ports are so low that the stiffness ratio α has a value close to 0.2, gradually approaching the condition of a flat plate. It is not applicable when $\alpha = 0$. If it is to be used in the latter condition, however, we can assume that part of the slab in the column region acts as a beam. Thickness h cannot be less than the following values:

Slabs without beams or drop panels	5 in.
Slabs without beams, but with drop panels	4 in.
Slabs with beams on all four edges with a value of α_{fm} at least equal to 2.0	3.5 in.

h also has to be increased by at least 10% for flat-plate floors if the end panels have no edge beams and by 45% for corner panels.

In addition, in the equations above,

α = ratio of flexural stiffness of beam section to flexural stiffness of a width of slab bounded laterally by the center line of the adjacent panel (if any) on each side of the beam

α_{fm} = average value of α for all beams on edges of a panel

β = ratio of clear spans in long to short direction of two-way slabs

It has to be emphasized that a deflection check is critical for the construction loading condition. Shoring and reshoring patterns can result in dead-load deflection in excess of the normal service-load state at a time when the concrete has only a 7-day strength or less and not the normal design 28-day strength. The stiffness EI in such a state is less than the design value. Flexural cracking lowers further the stiffness values of the two-way slab or plate, with a possible increase in long-term deflection several times the anticipated design deflection. Consequently, reinforced concrete two-way slabs and plates have to be constructed with a camber of $\frac{3}{8}$ in. in 10-ft span or more and crack control exercised as in Section 11.9 in order to counter the effects of excessive deflection at the construction loading stage. An analysis for the construction load stresses and deflections is important in most cases.

It should be noted that while the ACI Code stipulates the use of $f_r = 7.5\sqrt{f_c'}$ for the modulus of rupture in computing the cracking moment, M_{cr}, it is advisable that a lower value than 7.5 in the expression for f_r be used, such as 4.0–4.5. In this manner, the possibility is avoided of unanticipated deflection of a two-way slab larger than what the ACI deflection tables present.

11.5 DESIGN AND ANALYSIS PROCEDURE: DIRECT DESIGN METHOD

11.5.1 Operational Steps

Figure 11.9 gives a logic flowchart for the following operational steps:

1. Determine whether the slab geometry and loading allow the use of the direct design method as listed in Section 11.3.1.
2. Select slab thickness to satisfy deflection and shear requirements. Such calculations require a knowledge of the supporting beam or column dimensions. A reasonable value of such a dimension of columns or beams would be 8 to 15% of the average of the long and short span dimensions, that is, $\frac{1}{2}(l_1 + l_2)$. For shear check, the critical section is at a distance $d/2$ from the face of the support. If the thickness shown for deflection is not adequate to carry the shear, use one or more of the following:
   (a) Increase the column dimension.
   (b) Increase concrete strength.
   (c) Increase slab thickness.

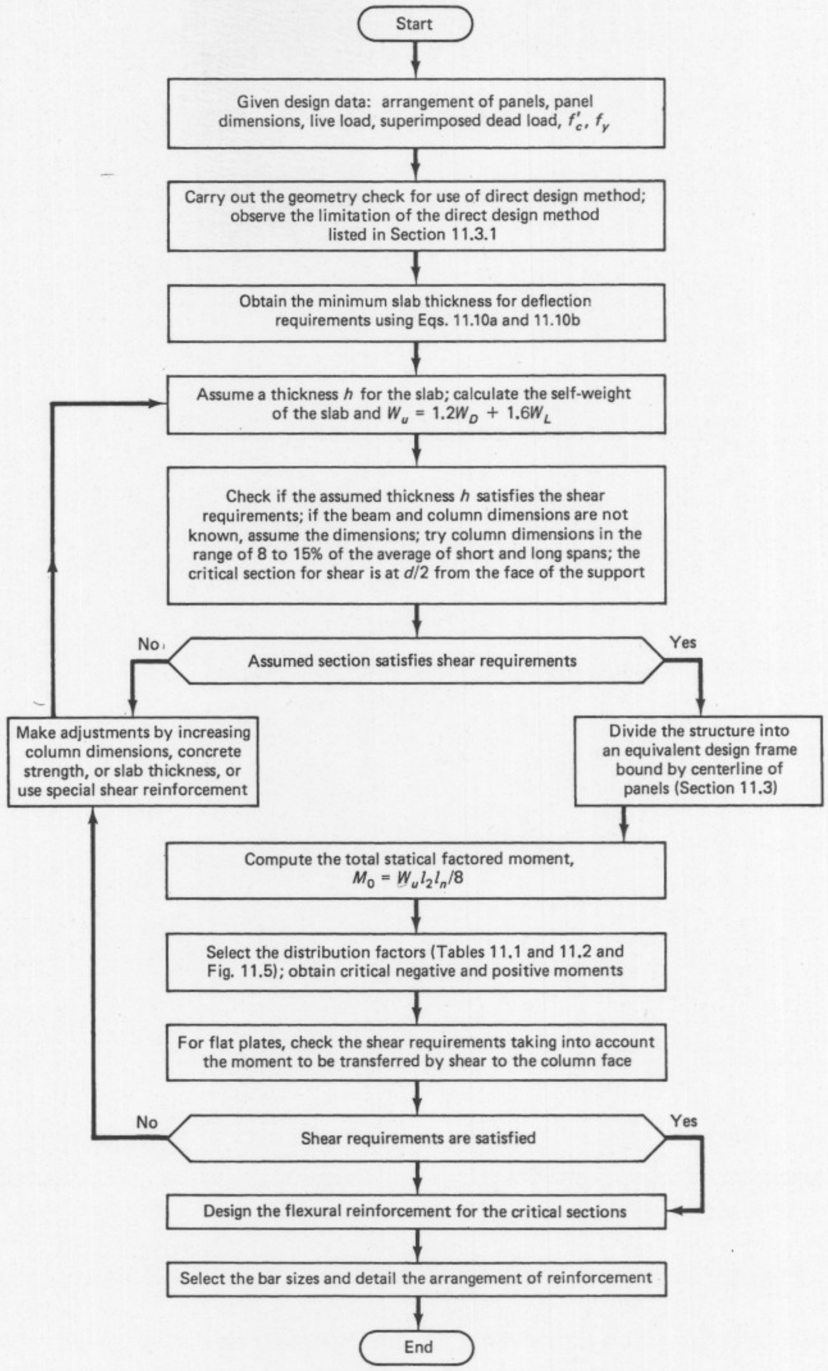

Figure 11.9 Flowchart for design sequence in two-way slabs and plates by the direct design method.

