

CHAPTER 2

TOLERANCES AND FITS

Quality in a product starts at the specification stage.

Preferred Numbers

In order to reduce variety, it is a standard practice to select the sizes of objects from a preferred series of numbers, which follow a geometric progression.

Charles Renard, a French army captain, developed a set of these numbers in 1877 and reduced different sizes of rope for military balloons from 425 to 17.

The series of numbers are designated R5, R10, R20, and R40 series (R for Renard).

R10 series has a progression ratio of $\sqrt[10]{10} = 1.2589254117\dots(\sim 1.25)$, and so on.

The numbers are calculated using these exact ratios and then rounded. The preferred numbers are included in the standards ANSI Z17.1-1973 and ISO 3-1973. Four basic series of preferred numbers are given here.

R5: 10, 16, 25, 40, 63, 100.

R10: 10, 12.5, 16, 20, 25, 31.5, 40, 50, 63, 80, 100.

R20: 10, 11.2, 12.5, 14, 16, 18, 20, 22.4, 25, 28, 31.5, 35.5, 40, 45, 50, 56, 63, 71, 80, 90, 100.

R40: 10, 10.6, 11.2, 11.8, 12.5, 13.2, 14, 15, 16, 17, 18, 19, 20, 21.2, 22.4, 23.6, 25, 26.5, 28, 30, 31.5, 33.5, 35.5, 37.5, 40, 42.5, 45, 47.5, 50, 53, 56, 60, 63, 67, 71, 75, 80, 85, 90, 95, 100.

We note that multiples of 10, 100, ... and fractions 1/10, 1/100,... are also included within each of the series.

Other rounded numbers given below are suggested for preferred metric sizes. These may be used for choosing dimensions, capacities and so on.

Preferred metric sizes:

First Choice	10	16	25	40	60	100				
Second Choice	12	20	30	50	80					
Third Choice	11	14	18	22	28	35	45	55	70	90

As an example, the automotive engine capacity designations 1 liter, 1.6 liter, 2.5 liter, 4 liter, etc, are numbers originating from this series. The rounded off preferred numbers here may be used as preliminary values for design. For more detailed use, we recommend the use of values from ISO or ANSI standards.

Use of preferred sizes is a step toward standardization. Standardization reduces variety.

Another example is the spindle speeds in machine tools. Geometric speed steps are used in multiple speed gear boxes.

Part sizes and tolerance values are chosen from the preferred numbers. Tolerance on a part is a measure of quality. The specification of tolerance plays an important role in making economic decisions.

Tolerances and Fits

No engineering component can be manufactured to an exact size repeatedly.

If a number of components of specified dimensions are made by the same process, their sizes will vary.

These variations will be random in nature. These variations may be reduced by controlling the variables of the process. The designer must make a decision about this variation based on the function of the part

Tolerance is the difference between the maximum and minimum size limits on a part.

The tolerance is specified by the international tolerance grade, for example IT7.

International tolerance grades range from IT01-IT16. The tolerance values for various grades are discussed after introducing some important definitions.

Basic size is the size to which tolerance and deviations are assigned. Basic size is the same for the hole and its mating shaft.

Deviation is the algebraic difference between a size and the corresponding basic size.

Upper deviation is the algebraic difference between the maximum size limit and the basic size.

Lower deviation refers to the algebraic difference between the minimum size limit and the basic size.

Fundamental deviation is the deviation of the size closer to the basic size. The fundamental deviation is specified by a letter. Upper case letters are assigned for holes and lower case letters for shafts.

The tolerance on a part is completely specified by specifying the basic size, fundamental deviation, and the tolerance grade, for example 50H7 specifies the tolerance on a hole of basic size 50mm and 50g6 specifies the tolerance on a shaft of basic size 50mm.

The maximum size of a shaft and the minimum size of the hole defining a fit are referred to as the *maximum material condition* (MMC). It is clear that the part has the most material in it at MMC. MMC represents the most severe condition for a fit.

