

For every OFDM frame, the ARMA coefficients are estimated and then transmitted to the receiver where the difference signal  $e[n]$  is regenerated from a white Gaussian noise (WGN) process with some acceptable error using the inverse of the transmitter's ARMA model. This process is similar to the classical linear prediction coding (LPC) of speech signals, where speech frames are generated at the receiver using a few LPC coefficients and a WGN process. Now, instead of decompressing the received OFDM envelope, the regenerated difference signal  $\hat{e}[n]$  is added to the received companded envelope to recover the uncompanded OFDM signal. Figure 1(b) provides a block diagram of the proposed system.

As an example, consider an OFDM frame of 64 sub-carriers and an oversampling factor of 4 with QPSK modulation. Figure 2(a) shows the uncompanded and companded envelopes of the OFDM signal, where the error function (erf) is used to perform signal companding. It is clear that high peaks are attenuated, while low amplitudes are enhanced. Figure 2(b) shows the difference signal between the uncompanded and companded OFDM envelopes in Fig. 2(a) (transmitter side) and the reconstructed difference signal using an ARMA(2,2) model (receiver side). Observe that the ARMA model provides a fairly accurate representation of the difference signal.

Theoretically, the proposed scheme requires sending the ARMA model parameters as side information. However, in practice, the coefficients are quantized and represented by finite length binary words. Therefore, there exists only a finite number of possible quantized ARMA parameters. Hence, it is possible to use lookup tables at both sides and transmit only the index parameter corresponding to the ARMA coefficients in use.

Consider the proposed scheme with an ARMA(2,2) model, where each parameter is represented by eight bits and QPSK modulation (two bits per sub-carrier) is used. Table I shows the percentage of sub-carriers dedicated to send side information for different numbers of sub-carriers.

In the following proposition, we show that under a mild sufficient condition, the proposed scheme outperforms the

TABLE I  
PERCENTAGE OF SIDE INFORMATION OVERHEAD

Number of sub-carriers	Side information percentage
64	25 %
128	12.5 %
256	6.25 %
512	3.125 %
1024	1.56 %
2048	0.78 %

conventional companding methods with respect to SER.

**Proposition 1.** Consider the envelope  $z$  of a sampled OFDM signal, companded by the function  $C$ . For a white noise channel, the proposed system results in a smaller error at the receiver compared to the conventional system if

$$|C'(z)| \leq 1, \quad (6)$$

where  $C'$  denotes the derivative of the function  $C$ .

**Proof 1.** For simplicity, we write  $z$  to denote the discrete-time signal  $z[n]$ . Let  $w$  be the channel's noise. In the proposed scheme, the received signal  $y$  is given by (see Fig. 1(b)).

$$y = C(z) + w + [z - C(z)] = z + w, \quad (7)$$

where we have neglected the ARMA reconstruction error of the difference signal at the receiver with respect to channel's noise. In particular, this is true for low signal-to-noise ratio (SNR). Therefore, the absolute received error for the proposed scheme is

$$|e_p| = |y - z| = |w|. \quad (8)$$

In the conventional scheme, the absolute value of the received error is given by

$$|e_c| = |C^{-1}(C(z) + w) - z|. \quad (9)$$

Let us use Taylor's series expansion up to the first order derivative for the function  $C^{-1}[C(z) + w]$  about the point  $C(z)$ . By noting that  $(C^{-1})'(C(z)) = \frac{1}{C'(z)}$ , we obtain

$$\begin{aligned} C^{-1}[C(z) + w] &= C^{-1}[C(z)] + w \frac{1}{C'(z)} + o(w^2) \\ &= z + \frac{w}{C'(z)} + o(w^2). \end{aligned} \quad (10)$$

Hence

$$|e_c| = \left| \frac{w}{C'(z)} \right| + o(w^2). \quad (11)$$

Therefore, a sufficient condition to have  $|e_p| \leq |e_c|$  is  $|C'(z)| \leq 1$ .

It is important to emphasize that the condition given in Eq. (6) is sufficient but not necessary to ensure the superiority of the proposed scheme over the conventional companding schemes. In particular, one might obtain tighter conditions for specific companding systems by considering higher order Taylor series expansion.