(d) Use special shear reinforcement.
(e) Use drop panels or column capitals to improve shear strength.

3. Divide the structure into equivalent design frames bound by center lines of panels on each side of a line of columns.
4. Compute the total statical factored moment $M_0 = (w_u l_2 l_{n1}^2)/8$.
5. Select the distribution factors of the negative and positive moments to the exterior and interior columns and spans as in Figure 11.3b and Table 11.1 and calculate the respective factored moments.
6. Distribute the factored equivalent frame moments from step 5 to the column and middle strips.
7. Determine whether the trial slab thickness chosen is adequate for moment-shear transfer in the case of flat plates at the column junction computing that portion of the moment transferred by shear and the properties of the critical shear section at distance $d/2$ from column face.
8. Design the flexural reinforcement to resist the factored moments in step 6.
9. Select the size and spacing of the reinforcement to fulfill the requirements for crack control, bar development lengths, and shrinkage and temperature stresses.

11.5.2 Example 11.1: Design of Flat Plate without Beams by the Direct Design Method

A three-story building is four panels by four panels in plan. The clear height between the floors is 12 ft, and the floor system is a reinforced concrete flat-plate construction with no edge beams. The dimensions of the end panels as well as the size of the supporting columns are shown in Figure 11.10. Given:

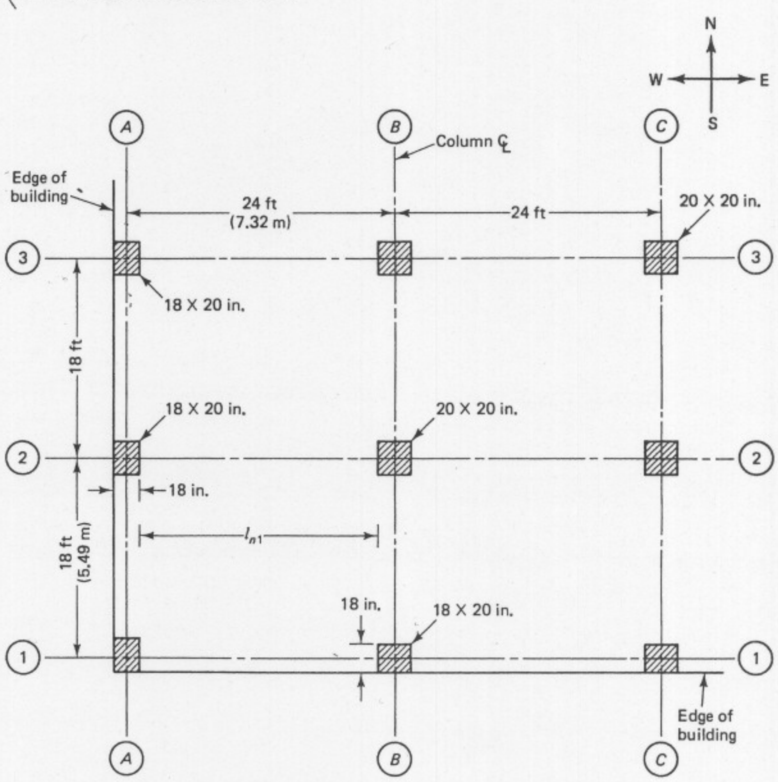

Figure 11.10 Floor plan of end panels in a three-story building.

$$\text{live load} = 70 \text{ psf } (3.35 \text{ kPa})$$

$$f'_c = 4000 \text{ psi } (27.6 \text{ MPa}), \text{ normal-weight concrete}$$

$$f_y = 60{,}000 \text{ psi } (414 \text{ MPa})$$

The building is not subject to earthquake; consider gravity loads only. Design the end panel and the size and spacing of the reinforcement needed. Consider flooring weight to be 10 psf in addition to the floor self-weight.

Solution: *Geometry check for use of direct design method (step 1)*

(a) Ratio longer span/shorter span = 24/18 = 1.33 < 2.0, hence two-way action
(b) More than three spans in each direction and successive spans in each direction the same and columns are not offset.
(c) Assume a thickness of 9 in. and flooring of 10 psf.

$$w_d = 10 + \frac{9}{12} \times 150 = 122.5 \text{ psf} \qquad 2w_d = 245 \text{ psf}$$

$$w_l = 70 \text{ psf} < 2w_d \qquad \text{O.K.}$$

Hence the direct design method is applicable.

Minimum slab thickness for deflection requirement (step 2)

$$\text{E–W direction } l_{n1} = 24 \times 12 - \frac{18}{2} - \frac{20}{2} = 269 \text{ in. } (6.83 \text{ m})$$

$$\text{N–S direction } l_{n2} = 18 \times 12 - \frac{20}{2} - \frac{20}{2} = 196 \text{ in. } (4.98 \text{ m})$$

$$\text{ratio of longer to shorter clear span } \beta = \frac{269}{196} = 1.37$$

Minimum preliminary thickness h from Table 11.3 for a flat plate without edge beams or drop panels using f_y = 60,000-psi steel is $h = l_n/30$, to be increased by at least 10% when no edge beam is used.

$$\text{E–W:} \quad l_n = 269 \text{ in.}$$

$$h = \frac{269}{30} \times 1.1 = 9.86 \text{ in.}$$

Try a slab thickness h = 10 in. This thickness is larger than the absolute minimum thickness of 5 in. required in the code for flat plates; hence O.K. Assume that $d \simeq h - 1$ in. = 9 in.