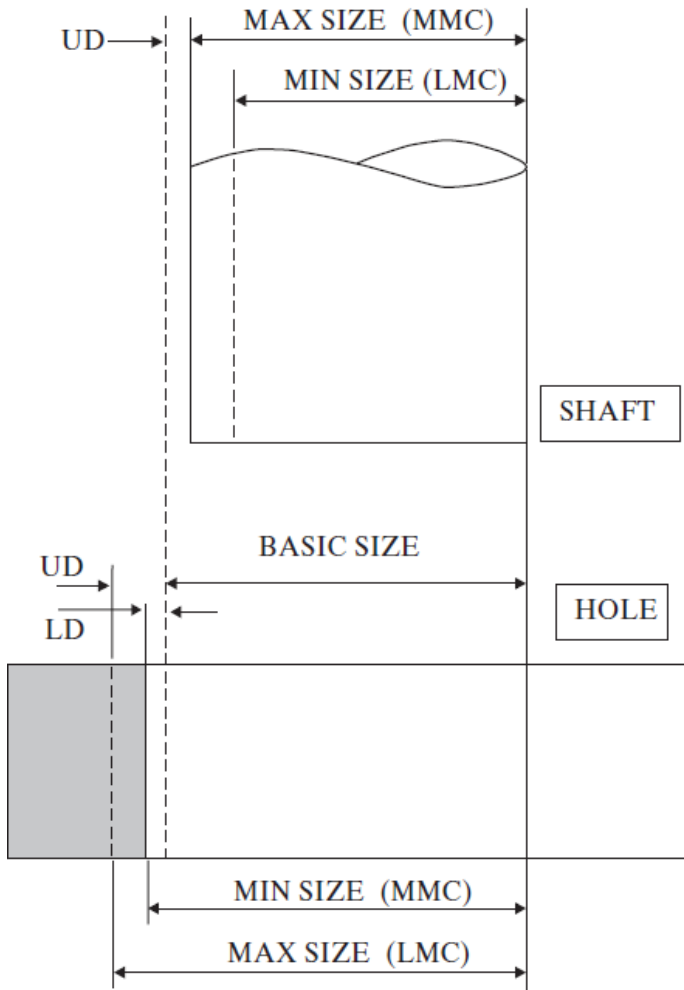


Figure 2.1.

Tolerance Grades

In defining the tolerance grades IT5 to IT16, we make use of the tolerance unit $i \mu\text{m}$

($1 \mu\text{m} = 10^{-6}\text{m}$ or 0.001 mm) defined by

$$i = 0.45D^{\frac{1}{3}} + 0.001D \quad \mu\text{m} \quad (2.1)$$

where D is the dimension in mm.

The tolerance values for various grades are given as

Grade	IT5	IT6	IT7	IT8	IT9	IT10	IT11	IT12	IT13	IT14	IT15	IT16
Tol.	$7i$	$10i$	$16i$	$25i$	$40i$	$64i$	$100i$	$160i$	$250i$	$400i$	$640i$	$1000i$

(2.2)

The tolerances for IT01, IT0, and IT1 are taken as $0.3 + 0.008D$, $0.5 + 0.012D$, and $0.8 + 0.020D$ respectively. IT2 to IT3 are placed between IT1 and IT5 to form a geometric progression.

Example . Calculate the tolerance grades IT01 to IT16 for the nominal size 40mm.

Solution:

The unit tolerance is calculated using $D = 40\text{mm}$

$$i = 0.45D^{\frac{1}{3}} + 0.001D = 1.578978 \mu\text{m}$$

For IT5, $7i = 11.1 \mu\text{m}$

For grades IT01, $0.3 + 0.008D = 0.6 \mu\text{m}$

For grades IT0, $0.5 + 0.012D = 1.0 \mu\text{m}$

For grades IT1, $0.8 + 0.020D = 1.6 \mu\text{m}$

IT6 to IT16 are calculated using Eq. 2.2. IT2,3,4 are placed to form a geometric series.

The final result is shown below:

IT GRADE	01	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
TOL μm	0.6	1	1.6	2.6	4.2	6.8	11.1	15.8	25.3	39.5	63.2	101	158	253	395	632	1011	1579

The calculations above are given to explain the rationale of tolerance grades. These calculated values are then rounded and are provided in a table, see Table 2.1.