### III. EXAMPLES

In this section, we apply the proposed scheme to three typical companding functions found in the literature and find a sufficient condition for each case that ensures superiority of the proposed scheme, in terms of SER, compared to the conventional one.

#### A. Hyperbolic tangent (*tanh*) companding

The hyperbolic tangent (*tanh*) companding function with two positive parameters  $k_1$  and  $k_2$  is defined by [6]

$$C(z) = k_1 \tanh(k_2 z), \quad (12)$$

where  $k_1$  and  $k_2$  control the extent of companding applied to the signal  $z$ . Figure 3 shows  $C(z) = k_1 \tanh(k_2 z)$  versus  $z$  for different values of  $k_1$  and  $k_2$ . The proper choice of  $k_1$  and  $k_2$  should map the dynamic range of  $z$  into the nonlinear region of the curve. The derivative of  $C(z)$  is given by

$$C'(z) = k_1 k_2 [1 - \tanh^2(k_2 z)]. \quad (13)$$

Given that  $[1 - \tanh^2(k_2 z)] \leq 1$ , for all signals  $z$ , a sufficient condition to have  $C'(z) < 1$  is  $k_1 k_2 \leq 1$ . In [6],  $k_1$  and  $k_2$  were chosen such that  $k_1 = 1/k_2$ , or  $k_1 k_2 = 1$ . Therefore, the proposed scheme performs no worse than the conventional system using the hyperbolic tangent companding function when  $k_1 k_2 = 1$ . For other values of  $k_1$  and  $k_2$  such that  $k_1 k_2 < 1$  the proposed scheme will perform better than that in [6].

#### B. Error function (*erf*) companding

The error function (*erf*) with two positive parameters  $k_1$  and  $k_2$  is defined by [7], [8]

$$C(z) = k_1 \operatorname{erf}(k_2 z). \quad (14)$$

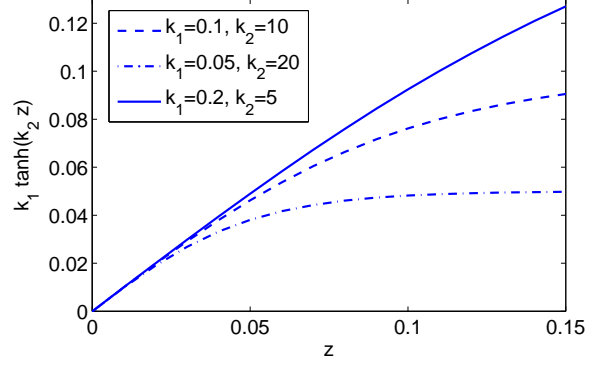


Fig. 3.  $k_1 \tanh(k_2 z)$  versus  $z$  curves for different values of  $k_1$  and  $k_2$ .

Hence,

$$C'(z) = \frac{2k_1 k_2}{\sqrt{\pi}} \exp[-(k_2 z)^2]. \quad (15)$$

Since both  $k_2$  and  $z$  are positive, the exponential part of this equation has its maximum value of 1 at  $z = 0$ . Therefore, a sufficient condition to obtain  $C'(z) < 1$  is easily found to be  $k_1 k_2 < \sqrt{\pi}/2$ . In [7], the authors used the values  $k_1 = 1$  and  $k_2 = 0.7071$ , hence  $k_1 k_2 = 0.7071 < \sqrt{\pi}/2$ . Consequently, the proposed scheme outperforms the conventional *erf* companding system presented in [7].

#### C. $\mu$ -law companding

The  $\mu$ -law companding function [9] is given by

$$C(z) = \frac{A \operatorname{sign}(z) \log \left[ 1 + \mu \left| \frac{z}{A} \right| \right]}{\log(1 + \mu)}, \quad (16)$$

where  $A$  is a normalization constant such that  $0 \leq |z/A| \leq 1$  and  $\mu$  is the companding parameter. Applying the condition in Eq. (6) to this function, we obtain

$$\log(1 + \mu) \left( \frac{1}{\mu} + \left| \frac{z}{A} \right| \right) > 1. \quad (17)$$

However, practical values used in OFDM systems do not satisfy Eq. (17). In order to obtain a condition for the superiority of the proposed scheme over the conventional methods, we extend the Taylor series expansion in the proof of proposition 1 to the second order to yield

$$\left| \frac{1 + \mu \frac{z}{A}}{\mu / \log(1 + \mu)} + \frac{w(1 + k_2 z)}{2k_1^2 k_2} \right| > 1. \quad (18)$$