$$\text{new } w_d = 10 + \frac{10}{12} \times 150 = 135.0 \text{ psf}$$

Therefore,

$$2w_d = 270 \text{ psf}$$

$$w_l = 70 \text{ psf} < 2w_d \qquad \text{O.K.}$$

Shear thickness requirement (step 2)

$$w_u = 1.6L + 1.2D = 1.6 \times 70 + 1.2 \times 135.0$$

$$= 274 \text{ psf } (13.12 \text{ kPa})$$

Interior column: The controlling critical plane of maximum perimetric shear stress is at a distance $d/2$ from the column faces; hence, the net factored perimetric shear force is

$$V_u = [(l_1 \times l_2) - (c_1 + d)(c_2 + d)]w_u$$

$$= \left(18 \times 24 - \frac{20 + 9}{12} \times \frac{20 + 9}{12}\right)274 = 116{,}768$$

$$V_n = \frac{V_u}{\phi} = \frac{116{,}768}{0.75} = 155{,}691 \text{ lb}$$

From Figure 11.11, the perimeter of the critical shear failure surface is

$$b_0 = 2(c_1 + d + c_2 + d) = 2(c_1 + c_2 + 2d)$$

$$\text{perimetric shear surface } A_c = b_0 d = 2d(c_1 + c_2 + 2d) = 2 \times 9.0(20 + 20 + 18)$$

$$= 116 \times 9 = 1044 \text{ in.}^2 \ (673{,}400 \text{ mm}^2)$$

Since moments are not known at this stage, only a preliminary check for shear can be made.

$$\beta = \text{ratio of longer to shorter side of columns} = \frac{20}{20} = 1.0$$

Available nominal shear V_c is the least of

$$V_c = \left(2 + \frac{4}{\beta}\right)\sqrt{f'_c}b_0 d = \left(2 + \frac{4}{1}\right)\sqrt{4000} \times 1044 = 369{,}170 \text{ lb } (1.64 \times 10^3 \text{ kN})$$

or

$$V_c = \left(\frac{\alpha_s d}{b_0} + 2\right)\sqrt{f'_c}b_0 d = \left(\frac{40 \times 9}{116} + 2\right)\sqrt{4000} \times 1044 = 336{,}972 \text{ lb } (1.5 \times 10^3 \text{ kN})$$

or

$$V_c = 4\sqrt{f'_c}\, b_0 d = 4\sqrt{4000} \times 1044 = 264{,}113 \text{ lb } (1.16 \times 10^3 \text{ kN})$$

$$\text{controlling } V_c = 264{,}113 \text{ lb} > \text{required } V_n = 155{,}691 \text{ lb}$$

Hence the floor thickness is adequate. Note that because the preliminary forgoing check for the shear V_c at this stage does not take into account the shear transferred by moment, it is prudent to recognize that the chosen trial slab thickness would have to be larger than what the gravity V_n requires. As a guideline, in the case of interior columns, a thickness based on about

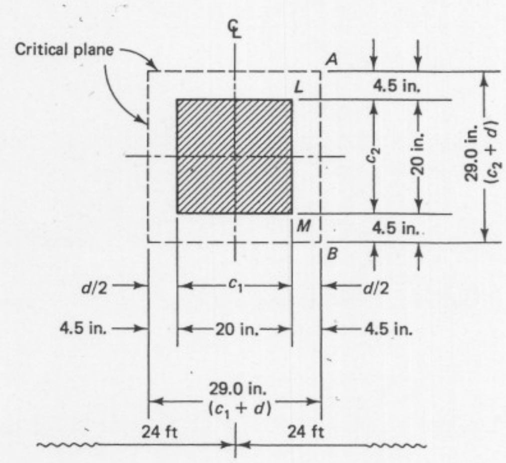

Figure 11.11 Critical plane for shear moment transfer in Ex. 11.1 interior column (line B–B, Fig. 11.10).

1.2 V_n applies in the case of interior columns. For end columns, a recommended multiplier for V_n might have to be as high as 1.6–1.8, and for corner columns, a higher value is applicable. Often, shear heads or drop panels are necessitated for corner columns to overcome too large a required thickness of the slab. As the serviceability tabulated values in Table 11.3 for minimum thickness of slabs apply only to the interior column zones, to be augmented by 10–15% for end columns and almost 50% for corner columns, they indirectly take into account the above stipulations for choosing trial slab thickness based on augmenting V_c, as was done at the outset in basing the choice of the slab thickness on augmenting the Table 11.3 value by 10 percent.

Exterior column: Include weight of exterior wall, assuming its service weight to be 270 plf. Net factored perimetric shear force is

$$V_u = \left[18 \times \left(\frac{24}{2} + \frac{18}{2 \times 12}\right) - \frac{(18 + 4.50)(20 + 9.0)}{144}\right]274$$

$$+ \left(18 - \frac{20}{12}\right) \times 270 \times 1.2 = 66{,}933 \text{ lb} \qquad V_n = \frac{66{,}933}{0.75} = 89{,}244 \text{ lb}$$

Consider the line of action of V_u to be at the column face LM in Figure 11.12 for shear moment transfer to the centroidal plane c–c. This approximation is adequate since V_u acts *perimetrically* around the column faces and not along line AB only. From Figure 11.12,

$$A_c = d(2c_1 + c_2 + 2d) = 9.0(2 \times 18 + 20 + 18) = 9 \times 74$$
$$= 666 \text{ in.}^2 \ (429{,}700 \text{ mm}^2)$$

Available nominal shear V_c is the least of

$$V_c = \left(2 + \frac{4}{\beta}\right)\sqrt{f'_c}\,b_0 d = \left(2 + \frac{4}{20/18}\right)\sqrt{4000} \times 666 = 235{,}881 \text{ lb } (1.05 \times 10^3 \text{ kN})$$

or

$$V_c = \left(\frac{\alpha_s d}{b_0} + 2\right)\sqrt{f'_c}\,b_0 d = \left(\frac{30 \times 9}{74} + 2\right)\sqrt{4000} \times 666 = 237{,}930 \text{ lb } (1.06 \times 10^3 \text{ kN})$$

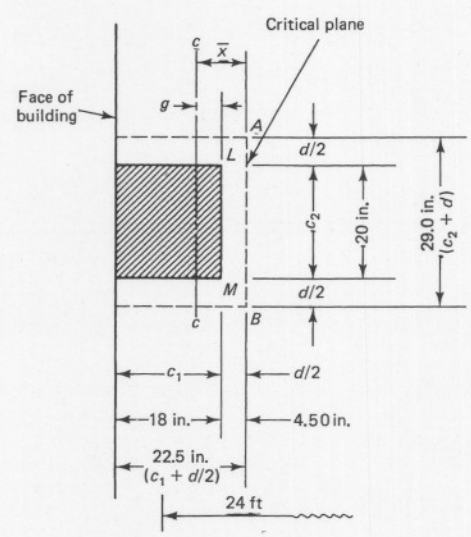

Figure 11.12 Centroidal axis for shear moment transfer in Ex. 11.1 end column (line A–A or 1–1, Fig. 11.10).

where $\alpha_s = 30$ for edge column

or

$$V_c = 4\sqrt{f'_c}b_0 d = 4\sqrt{4000} \times 666 = 168{,}486 \text{ lb } (0.75 \times 10^3 \text{ kN}) \qquad \text{controls}$$

$> \text{required } V_n = 89{,}244 \text{ lb}$ O.K.