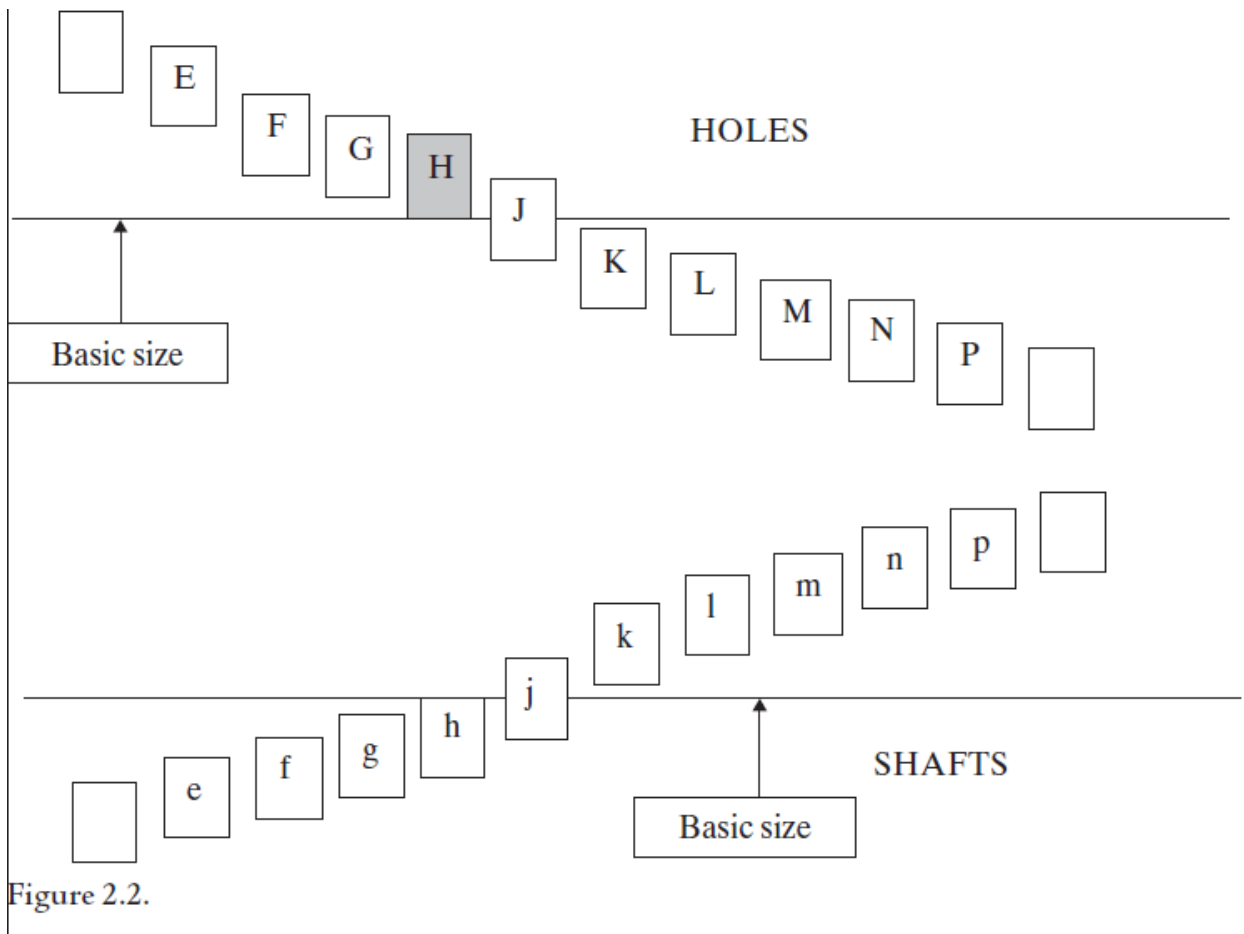


Figure 2.2.