Taking the expected value on both sides and noting that  $E[w] = 0$  and  $E[wz] = 0$  because  $w$  and  $z$  are independent, we obtain the following condition

$$1 + \frac{\mu E[z]}{A} > \frac{\mu}{\log(1 + \mu)}, \quad (19)$$

which yields  $\mu > 2.5$  using  $E[z] = 0.04$  and  $A = 0.1$ , which are typical values for 64 sub-carriers, QPSK modulated OFDM signal. This condition is weaker than the one in proposition 1 because it provides a sufficient condition for the proposed

scheme to outperform the conventional companding system on average, whereas the condition in Eq. (6) ensures that the error at the receiver of the proposed system is always smaller than the error at the receiver of the conventional system sample by sample.

#### IV. RESULTS AND DISCUSSION

##### A. Average power constraint

To keep the average power of the companded signal unchanged compared to the original un-companded signal, normalization is required after companding. However, we can choose the companding parameters such that the average power of the companded signal is unchanged. Let us take the hyperbolic tangent function and the error function with their companding parameters  $k_1$  and  $k_2$  as examples.

**Proposition 2.** Consider the envelope  $z$  of an OFDM signal of 64 sub-carriers with an oversampling factor of 4 and quadrature phase-shift keying (QPSK) modulation. Companding the envelope by the hyperbolic tangent function defined by  $C_{\tanh}(z) = k_1 \cdot \tanh(k_2 z)$ , or the error function defined by  $C_{\text{erf}}(z) = k_1 \cdot \text{erf}(k_2 z)$  can be carried out without changing the average power by setting  $k_1 \approx 1/k_2$ .

**Proof 2.** The envelope of the un-companded OFDM signal follows the Rayleigh distribution with the probability density function

$$f(z) = \frac{z}{\sigma^2} \exp\left(\frac{-z^2}{2\sigma^2}\right), \quad (20)$$

with mean and variance given by  $\sigma\sqrt{\pi/2}$  and  $\frac{4-\pi}{2}\sigma^2$ , respectively, where  $\sigma$  is an adjustable Rayleigh parameter. In practice, we use the maximum likelihood estimator (MLE) to compute  $\sigma$ ,

$$\hat{\sigma} = \sqrt{\frac{1}{2N} \sum_{i=1}^N z_i^2}, \quad (21)$$

where  $N$  is the total number of samples in the OFDM frame. To keep the average power of the companded signal unchanged, the following condition must be satisfied

$$\begin{aligned} E[z^2] &= E[C^2(z)] \\ 2\sigma^2 &= E^2[C(z)] + \text{var}[C(z)]. \end{aligned} \quad (22)$$

Using the approximate expressions for the mean and variance of a known function of a random variable with known distribution given by [10]

$$E[C(z)] \approx C[E(z)], \quad (23)$$

$$\text{var}[C(z)] \approx \left[\frac{d}{dz} C[E(z)]\right]^2 \text{var}(z), \quad (24)$$

we can find an approximate relation for  $k_1$  and  $k_2$  that satisfies the condition in Eq. (22). It is easy to show that for the

hyperbolic tangent and the error functions, these relations are given by

$$k_1^{\tanh} \approx \sqrt{\frac{2\sigma^2 / \tanh^2(1.25\sigma k_2)}{1 + 0.43\sigma^2 k_2^2 \left[\frac{1 - \tanh^2(1.25\sigma k_2)}{\tanh(1.25\sigma k_2)}\right]^2}}, \quad (25)$$

and

$$k_1^{\text{erf}} \approx \sqrt{\frac{2\sigma^2 / \text{erf}^2(1.25\sigma k_2)}{1 + 0.546 \left[\frac{\sigma k_2 e^{-(1.25\sigma k_2)^2}}{\text{erf}(1.25\sigma k_2)}\right]^2}}. \quad (26)$$