Statical moment computation (steps 3 to 5)

$$\text{E–W:} \quad l_{n1} = 269 \text{ in.} = 22.42 \text{ ft}$$

$$\text{N–S:} \quad l_{n2} = 196 \text{ in.} = 16.33 \text{ ft}$$

$$0.65l_1 = 0.65 \times 24 = 15.6 \text{ ft} \qquad \text{Use } l_{n1} = 22.4 \text{ ft}$$

$$0.65l_{n2} = 0.65 \times 18 = 11.7 \text{ ft} \qquad \text{Use } l_{n2} = 16.33 \text{ ft}$$

(a) *E–W direction*

$$M_0 = \frac{w_u l_2 l_{n1}^2}{8} = \frac{274 \times 18(22.42)^2}{8} = 309{,}888 \text{ ft-lb } (420 \text{ kN-m})$$

For end panel of a flat plate without end beams, the moment distribution factors as in Table 11.1 are

$$-M_u \text{ at first interior support} = 0.70M_0$$

$$+M_u \text{ at midspan of panel} = 0.52M_0$$

$$-M_u \text{ at exterior face} = 0.26M_0$$

$$\text{negative design moment } -M_u = 0.70 \times 309{,}888$$

$$= 216{,}922 \text{ ft-lb } (295 \text{ kN-m})$$

$$\text{positive design moment } +M_u = 0.52 \times 309{,}888$$

$$= 161{,}142 \text{ ft-lb } (218 \text{ kN-m})$$

$$\text{negative moment at exterior } -M_u = 0.26 \times 309{,}888$$

$$= 80{,}571 \text{ ft-lb } (109 \text{ kN-m})$$

(b) *N–S direction*

$$M_0 = \frac{w_u l_1 l_{n2}^2}{8} = \frac{274 \times 24(16.33)^2}{8} = 219{,}202 \text{ ft-lb } (298 \text{ kN-m})$$

$$\text{negative design moment } -M_u = 0.70 \times 219{,}202$$

$$= 153{,}441 \text{ ft-lb } (208 \text{ kN-m})$$

$$\text{positive design moment } +M_u = 0.52 \times 219{,}202$$

$$= 113{,}985 \text{ ft-lb } (77 \text{ kN-m})$$

$$\text{negative design moment at exterior face } -M_{u1} = 0.26 \times 219{,}202$$

$$= 56{,}993 \text{ ft-lb } (75 \text{ kN-m})$$

Note that the smaller moment factor 0.35 could be used for the positive factored moment in the N–S direction in this example if the exterior edge is fully restrained. For the N–S direction, panel BC12 was considered.

Moment distribution in the column and middle strips (steps 6 and 7)

At the exterior column there is no torsional edge beam; hence the torsional stiffness ratio β_t of an edge beam to the columns is zero. Hence $\alpha_1 = 0$. From the *exterior* factored moments tables for the column strip in Section 11.4.2, the distribution factor for the negative moment at the exterior support is 100%, the positive midspan moment is 60%, and the interior negative moment is 75%. Table 11.5 gives the moment values resulting from the moment distributions to the column and middle strips.

Table 11.5 Moment Distribution Operations Table

	E–W Direction			N–S Direction		
l_2/l_1:	18/24 = 0.75			24/18 = 1.33		
$\alpha_1(l_2/l_1)$:	0			0		
Column strip	Interior negative moment	Positive midspan moment	Exterior negative moment	Interior negative moment	Positive midspan moment	Exterior negative moment
M_u (ft-lb)	216,922	161,142	80,571	153,441	113,985	56,993
Distribution factor(%)	75	60	100	75	60	100
Column strip design moments (ft-lb)	0.75 × 216,922 = 162,692	0.60 × 161,142 = 96,685	1.0 × 80,571 = 80,571	0.75 × 153,441 = 115,081	0.60 × 113,985 = 68,391	1.0 × 56,993 = 56,993
Middle strip design moments (ft-lb)	216,922 − 162,692 = 54,230	161,142 − 96,685 = 64,457	80,571 − 80,571 = 0	153,441 − 115,081 = 38,360	113,985 − 68,391 = 45,594	56,993 − 56,993 = 0

Check the shear moment transfer capacity at the exterior column supports

$$-M_c \text{ at interior column 2–}B = 216{,}922 \text{ ft-lb}$$
$$-M_e \text{ at exterior column 2–}A = 80{,}571 \text{ ft-lb}$$
$$V_u = 66{,}933 \text{ lb acting at the face of the column}$$

The ACI Code stipulates that the nominal moment strength be used in evaluating the unbalanced transfer moment at the edge column; that is, use M_n based on $-M_e = 80{,}571$ ft-lb. Factored shear force at the edge column adjusted for the interior moment is

$$V_u = 66{,}933 - \frac{216{,}922 - 80{,}571}{24 - \dfrac{9 + 10}{12}} = 60{,}850 \text{ lb}$$

$V_n = 60{,}850/0.75 = 81{,}133$ lb, assuming that the design M_u has the same value as the factored M_u.

$$A_c \text{ from before} = 666 \text{ in.}^2$$

From Figures 11.7c and 11.12, taking the moment of area of the critical plane about axis *AB*,

$$d(2c_1 + c_2 + 2d)\bar{x} = d\left(c_1 + \frac{d}{2}\right)^2$$

where $\bar{x}$ is the distance to the centroid of the critical section or

$$(2 \times 18 + 20 + 18)\bar{x} = \left(18 + \frac{9.0}{2}\right)^2$$

$$\bar{x} = \frac{506.25}{74} = 6.84 \text{ in. (174 mm)}$$

and $g = 6.84 - 9.0/2 = 2.34$ in, where g is the distance from the column face to the centroidal axis of the section.