Table 2.1
METRIC $1 \mu\text{m} = 0.001 \text{ mm}$

D1 < D ≤ D2		INTERNATIONAL TOLERANCE SYSTEM																		Unit = 0.001 mm (1 μm)	
D1	D2	IT01	IT0	IT1	IT2	IT3	IT4	IT5	IT6	IT7	IT8	IT9	IT10	IT11	IT12	IT13	IT14	IT15	IT16		
> 0	≤ 3	0.3	0.5	0.8	1.2	2	3	4	6	10	14	25	40	60	100	140	250	400	600		
3	6	0.4	0.6	1	1.5	2.5	4	5	8	12	18	30	48	75	120	180	300	480	750		
6	10	0.4	0.6	1	1.5	2.5	4	6	9	15	22	36	58	90	150	220	360	580	900		
10	18	0.5	0.8	1.2	2	3	5	8	11	18	27	43	70	110	180	270	430	700	1100		
18	30	0.6	1	1.5	2.5	4	6	9	13	21	33	52	84	130	210	330	520	840	1300		
30	50	0.6	1	1.5	2.5	4	7	11	16	25	39	62	100	160	250	390	620	1000	1600		
50	80	0.8	1.2	2	3	5	8	13	19	30	46	74	120	190	300	460	740	1200	1900		
80	120	1	1.5	2.5	4	6	10	15	22	35	54	87	140	220	350	540	870	1400	2200		
120	180	1.2	2	3.5	5	8	12	18	25	40	63	100	160	250	400	630	1000	1600	2500		
180	250	2	3	4.5	7	10	14	20	29	46	72	115	185	290	460	720	1150	1850	2900		
250	315	2.5	4	6	8	12	16	23	32	52	81	130	210	320	520	810	1300	2100	3200		
315	400	3	5	7	9	13	18	25	36	57	89	140	230	360	570	890	1400	2300	3600		
400	500	4	6	8	10	15	20	27	40	63	97	155	250	400	630	970	1550	2500	4000		
500	630	-	-	-	-	-	-	-	44	70	110	175	280	440	700	1100	1750	2800	4400		
630	800	-	-	-	-	-	-	-	50	80	125	200	320	500	800	1250	2000	3200	5000		
800	1000	-	-	-	-	-	-	-	56	90	140	230	360	560	900	1400	2300	3600	5600		
1000	1250	-	-	-	-	-	-	-	66	105	165	260	420	660	1050	1650	2600	4200	6600		
1250	1600	-	-	-	-	-	-	-	78	125	195	310	500	780	1250	1950	3100	5000	7800		
1600	2000	-	-	-	-	-	-	-	92	150	230	370	600	920	1500	2300	3700	6000	9200		
2000	2500	-	-	-	-	-	-	-	110	175	280	440	700	1100	1750	2800	4400	7000	11000		
2500	3150	-	-	-	-	-	-	-	135	210	330	540	860	1350	2100	3300	5400	8600	13500		

We observe that H for holes and h for shafts are with fundamental deviation zero. Holes with H designation are of basic size and larger and shafts with h designation are basic size and smaller.

We provide a table for typical recommended fits in Table 2.3.

The tolerance conditions on the hole and the shaft result in a *fit* in an assembly. If clearance occurs under all tolerance conditions we call it the *clearance fit*. If interference occurs under all tolerance conditions we call it the *interference fit*. If clearance or interference is determined by the actual size within the permissible tolerances, we call it the *transition fit*.

A fit between a hole and a shaft is conveniently determined by fixing the tolerance zone for one of the members and choosing different tolerance zones for the other. In the *hole basis* system all holes are designated H, and different shaft tolerances are used for the desired fit. In the *shaft basis* system, the shafts are fixed as h and the holes of various designations are used for different fits.

Suggested fits from ISO are given in Table 2.3.

Table 2.3: Description of Preferred Fits (from ANSI B4.2)

Hole Basis	Shaft Basis	Description
H11/c11	C11/h11	<i>Loose running</i> fit for wide commercial tolerances or allowances on external members.
H9/d9	D9/h9	<i>Free running</i> fit for use where accuracy is essential, but good for large temperature variations, high running speeds, or heavy journal pressures.
H8/f7	F8/h7	<i>Close running</i> fit for running on accurate machines and for accurate location at moderate speeds and journal pressures.
H7/g6	G7/h6	<i>Sliding</i> fit not intended to run freely, but to move and turn freely and locate accurately.
H7/h6	H7/h6	<i>Locational clearance</i> fit provides snug fit for locating stationary parts, but can be freely assembled and disassembled.
H7/k6	K7/h6	<i>Locational transition</i> fit for accurate location, a compromise between clearance and interference.

H7/n6	N7/h6	<i>Locational transition</i> fit for more accurate location where greater interference is permissible.
H7/p6	P7/h6	<i>Locational interference</i> fit for parts requiring rigidity and alignment with prime accuracy of location but without special bore pressure requirements.
H7/s6	S7/h6	<i>Medium drive</i> fit for ordinary steel parts or shrink fits on light sections, the tightest fit usable with cast iron.
H7/u6	U7/h6	<i>Force</i> fit suitable for parts which can be highly stressed or for shrink fits where the heavy pressing forces required are impractical.

The tolerances for these recommended fits are given in Tables 2.3A and 2.3B.