For an OFDM signal of 64 sub-carriers, oversampling factor of 4 and modulated by a QPSK scheme, we found that the average value of  $\hat{\sigma}$  over  $10^5$  OFDM frames is  $\hat{\sigma} \approx 0.03$ . From Fig. 3, we see that appropriate values of  $k_2$ , which map the dynamic range of  $z$  to the nonlinear region of the companding function, belong to the segment [1,25]. Therefore,  $(1.25\sigma k_2) \leq (1.25\sigma \times 25) < 1$ . Hence both  $\text{erf}(1.25\sigma k_2)$  and  $\tanh(1.25\sigma k_2)$  can be fairly approximated by  $(1.25\sigma k_2)$ . From our extensive simulations, we found that we can fairly approximate  $e^{-(1.25\sigma k_2)^2}$  by 1. Therefore, Eqs. (25) and (26) become, respectively,

$$\begin{aligned} k_1^{\tanh} &\approx \sqrt{\frac{2\sigma^2 / (1.25\sigma k_2)^2}{1 + 0.43\sigma^2 k_2^2 \left[\frac{1 - (1.25\sigma k_2)^2}{(1.25\sigma k_2)}\right]^2}} \\ &\approx \frac{1}{k_2}. \end{aligned} \quad (27)$$

and

$$k_1^{\text{erf}} \approx \sqrt{\frac{2\sigma^2 / (1.25\sigma k_2)^2}{1 + 0.546 \left[\frac{1}{(1.25\sigma k_2)}\right]^2}} \approx \frac{1}{k_2}. \quad (28)$$

Interestingly, the average power constraint, for the hyperbolic tangent and the error functions, leads to design parameters  $k_1$  and  $k_2$  that also satisfies the sufficient condition for the superiority of the proposed ARMA-based scheme over the conventional companding methods. In particular, this choice of design parameters avoids the normalization step required after companding to keep the average power unchanged. Thus, the proposed scheme outperforms the conventional schemes in SER performance without changing the average power of the OFDM signal.

##### B. Simulation results

Consider a baseband OFDM system like the one shown in Fig. 1 and compare the conventional and proposed methods. We use a QPSK modulation scheme with 64 sub-carriers and an oversampling factor of 4. A solid state power amplifier (SSPA) model is used to model the HPA. SSPA produces amplitude conversion without phase distortion and its output is given by

$$z_{\text{out}} = \frac{z_{\text{in}}}{\left[1 + \frac{z_{\text{in}}^{2p}}{A}\right]^{1/2p}}, \quad (29)$$

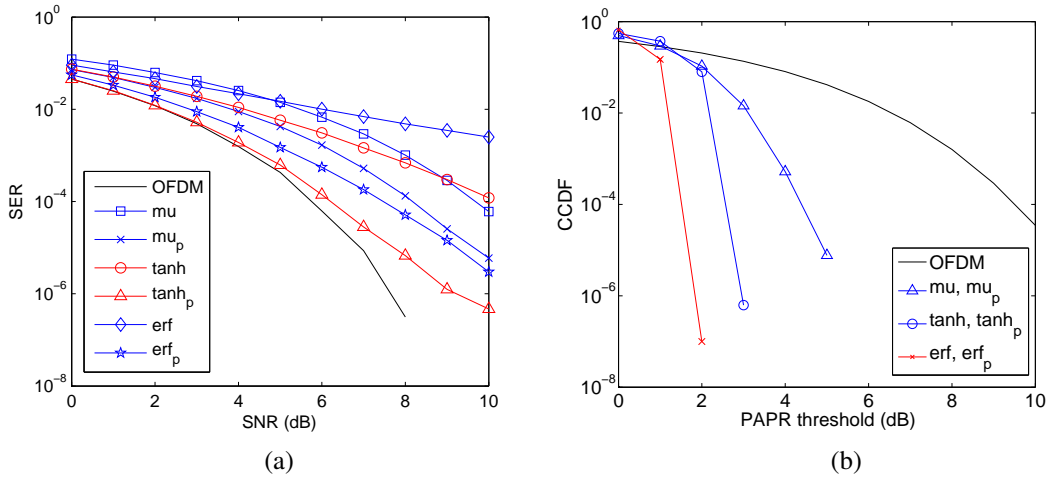


Fig. 4. Performance comparison between the conventional and proposed OFDM systems, where  $C$  and  $C_p$  denote the conventional and proposed systems, respectively: (a) SER performance for different companding functions; (b) CCDF performance for different companding functions. Note that the CCDF of the proposed system is identical to the conventional one.

where  $p$  is a parameter controlling the nonlinearity of the power amplifier and  $A$  is a normalization factor specifying the saturation level of the power amplifier. In our simulations, we set  $p = 2$ , and  $A = 0.2$ . For the  $\mu$ -law companding simulations,  $\mu$  is set to 16. For the hyperbolic tangent function, we use  $k_1 = 0.05$  and  $k_2 = 18$ . For the error function, values of  $k_1$  and  $k_2$  were set to 0.04 and 23.