To transfer the shear V_u from the face of column to the centroid of the critical section adds an additional moment to the value of $M_e = 80{,}571$ ft-lb. Therefore, the total external factored moment $M_{ue} = 80{,}571 + 60{,}850(2.34/12) = 92{,}437$ ft-lb. Total required minimum unbalanced moment strength is

$$M_n = \frac{M_{ue}}{\phi} = \frac{92{,}437}{0.90} = 102{,}708 \text{ ft-lb}$$

The fraction of nominal moment strength M_n to be transferred by shear is

$$\gamma_v = 1 - \frac{1}{1 + \frac{2}{3}\sqrt{b_1/b_2}} = 1 - \frac{1}{1 + 0.59} = 0.37$$

where $b_1 = c_1 + d/2 = 18 + 4.5 = 22.5$ in. and $b_2 = c_2 + d = 20 + 9 = 29$ in. It should be noted that the dimension $c_1 + d$ for the end column in the above expression becomes $c_1 + d/2$. Hence $M_{nv} = 0.37\ M_n$. Moment of inertia of sides parallel to the moment direction about *N–S* axis is

$$I_1 = \left(\frac{bh^3}{12} + Ar^2 + \frac{hb^3}{12}\right)2 \qquad \text{for both faces}$$

$$I_1 = \left[\frac{9.0(22.5)^3}{12} + (9.0 \times 22.5)\left(\frac{22.50}{2} - 6.84\right)^2 + \frac{22.5(9.0)^3}{12}\right]2$$

$$= (8543 + 3938 + 1367)2 = 27{,}696 \text{ in.}^4$$

Moment of inertia of sides perpendicular to the moment direction about *N–S* axis is

$$I_2 = A(\bar{x})^2 = [(20 + 9.0)9.0](6.84)^2 = 12{,}211 \text{ in.}^4$$

Therefore,

$$\text{torsional moment of inertia } J_c = 27{,}696 + 12{,}211$$
$$= 39{,}907 \text{ in.}^4$$

If Eq. 11.7e is used instead from first principle calculations, as shown above, the same value $J_c = 39{,}907$ in.^{4} is obtained.

Shearing stress due to perimeter shear, effect of M_n, and weight of wall is

$$v_n = \frac{V_u}{\phi A_c} + \frac{\gamma_v c_{AB} M_n}{J_c}, \qquad \text{where } M_{nv} = \gamma_v \times M_n$$

$$= \frac{60{,}850}{0.75 \times 666} + \frac{0.37 \times 6.84 \times 102{,}708 \times 12}{39{,}907}$$

$$= 121.8 + 78.2 = 200.0 \text{ psi}$$

From before, maximum allowable $v_c = 4\sqrt{f'_c} = 4\sqrt{4000} = 253.0$ psi and

$$v_n < v_c$$

Therefore, accept plate thickness. For the corner panel column, special shear-head provision or an enlarged column or capital might be needed to resist the high-shear stresses at that location.

Check for shear-moment transfer at the interior column joint:

$$w_{nl} = 1.6 \times 70 = 112 \text{ psf.}$$

From Eq. 11.8 (c):

$$M_{ue} = 0.07\,(0.5 \times 112 \times 18.0)(22.42)^2 = 35{,}467 \text{ ft-lb}$$

$$V_n = 155{,}691 + \frac{66{,}933 - 60{,}850}{0.75} = 163{,}802 \text{ lb at the face of the column}$$

$$J_c = \frac{9(29)^3}{6} + \frac{(9)^3 \times 29}{6} + \frac{9 \times 29(29)^2}{2} = 149{,}858 \text{ in.}^4$$

$$g = 20/2 = 10 \text{ in.}$$

$$\text{Total unbalanced moment } M_n = \frac{35{,}467 \times 12}{0.9} + 163{,}802 \times 10 = 2{,}110{,}913 \text{ in.-lb}$$

$$\gamma_v = 1 - \frac{1}{1 + 2/3\sqrt{1}} = 0.4$$

$$\text{Hence, actual } v_n = \frac{163{,}802}{1044} + \frac{0.40 \times 2{,}110{,}913 \times (29/2)}{149{,}858} = 156.9 + 81.7 = 238.6 \text{ psi} \angle 253 \text{ psi,}$$

O.K.

Then proceed in the same manner as that used for the end column for choosing the concentrated reinforcement in the column zone at the slab top to account for the $\gamma_f M_n$ moment to be transferred in flexure. Use straight bars over the column extending over the two adjacent spans with full development length.

Design of reinforcement in the slab area at column face for the unbalanced moment transferred to the column by flexure

From Eq. 11.6b,

$$\gamma_f = 1 - \gamma_v = 1 - 0.37 = 0.63$$

$$M_{nf} = \gamma_f M_n = 0.63 \times 102{,}708 \times 12 = 776{,}472 \text{ in.-lb}$$

This moment has to be transferred within $1.5h$ on each side of the column as in Figure 11.7d.

$$\text{transfer width} = (1.5 \times 10.0)2 + 20 = 50.0 \text{ in.}$$

$$M_{nf} = A_s f_y\left(d - \frac{a}{2}\right) \qquad \text{assume that} \left(d - \frac{a}{2}\right) \simeq 0.9d$$

or $776{,}472 = A_s \times 60{,}000(9.0 \times 0.9)$ gives

$$A_s = 1.60 \text{ in.}^2 \text{ over a strip width} = 50.0 \text{ in.}$$

Verifying A_s,

$$a = \frac{1.60 \times 60{,}000}{0.85 \times 4{,}000 \times 50.0} = 0.56 \text{ in.}$$

Therefore,

$$776{,}472 = A_s \times 60{,}000\left(9.0 - \frac{0.56}{2}\right)$$

$$A_s = 1.48 \text{ in.}^2 \simeq 5 \text{ No. 5 bars at 4 in. c-c in the moment-transfer zone}$$

For the total negative M_e (assigned totally to the negative strip – see Table 11.6), $M_n = 80{,}571 / 0.9 = 89{,}523$ ft-lb. The computed $(d \times a/2) = 8.6$ in. Hence $A_s = 89{,}523 \times 12 / (60{,}000 \times 8.6) = 2.08$ in.^{2} Consequently, reinforcement outside the column moment-transfer 50 in. wide zone is $A_s = 2.08 - 1.48 = 0.60$ in.^{2} over a segment = 9 ft less 50 in. = 4.83 ft. Minimum temperature reinforcement = 0.0018 b h / ft = $0.0018 \times 12 \times 10 = 0.216$ in.^{2}, requiring 5 No. 4 bars at the right and left sides of the end column. The 5 No. 5 bars for the M_{nf} portion of the total end negative moment is concentrated within the 20 in. colmun width in order to efficiently develop the required development length (see Fig. 11.13).