Table 2.3A. Tolerance Zones for Shafts (all units in μm $1 \mu\text{m} = 0.001 \text{ mm}$)

TOLERANCE ZONES FOR SHAFTS (FOR HOLE BASIS FITS)											
OVER	TO	c11	d9	f7	g6	h6	k6	n6	p6	s6	u6
0	3	-120/-60	-45/-20	-16/-6	-8/-2	-6/0	0/+6	+4/+10	+6/+12	+14/+20	+18/+24
3	6	-145/-70	-60/-30	-18/-10	-12/-4	-8/0	+1/+9	+8/+16	+12/+20	+19/+27	+23/+31
6	10	-170/-80	-76/-40	-28/-13	-14/-5	-9/0	+1/+10	+10/+19	+15/+24	+23/+32	+28/+37
10	14	-205/-95	-93/-50	-34/-16	-17/-6	-11/0	+1/+12	+12/+23	+18/+29	+28/+39	+33/+44
14	18	-205/-95	-93/-50	-34/-16	-17/-6	-11/0	+1/+12	+12/+23	+18/+29	+28/+39	+33/+44
18	24	-240/-110	-117/-65	-41/-20	-20/-7	-13/0	+2/+15	+15/+28	+22/+35	+35/+48	+41/+54
24	30	-240/-110	-117/-65	-41/-20	-20/-7	-13/0	+2/+15	+15/+28	+22/+35	+35/+48	+48/+61
30	40	-280/-120	-142/-80	-50/-25	-25/-9	-16/0	+2/+18	+17/+33	+26/+42	+43/+59	+60/+76
40	50	-290/-130	-142/-80	-50/-25	-25/-9	-16/0	+2/+18	+17/+33	+26/+42	+43/+59	+70/+86
50	65	-330/-140	-174/-100	-60/-30	-29/-10	-19/0	+2/+21	+20/+39	+32/+51	+53/+72	+87/+106
65	80	-340/-150	-174/-100	-60/-30	-29/-10	-19/0	+2/+21	+20/+39	+32/+51	+59/+78	+102/+121
80	100	-390/-170	-207/-120	-71/-36	-34/-12	-22/0	+3/+25	+23/+45	+37/+59	+71/+93	+124/+146
100	120	-400/-180	-207/-120	-71/-36	-34/-12	-22/0	+3/+25	+23/+45	+37/+59	+79/+101	+144/+166
120	140	-450/-200	-245/-145	-83/-43	-39/-14	-25/0	+3/+28	+27/+52	+43/+68	+92/+117	+170/+195
140	160	-460/-210	-245/-145	-83/-43	-39/-14	-25/0	+3/+28	+27/+52	+43/+68	+100/+125	+190/+215
160	180	-480/-230	-245/-145	-83/-43	-39/-14	-25/0	+3/+28	+27/+52	+43/+68	+108/+133	+210/+235
180	200	-530/-240	-285/-170	-96/-50	-44/-15	-29/0	+4/+33	+31/+60	+50/+79	+122/+151	+236/+265
200	225	-550/-260	-285/-170	-96/-50	-44/-15	-29/0	+4/+33	+31/+60	+50/+79	+130/+159	+258/+287
225	250	-570/-280	-285/-170	-96/-50	-44/-15	-29/0	+4/+33	+31/+60	+50/+79	+140/+169	+284/+313
250	280	-620/-300	-325/-190	-108/-56	-49/-17	-32/0	+4/+36	+34/+66	+56/+88	+158/+190	+315/+347
280	315	-650/-330	-325/-190	-108/-56	-49/-17	-32/0	+4/+36	+34/+66	+56/+88	+170/+202	+350/+382
315	355	-720/-360	-350/-210	-119/-62	-54/-18	-36/0	+4/+40	+37/+73	+62/+98	+190/+226	+390/+426
355	400	-760/-400	-350/-210	-119/-62	-54/-18	-36/0	+4/+40	+37/+73	+62/+98	+208/+244	+435/+471
400	450	-840/-440	-385/-230	-131/-68	-60/-20	-40/0	+5/+45	+40/+80	+68/+108	+232/+272	+490/+530
450	500	-880/-480	-385/-230	-131/-68	-60/-20	-40/0	+5/+45	+40/+80	+68/+108	+252/+292	+540/+580
		(Ref. ANSI B4.2-1978)				See Sheet 2 for Tolerance Zones for Holes					

Table 2.3B. Tolerance Zones for Holes (all units in μm $1 \mu\text{m} = 0.001 \text{ mm}$)