Figure 4(a) shows the SER performance for the conventional and proposed companding schemes using the three companding functions presented in the examples. Simulation results show that, on the average, the proposed scheme provides a gain of 1.5dB for the  $\mu$ -law function at a SER of  $10^{-4}$ , 4dB for the hyperbolic tangent function at a SER of  $10^{-4}$  and 3.5dB for the error function for a SER of  $10^{-2}$ .

For any scheme, PAPR reduction capability is measured by the empirical complementary cumulative distributive function (CCDF), which indicates the probability that PAPR is above a certain threshold. Figure 4(b) shows the CCDF curves for the three companding functions. It is worth mentioning that the purpose of Fig. 4(b) is not to compare between different companding functions in terms of PAPR reduction capability, but to show that the proposed scheme achieves a better BER without changing the CCDF performance.

## V. CONCLUSIONS

In this paper, we propose a nonlinear companding scheme for OFDM signals with enhanced SER performance compared to the conventional companding methods. We prove analytically that, under a mild sufficient condition, the proposed scheme outperforms the conventional companding schemes using the same nonlinear companding functions, in the sense of reducing the symbol error rate at the receiver. Moreover, we consider choosing suitable companding parameters that keep the average power of the companded signal unchanged so as to eliminate the required normalization process after companding.

The proposed system does not degrade PAPR reduction capability because the transmitted companded signal remains unchanged as in conventional schemes. Our simulation results for the three most popular companding functions confirm our theoretical results.

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## REFERENCES

- [1] R. V. Nee, G. Awater, M. Morikura, H. Takahashi, M. Webster, and K. W. Halford, "New high-rate wireless LAN standards," *IEEE Commun. Mag.*, vol. 37, no. 12, pp. 82–88, December 1999.
- [2] M. Schwartz, *Mobile Wireless Communications*. Cambridge University Press, 2005.
- [3] M. Sharif, M. Gharavi-Alkhansari, and B. H. Khalaj, "On the peak-to-average power of OFDM signals based on oversampling," *IEEE Trans. Commun.*, vol. 51, no. 1, pp. 72–78, January 2003.
- [4] C. Tellambura, "Computation of the continuous-time PAR of an OFDM signal with BPSK subcarriers," *IEEE Commun. Lett.*, vol. 5, no. 3, pp. 185–187, May 2001.
- [5] S. H. Han and J. H. Lee, "An overview of peak-to-average power ratio reduction techniques for multicarrier transmission," *IEEE Wireless Commun. Magazine*, vol. 12, no. 2, pp. 56–65, April 2005.
- [6] D. Lowe and X. Huang, "Optimal adaptive hyperbolic companding for OFDM," in *Proc. 2nd Int. Conf. on Wireless Broadband and Ultra Wideband Commun.*, August 2007, pp. 24–24.
- [7] T. Jiang and G. Zhu, "Nonlinear companding transform for reducing peak-to-average power ratio of OFDM signals," *IEEE Trans. Broadcasting*, vol. 50, no. 3, pp. 342–346, September 2004.
- [8] T. Jiang, W. Xiang, P. C. Richardson, D. Qu, and G. Zhu, "On the nonlinear companding transform for reduction in PAPR of MCM signals," *IEEE Trans. Wireless Commun.*, vol. 6, no. 6, pp. 2017–2021, June 2007.
- [9] X. Wang, T. T. Tjhung, and C. S. Ng, "Reduction of peak-to-average power ratio of OFDM system using a companding technique," *IEEE Trans. Broadcasting*, vol. 45, no. 3, pp. 303–307, September 1999.
- [10] S. Selvin, *Survival Analysis for Epidemiologic and Medical Research (Practical Guides to Biostatistics and Epidemiology)*. Cambridge University Press, 2008.