Checks have to be made in a similar manner for the shear-moment transfer at the face of the interior column *C*. As also described in Section 11.4.5.2, checks are sometimes necessary for pattern loading conditions and for cases where adjoining spans are not equal or not equally loaded.

Proportioning of the plate reinforcement (steps 8 and 9)

(a) *E–W direction (long span)*

1. *Summary of moments in column strip (ft-lb)*

$$\text{interior column negative } M_n = \frac{162{,}692}{\phi = 0.9} = 180{,}769$$

$$\text{midspan positive } M_n = \frac{96{,}685}{0.9} = 107{,}428$$

$$\text{exterior column negative } M_{ne} = \frac{80{,}571}{0.9} = 89{,}523$$

2. *Summary of moments in middle strip (ft-lb)*

$$\text{interior column negative } M_n = \frac{54{,}230}{0.9} = 60{,}256$$

$$\text{midspan positive } M_n = \frac{64{,}457}{0.9} = 71{,}619$$

$$\text{exterior column negative } M_n = 0$$

3. *Design of reinforcement for column strip*

$$-M_n = 180{,}769 \text{ ft-lb acts on a strip width of } 2(0.25 \times 18) = 9.0 \text{ ft}$$

$$\text{unit } -M_n \text{ per 12-in.-wide strip} = \frac{180{,}769 \times 12}{9.0} = 241{,}025 \text{ in.-lb}$$

$$\text{unit } + M_n = \frac{107{,}428 \times 12}{9.0} = 143{,}427 \text{ in.-lb/12-in.-wide strip}$$

minimum A_s for two-way plates using $f_y = 60{,}000$-psi steel $= 0.0018bh$

$$= 0.0018 \times 10 \times 12 = 0.216 \text{ in.}^2 \text{ per 12-in. strip}$$

Negative steel:

$$M_n = A_s f_y\left(d - \frac{a}{2}\right) \quad \text{or} \quad 241{,}025 = A_s \times 60{,}000\left(9.0 - \frac{a}{2}\right)$$

Assume that moment arm $d - a/2 \simeq 0.9d$ for first trial and $d = h - \frac{3}{4}$ in. $- \frac{1}{2}$ diameter of bar $\simeq 9.0$ in. for all practical purposes. Therefore,

$$A_s = \frac{241{,}025}{60{,}000 \times 0.9 \times 9.0} = 0.50 \text{ in.}^2$$

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{0.50 \times 60{,}000}{0.85 \times 4000 \times 12} = 0.74 \text{ in.}$$

For the second trial-and-adjustment cycle,

$$241{,}025 = A_s \times 60{,}000\left(9.0 - \frac{0.74}{2}\right)$$

Therefore, required A_s per 12-in.-wide strip = 0.47 in.^{2}. Try No. 5 bars (area per bar = 0.305 in.^{2}).

$$\text{spacing } s = \frac{\text{area of one bar}}{\text{required } A_s \text{ per 12-in. strip}}$$

Therefore,

$$\begin{array}{c}s \text{ for negative moment} \\ (\text{No. 5 bars})\end{array} = \frac{0.305}{0.47/12} = 7.79 \text{ in. c-c (194 mm)}$$

$$s \text{ for positive moment} = 7.79 \times \frac{241{,}025}{143{,}237} = 13.11 \text{ in. c-c (326 mm)}$$

The maximum allowable spacing = $2h = 2 \times 10 = 20$ in. (508 mm). Try No. 4 bars for positive moment ($A_s = 0.20$ in.^{2}).

$$A_s = \frac{143{,}237}{241{,}025} \times 0.47 = 0.28 \text{ in.}^2 \text{ per 12-in. strip}$$

$$\begin{aligned}\text{minimum temperature reinforcement} &= 0.0018\,bh = 0.0018 \times 12 \times 10 \\ &= 0.216 \text{ in.}^2/\text{ft} < 0.28 \text{ in.}^2 \quad \text{O.K.}\end{aligned}$$

$$s = \frac{0.20}{0.28/12} = 8.57 \text{ in. c-c (218 mm)}$$

For an external negative moment, use No. 4 bars.

$$\text{Moment} = \frac{89{,}523 \times 12}{9.0} = 119{,}364 \text{ in. -lb}$$

$$s = 8.57 \times \frac{143{,}237}{119{,}364} = 10.28 \text{ in. c-c}$$

Use 14 No. 5 bars at $7\frac{1}{2}$ in. center to center for negative moment at interior column side, 12 No. 4 bars at $8\frac{1}{2}$ in. center to center for positive moment; and 10 No. 4 bars at 10 in. center to center for the exterior negative moment M_e with 8 of these bars to be placed outside the shear moment transfer band width 50 in., as seen in Figure 11.13b.

4. *Design of reinforcement for middle strip*

$$\text{unit } -M_n = \frac{54{,}230}{0.9} = 60{,}256 \text{ acting on a strip width of } 18.0 - 9.0 = 9.0 \text{ ft}$$

$$\text{unit } -M \text{ per 12-in.-width strip} = \frac{60{,}276 \times 12}{9} = 80{,}341 \text{ lb-in.}$$

$$80{,}341 = A_s \times 60{,}000(9.0 \times 0.9)$$

$$A_s = 0.17 \text{ in.}^2 \qquad a = \frac{0.17 \times 60{,}000}{0.85 \times 4000 \times 12} = 0.25 \text{ in.}$$