TOLERANCE ZONES FOR HOLES (FOR SHAFT BASIS FITS)											
OVER	TO	C11	D9	F8	G7	H7	K7	N7	P7	S7	U7
0	3	+60/+120	+20/+45	+6/+20	+2/+12	0/+10	-10/0	-14/-4	-16/-6	-24/-14	-28/-18
3	6	+70/+145	+30/+60	+10/+28	+4/+16	0/+12	-9/+3	-16/-4	-20/-8	-27/-15	-31/-19
6	10	+80/+170	+40/+76	+13/+35	+5/+20	0/+15	-10/+5	-19/-4	-24/-9	-32/-17	-37/-22
10	14	+95/+205	+50/+93	+16/+43	+6/+24	0/+18	-12/+6	-23/-5	-29/-11	-39/-21	-44/-26
14	18	+95/+205	+50/+93	+16/+43	+6/+24	0/+18	-12/+6	-23/-5	-29/-11	-39/-21	-44/-26
18	24	+110/+240	+65/+117	+20/+53	+7/+28	0/+21	-15/+6	-28/-7	-35/-14	-48/-27	-54/-33
24	30	+110/+240	+65/+117	+20/+53	+7/+28	0/+21	-15/+6	-28/-7	-35/-14	-48/-27	-61/-40
30	40	+120/+280	+80/+142	+25/+64	+9/+34	0/+25	-18/+7	-33/-8	-42/-17	-59/-34	-76/-51
40	50	+130/+290	+80/+142	+25/+64	+9/+34	0/+25	-18/+7	-33/-8	-42/-17	-59/-34	-86/-61
50	65	+140/+330	+100/+174	+30/+76	+10/+40	0/+30	-21/+9	-39/-9	-51/-21	-72/-42	-106/-76
65	80	+150/+340	+100/+174	+30/+76	+10/+40	0/+30	-21/+9	-39/-9	-51/-21	-78/-48	-121/-91
80	100	+170/+390	+120/+207	+36/+90	+12/+47	0/+35	-25/+10	-45/-10	-59/-24	-93/-58	-146/-111
100	120	+180/+400	+120/+207	+36/+90	+12/+47	0/+35	-25/+10	-45/-10	-59/-24	-101/-66	-166/-131
120	140	+200/+450	+145/+245	+43/+106	+14/+54	0/+40	-28/+12	-52/-12	-68/-28	-117/-77	-195/-155
140	160	+210/+460	+145/+245	+43/+106	+14/+54	0/+40	-28/+12	-52/-12	-68/-28	-125/-85	-215/-175
160	180	+230/+480	+145/+245	+43/+106	+14/+54	0/+40	-28/+12	-52/-12	-68/-28	-133/-93	-235/-195
180	200	+240/+530	+170/+285	+50/+122	+15/+61	0/+46	-33/+13	-60/-14	-79/-33	-151/-105	-265/-219
200	225	+260/+550	+170/+285	+50/+122	+15/+61	0/+46	-33/+13	-60/-14	-79/-33	-159/-113	-287/-241
225	250	+280/+570	+170/+285	+50/+122	+15/+61	0/+46	-33/+13	-60/-14	-79/-33	-169/-123	-313/-267
250	280	+300/+620	+190/+320	+56/+137	+17/+69	0/+52	-36/+16	-66/-14	-88/-36	-190/-138	-347/-255
280	315	+330/+650	+190/+320	+56/+137	+17/+69	0/+52	-36/+16	-66/-14	-88/-36	-202/-150	-382/-330
315	355	+360/+720	+210/+350	+62/+151	+18/+75	0/+57	-40/+17	-73/-16	-98/-41	-226/-169	-426/-369
355	400	+400/+760	+210/+350	+62/+151	+18/+75	0/+57	-40/+17	-73/-16	-98/-41	-244/-187	-471/-414
400	450	+440/+840	+230/+285	+68/+165	+20/+83	0/+63	-45/+18	-80/-17	-108/-45	-272/-209	-530/-467
450	500	+480/+880	+230/+285	+68/+165	+20/+83	0/+63	-45/+18	-80/-17	-108/-45	-292/-229	-580/-517
		(Ref. ANSI B4.2-1978)				See Sheet 2 for Tolerance Zones for Shafts					

Example. Make your choice for a sliding fit for basic size 50mm and provide the tolerances for the hole basis and shaft basis approaches.

Solution: The choice is 50 H7/g6 for hole basis and 50 G7/h6 for shaft basis.

Hole basis:

50 H7/g6 tolerances: From Table 2.3B, tolerance for 50H7 is +0 to +25 μm

From Table 2.3A, 50g6 is -9 to -25 μm

Shaft basis:

50 G7/h6 tolerances: From Table 2.3B, tolerance for 50G7 is +9 to +34 μm

From Table 2.3A, 50h6 is 0 to -16 μm

The tolerances on parts are related to the manufacturing processes.

