Gauss- Legendre Numerical Integration T.R. Chandrupatla

We convert integral on interval [a b] for *x* to interval [-1 1] for ξ.

$$x = \frac{b+a}{2} + \left(\frac{b-a}{2}\right)\xi$$

$$dx = \left(\frac{b-a}{2}\right)d\xi$$

Integral

$$I = \int_a^b f(x)dx = \frac{b-a}{2}\int_{-1}^{1} f\left(\frac{b+a}{2} + \left(\frac{b-a}{2}\right)\xi\right)d\xi$$

Integration Table

Number of Points	Location ξ	Weights
1	0	2.0
2	+- 0.57735027	1.0
3	+- 0.77459667 0	0.5555556 0.8888889

n integraion points $\xi_1, \xi_2, \ldots, \xi_n,$ and weights $w_1, w_2, \ldots, w_n$.

Approximate integral is given by

$$I = \frac{b-a}{2}\left(w_1 f\left(\frac{b+a}{2} + \left(\frac{b-a}{2}\right)\xi_1\right) + w_2 f\left(\frac{b+a}{2} + \left(\frac{b-a}{2}\right)\xi_2\right) + \ldots + w_n f\left(\frac{b+a}{2} + \left(\frac{b-a}{2}\right)\xi_n\right)\right)$$

Example:

Find $I = \int_0^\pi \sin(\theta)d\theta$ using 2 point integration

Solution: $$I = \frac{\pi - 0}{2}\left(1.\sin\left(\frac{\pi}{2} - 0.57735027\frac{\pi}{2}\right) + 1.\sin\left(\frac{\pi}{2} + 0.57735027\frac{\pi}{2}\right)\right)$$

$$= 1.93582$$

The exact solution is $I = -\cos\theta_0^p = 1 + 1 = 2$.

Error is (1.93582-2)/2 = -3.21%