Second cycle:

$$80{,}341 = A_s \times 60{,}000\left(9.0 - \frac{0.25}{2}\right)$$

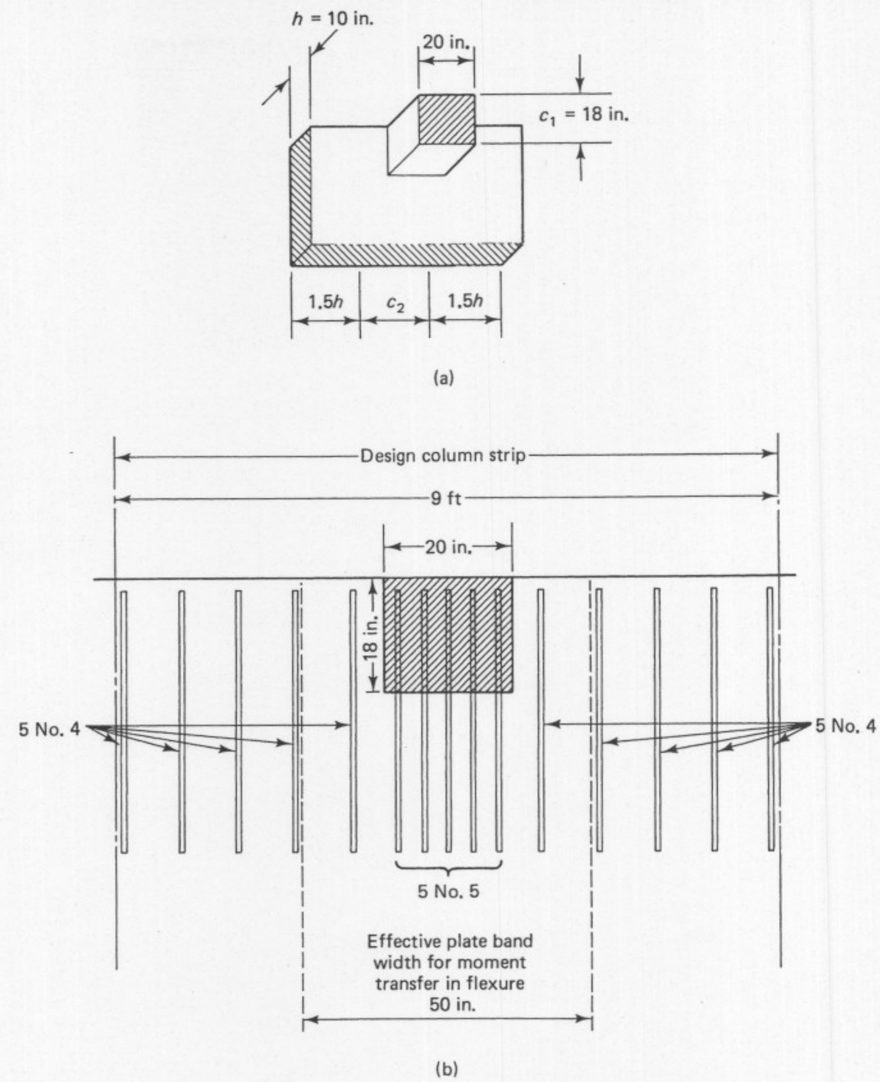

Figure 11.13 Shear moment transfer zone: (a) effective bandwidth; (b) reinforcing details.

$$A_s = 0.15 \text{ in.}^2/12\text{-in. strip} \qquad \text{minimum } A_s = 0.216$$

$$\text{use } A_s = 0.216 \text{ in.}^2/12\text{-in. strip}$$

Try No. 3 bars ($A_s = 0.11$ in.^{2} per bar).

$$\text{unit } +M = \frac{64{,}457 \times 12}{9 \times 0.9} = 95{,}492 \text{ in.-lb per 12-in. strip}$$

$$A_s = \frac{95{,}492}{60{,}000 \times 8.875} = 0.18 \text{ in.}^2 \qquad \text{use minimum } A_s = 0.216 \text{ in.}^2/12 \text{ in.}$$

Hence use negative and positive steel spacing $s = 0.11/(0.216/12) = 6.11$ in. (No. 3 at 6 in. center to center). Use No. 3 bars at 6 in. center to center for both the negative moment and positive moments.

(b) *N–S direction (short span):* The same procedure has to be followed as for the E–W direction. The width of the column strip on one side of the column $= 0.25l_1 = 0.25 \times 24 = 6$ ft,

Table 11.6 Moments, Bar Sizes, and Distribution

Strip	Moment Type	E–W Moment (lb-in./12 in.)	E–W A_s Req'd	E–W Bar Size and Spacing	N–S Moment (lb-in./12 in.)	N–S A_s Req'd	N–S Bar Size and Spacing
Column	Interior negative	241,025	0.47	No. 5 at $7\frac{1}{2}$	170,490	0.37	No. 4 at 6
	Exterior negative	119,364[c]	0.23	No. 4 at 10	84,434[b]	0.18	No. 3 at 6
	Midspan positive	143,237	0.28	No. 4 at $8\frac{1}{2}$	101,320	0.22	No. 3 at 6
Middle	Interior negative	80,341	0.15	No. 3 at 6	34,098	0.07	No. 3 at 6
	Exterior negative	0	0.22	No. 3 at 6[a]	0	0.22	No. 3 at 6
	Midspan positive	95,492	0.18	No. 3 at 6	40,528	0.09	No. 3 at 6

[a]Minimum temperature steel = $0.0018bh$ where it controls.

[b]For panel BC12 (see comment on page 494).

[c]63% of this moment is used for the shear-moment transfer negative reinforcement M_e of the end column zone (see Fig. 11.13).

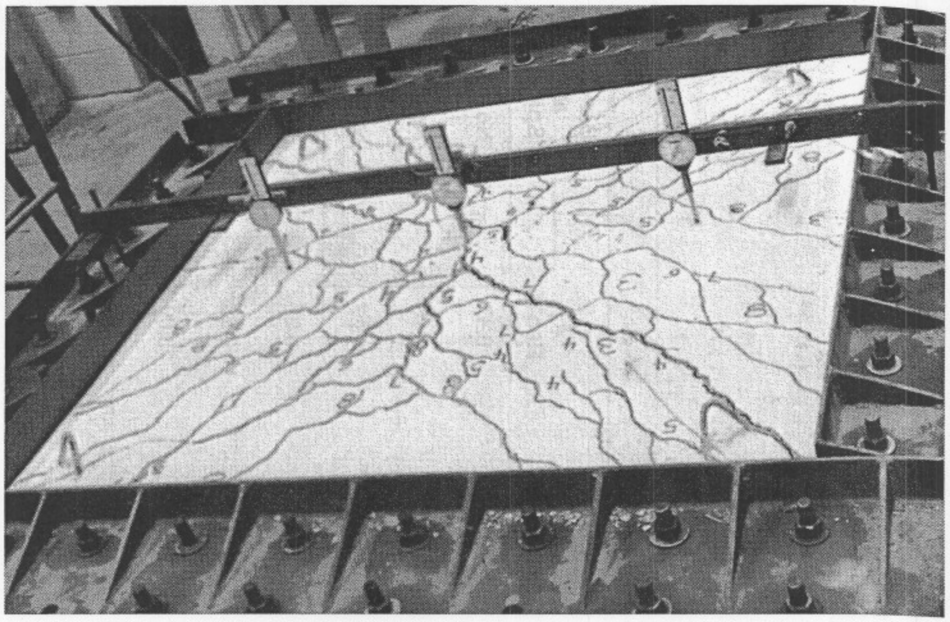

Photo 11.3 Flexural cracking in restrained one-panel reinforced concrete slab. (Tests by Nawy et al.)