Manufacturing Processes and Tolerances

Table 2.4. Manufacturing Processes and IT Grades (Ref: ANSI B4.2)

PROCESS	IT GRADE RANGE
Lapping and Honing	4 – 5
Grinding	5 – 8
Precision Turning and Boring	5 – 7
Broaching	5 – 8
Reaming	6 – 10
Powder Metallurgy (Sintered)	7 – 10
Turning and Boring	7 – 11
Milling, Planing, Shaping	10 – 11
Drilling	10 – 11
Punching, Blanking	10 – 11
Die Casting	10 – 12
Casting	14 – 16

Tolerance Selection in Assemblies

The tolerances on components are chosen at the design stage. An assembly has several components. If the components are stacked as shown in Fig. 2.3, the resulting dimension Y is the sum of the component dimensions.

$$Y = X_1 + X_2 + X_3 \quad (2.3)$$

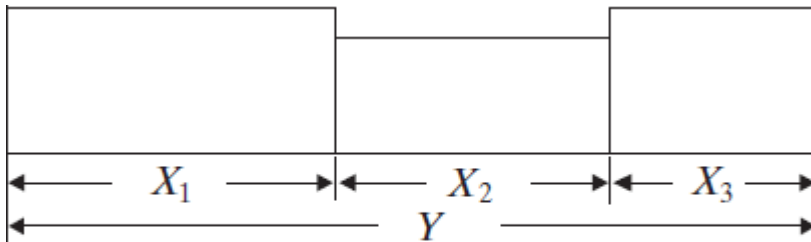


Figure 2.3.

Fig. 2.4 shows Y as a gap formed by the difference of dimensions X_1 and X_2 .

$$Y = X_1 - X_2 \quad (2.4)$$

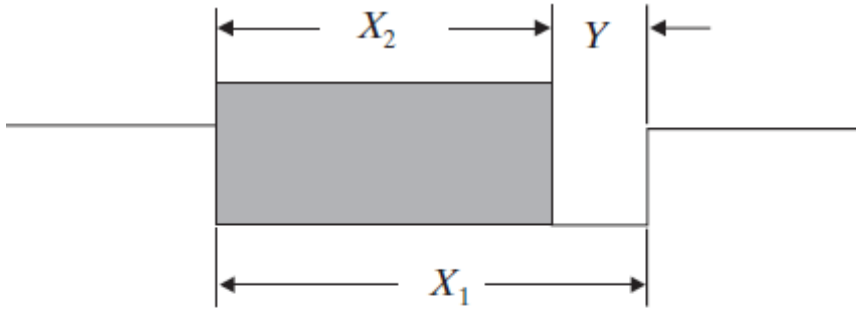


Figure 2.4.

We now denote t_Y as the tolerance on Y and t_1 , t_2 , and t_3 as tolerances on X_1 , X_2 , and X_3 .

$$t_Y^2 = t_1^2 + t_2^2 + t_3^2$$

For the assembly shown in Fig. 2.4, the tolerance relationship is

$$t_Y^2 = t_1^2 + t_2^2$$

We observe that *the square of the resulting tolerance is the sum of squares of the component tolerances*. Whether a dimension is added or subtracted in obtaining Y , the square of its tolerance is added to the right hand side of t_Y^2 . We now state this for a general case.

Let X_1, X_2, \dots, X_m be the m member dimensions. Let Y be the resulting dimension obtained by a linear combination of the member dimensions with coefficients $+1$ or -1 .

The resulting tolerance is

$$t_Y^2 = t_1^2 + t_2^2 + \dots + t_m^2 \quad (2.5)$$

Tolerance related problems may be classified into two categories.

- 1) If component tolerances t_1, t_2, \dots are known, what is the resulting tolerance t_Y ?

2) If final tolerance t_Y is known, how do we allocate it to the component tolerances?

The solution of the problem of the first type is straightforward and follows from (2.5).

We illustrate it by means of an example.

Example. Dimensions Y, X_1, X_2, X_3, X_4 are related by the relationship

$Y = X_1 + X_2 - X_3 + X_4$. If X_1, X_2, X_3 , and X_4 are specified as 20 ± 0.025 mm,

40 ± 0.045 mm, 30 ± 0.035 mm, 16 ± 0.015 mm, find the mean dimension of Y and specify its tolerance.

Solution: Since the mean dimensions of X_1, X_2, X_3, X_4 are given, we obtain the mean dimension of Y as

$$Y = 20 + 40 - 30 + 16 = 46 \text{ mm}$$

The tolerances on the four dimensions are $t_1 = 0.05$ mm, $t_2 = 0.09$ mm, $t_3 = 0.07$ mm, $t_4 = 0.03$ mm. Using equation (2.5),

$$t_Y = \sqrt{t_1^2 + t_2^2 + t_3^2 + t_4^2} = \sqrt{0.05^2 + 0.09^2 + 0.07^2 + 0.03^2} = 0.1281$$

The tolerance on Y may be specified as $Y \pm t_Y/2$, which is 46 ± 0.064 mm.