which is greater than $0.25l_2 = 4.5$ ft; hence a width of 4.5 ft controls. The total width of the column strip in the N–S direction = 2 × 4.5 = 9.0 ft. The width of the middle strip = 24.0 – 9.0 = 15.0 ft. Also, the effective depth d_2 would be smaller; $d_2 = (h - \frac{3}{4}$-in. cover – 0.5 in. – 0.5/2) = 8.5 in. The moment values and the bar size and distribution for the panel in the N–S direction as well as the E–W directions are listed in Table 11.6. It is recommended for crack-control purposes that a minimum of No. 3 bars at 12 in. center to center be used and that bar spacing not exceed 12 in. center to center. In this case, the minimum reinforcement required by the ACI Code for slabs reinforced with $f_y = 60{,}000$-psi steel = 0.0018 bh = No. 3 at $6\frac{1}{2}$ in. on centers. Space at 6 in. on centers.

The choice of size and spacing of the reinforcement is a matter of engineering judgment. As an example, the designer could have chosen for the positive moment in the middle strip No. 4 bars at 12 in. center to center, instead of No. 3 bars at 6 in. center to center, as long as the maximum permissible spacing is not exceeded and practicable bar sizes are used for the middle strip.

The placing of the reinforcement is schematically shown in Figure 11.14. The minimum cutoff of reinforcement for bond requirements in flat-plate floors is given in Figure 11.15. The exterior panel negative steel at outer edges, if no edge beams are used, has to be bent into full hooks in order to ensure sufficient anchorage of the reinforcement. The floor reinforcement plan gives the E–W steel for panel AB23 and N–S steel for panel BC12 of Figure 11.10.

11.5.3 Example 11.2 Design of Two-way Slab on Beams by Direct Design Method (DDM)

A two-story factory building is three panels by three panels in plan, monolithically supported on beams. Each panel is 18 ft (5.49 m) center to center in the N–S direction and 24 ft (7.32 m) center to center in the E–W direction, as shown in Figure 11.16. The clear height between the floors is 16 ft. The dimensions of the supporting beams and columns are also shown in Fig. 11.16, and the building is subject to gravity loads only. Given:

live load = 135 psf (6.40 kPa)

f'_c = 4000 psi (27.6 MPa), normal-weight concrete

f_y = 60,000 psi (414 MPa)

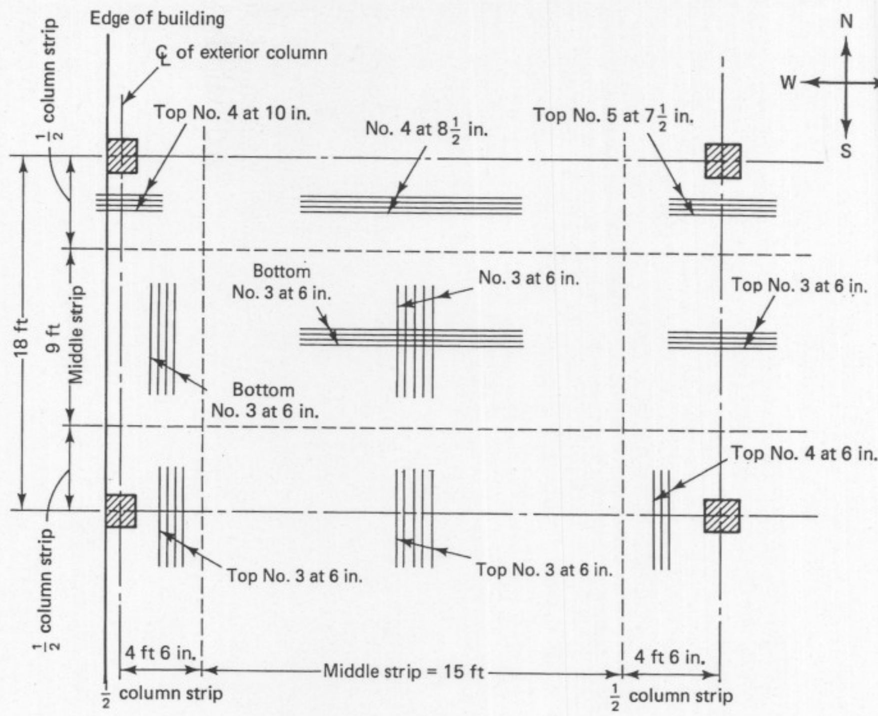

Figure 11.14 Schematic reinforcement distribution.

Assume that $\beta_c > 2.5$.

Design the interior panel and the size and spacing of reinforcement needed. Consider flooring weight to be 14 psf in addition to the slab self-weight.

Solution: *Geometry check for use of direct design method (step 1)*

(a) Ratio longer span/shorter span = 24/18 = 1.33 < 2.0; hence two-way action.
(b) More than three panels in each direction.
(c) Assume a thickness of 7 in.

$$w_d = 14 + \frac{7}{12} \times 150 = 101.5 \text{ psf}$$

$$2w_d = 203 \text{ psf}$$

$$w_l = 135 \text{ psf} < 2w_d$$

Hence the direct design method is applicable.

Minimum slab thickness for deflection requirement (step 2)

$$l_n \text{ (E–W)} = 24 \times 12 - 2 \times 8 = 272 \text{ in.}$$

$$l_n \text{ (N–S)} = 18 \times 12 - 2 \times 8 = 200 \text{ in.}$$

$$\beta = \frac{272}{200} = 1.36$$

For a preliminary estimate of the required total thickness h, using Eq. 11.10,

$$h = \frac{l_n\left(0.8 + \dfrac{f_y}{200{,}000}\right)}{36 + 9\beta}$$