The problems of the second type where t_Y is known and the component tolerances are to be found need special considerations. We first present an approach which is recommended when the component dimensions are nearly equal, say the largest

dimension is about 20% bigger than the smallest. We call this the equal tolerance approach. The second approach is more rigorous and is applicable for wider variations in tolerances.

Equal Tolerance Approach

In this approach we set all component tolerances equal

$$t = t_1 = t_2 = \dots = t_m \quad (2.6)$$

Then from (2.7) we get

$$t = \frac{t_Y}{\sqrt{m}} \quad (2.7)$$

Example. A gap of 0.2mm is to be maintained in an assembly with a tolerance of $\pm 16 \mu\text{m}$. The gap is arrived at as a combination of two dimensions as follows

$$0.2 = 30.5 - 30.3$$

determine the tolerances on each dimension.

Solution: The component dimensions are nearly equal in this problem. We use the equal allocation strategy. We have $t_Y = 32 \mu\text{m}$. The number of component dimensions $m = 2$.

From (2.7), we get

$$t = \frac{t_Y}{\sqrt{m}} = \frac{32}{\sqrt{2}} = 22.63 \mu\text{m}$$

The tolerances may be set as $30.5 \pm 11.3 \mu\text{m}$, and $30.3 \pm 11.3 \mu\text{m}$

Unit Tolerance Approach

Equation (2.1) defines the tolerance unit $i = 0.45D^{\frac{1}{3}} + 0.001D$ μm used for a dimension D mm. We use the following notation

$$i_k = 0.45X_k^{\frac{1}{3}} + 0.001X_k \quad X_k, k = 1, 2, \dots, m \quad (2.8)$$

We assume that all members have the same the international tolerance grades. We may then assume that the tolerance of a dimension is proportional to the unit tolerance. Let C be the constant of proportionality so that

$$t_k = Ci_k \quad k = 1, 2, \dots, m \quad (2.9)$$

We evaluate C by substituting in (2.5) and rearranging the terms,

$$C = \frac{t_Y}{\sqrt{i_1^2 + i_2^2 + \dots + i_m^2}} \quad (2.10)$$

Tolerances are then placed appropriately with respect to the basic dimensions. Note that C is a constant of proportionality so that the tolerance will get the same units as t_Y , however the dimensions must be in mm units for the applicability of (2.8).

This weighted approach gives a good allocation that is consistent with the international tolerance system.

Example. Dimension Y is obtained as the composite of dimensions X_1, X_2, \dots, X_4 , as

$$Y = X_1 + X_2 - X_3 + X_4$$

The tolerance on Y desired is ± 0.4 mm and $X_1 = 30$ mm, $X_2 = 40$ mm, $X_3 = 20$ mm, $X_4 = 10$ mm. Determine the tolerances on each dimension.

Solution: The smallest dimension is 10 mm and the largest is 40 mm. Since the variation is large, we use the unit tolerance approach for the tolerance allocation. The unit tolerance values for the dimensions are

$$i_1 = 0.45\sqrt[3]{30} + 0.001 \times 30 = 1.4283$$

$$i_2 = 0.45\sqrt[3]{40} + 0.001 \times 40 = 1.579$$

$$i_3 = 0.45\sqrt[3]{20} + 0.001 \times 20 = 1.2415$$

$$i_4 = 0.45\sqrt[3]{10} + 0.001 \times 10 = 0.9795$$

Now from (2.10), we get

$$C = \frac{t_y}{\sqrt{i_1^2 + i_2^2 + \dots + i_m^2}} = \frac{0.8}{\sqrt{1.4283^2 + 1.579^2 + 1.2415^2 + 0.9795^2}} = 0.3031$$

The tolerances are obtained using (2.9),

$$t_1 = Ci_1 = 0.432 \text{ mm}$$

$$t_2 = Ci_2 = 0.4785 \text{ mm}$$

$$t_3 = Ci_3 = 0.3763 \text{ mm}$$

$$t_4 = Ci_4 = 0.297 \text{ mm}$$

We use the symmetric placement to arrive at the dimensions $30 \pm 0.216 \text{ mm}$,

$40 \pm 0.24 \text{ mm}$, $20 \pm 0.19 \text{ mm}$, $10 \pm 0.15 \text{ mm}$

Conclusion