Chapter 5 ■ Exponential and Logarithmic Functions

1. Evaluate (a) $3^{-2} \cdot 3^5$ **Answer: 27** (b) $\left(\dfrac{1}{4}\right)^{-1/2}\left(\dfrac{1}{3}\right)^{-2}$ **Answer: 18**

   c) $9^{3/2}27^{-1/3}$ **Answer: 9** (d) $3^{1/3}\left(9^{1/6}\right)^2$ **Answer: 3**

2. Simplify

   (a) $\left(16x^4\right)^{1/2}$ **Answer:** $4x^2$

   (b) $\left(2x^2 y^3\right)^2$ **Answer:** $4x^4 y^6$

   (c) $\left(2x^4\right)\left(-3x^{-1}\right)$ **Answer:** $-6x^3$

   (d) $\dfrac{8a^{-4}}{2a^{-2}}$ **Answer:** $\dfrac{4}{a^2}$

   (e) $\dfrac{7^0}{\left(3^{-2}x^{-1}y\right)^2}\left(\dfrac{81x^2}{y^2}\right)$

   (f) $\dfrac{\left(a^{-m}\cdot a^n\right)^2}{\left(a^{m-n}\right)^{-2}}$ **Answer: 1**

3. Solve the equation $5^{2x}=5^4$ for x. **Answer:** $x=2$

4. Solve the equation $2.4^{-2x+1}=2.4^{-3}$ for x. **Answer:** $x=2$

5. Express in logarithmic form: $4^2=16$. **Answer:** $\log_4 16 = 2$

6. Express in logarithmic form: $\left(\dfrac{1}{3}\right)^4 = \dfrac{1}{81}$. **Answer:** $\log_{1/3}\dfrac{1}{81}=4$

7. Use the fact that $\log_6 5 = 0.8982$ and $\log_6 3 = 0.6131$ to find $\log_6 15$. **Answer: 1.5113**

8. Use the laws of logarithms to expand and simplify the expression: $\log x(x-1)^2$.
   Answer: $\log(x)+2\log(x-1)$

9. Use the laws of logarithms to expand and simplify the expression: $\log\dfrac{\sqrt{x-2}}{x^2-3}$.

   Answer: $\dfrac{1}{2}\log(x-2)-\log(x^2-3)$

10. Use the laws of logarithms to simplify the expression: $\log x^3\left(x^3+2\right)^{1/3}$.

    Answer: $3\log x+\dfrac{1}{3}\log(x^3+2)$

11. Use the laws of logarithms to simplify the expression: $\ln(xe^x)$. **Answer:** $\ln x + x$

12. Use the laws of logarithms to simplify the expression: $\ln\left(\dfrac{3e^x}{x}\right)$. **Answer:** $\ln 3 + x - \ln x$

13. Sketch the graph of (a) $f(x)=\log_2 x$. (b) $g(x)=\ln 3x$ (c) $g(x)=\log_{2/5} x$

14. Use the laws of logarithms to solve the equation $\log_3 x = 3$. **Answer:** $x=27$

15. Use the laws of logarithms to solve the equation $\log_2 \dfrac{1}{4}=x$. **Answer:** $x=-2$

16. Use the laws of logarithms to solve $\log(x+2)-\log(x-1)=\log 4$. **Answer:** $x=2$

17. Use the laws of logarithms to solve the equation $e^{3x-1} = 5$. **Answer:** $x = \dfrac{1}{3} + \dfrac{1}{3}\ln 5$

18. Use the laws of logarithms to solve the equation $5e^{-0.3t} - 3 = 7$. **Answer:** $t = -\dfrac{10\ln 2}{3}$

19. What is the interest from $20,000, invested at 6.5% for 7 years, and compounded annually? **Answer: $11,079.73**

20. What is the interest from $1 million, invested at 18% for 4 years, and compounded annually? **Answer: $938,777.76**

21. What is the future value of $1250, invested at 9.5% for 5 years, if it is compounded semiannually? **Answer: $1988.16**

22. What is the present value of $25,000 in 2 years, if it is invested at 12% compounded monthly? **Answer: $19,689.15**

22. Find the accumulated amount after 5 years if $1800 is invested at 8% per year compounded quarterly. **Answer: $2674.71**

23. Find the accumulated amount after 5 years if $3200 is invested at 7% per year compounded continuously. **Answer: $4541.02**

24. A father wants to be able to provide his newborn baby with a college education. To do this, the father estimates that he will need $120,000 when his child turns 18. How much money should the father invest in an account that pays 7% interest per year compounded daily so that the account is worth $120,000 in 18 years? **Answer: $34,042.60**

25. The temperature of a mug of coffee after t minutes is given by
$T = 80 + 100e^{-0.182t}$ where T is measured in degrees Fahrenheit.
(a) What is the initial temperature of the coffee? **Answer: 180° F**
(b) When (to the nearest hundredth) will the coffee be at 100°? **Answer: 8.84 min**

26. Find the derivative of the function $f(x) = e^{4x}$. **Answer:** $4e^{4x}$

27. Find the derivative of the function $f(x) = 3e^x - x^4$. **Answer:** $3e^x - 4x^3$

28. Find the derivative of the function $f(x) = x^2 e^{4x}$. **Answer:** $4x^2 e^{4x} + 2xe^{4x}$

29. Find the derivative of the function $f(x) = \dfrac{2x}{e^{2x}}$. **Answer:** $\dfrac{2-4x}{e^{2x}}$

30. Find the derivative of the function $f(t) = 18e^{0.5t} + 2$. **Answer:** $9e^{0.5t}$

31. Find the derivative of the function $f(x) = 3e^{2x+2}$. **Answer:** $6e^{2x+2}$

32. Find the derivative of the function $f(x) = \left(e^{2x} + 1\right)^{12}$. **Answer:** $24e^{2x}\left(e^{2x} + 1\right)^{11}$

33. Find the derivative of the function $f(x) = \dfrac{e^{2x} + 1}{e^{2x} - 1}$. **Answer:** $-4\dfrac{e^{2x}}{\left(e^{2x} - 1\right)^2}$

34. Find the second derivative of the function $f(t) = 3e^{-3t} + 4e^{-2t}$. **Answer:** $27e^{-3t} + 16e^{-2t}$

35. Find the interval(s) where $h(x) = xe^x$ is increasing and decreasing.
Answer: Increasing: $(-1, \infty)$; **decreasing:** $(-\infty, -1)$

36. Find the equation of the tangent line to the graph of the function $y = e^{3x-1}$ at the point $(1/3, 1)$.　　　　**Answer:** $y = 3x$

37. Find the derivative of the function $f(x) = \ln(x^2 + 3)$.　　　**Answer:** $\dfrac{2x}{x^2 + 3}$

38. Find the derivative of the function $f(x) = \dfrac{1}{\ln x}$.　　**Answer:** $\dfrac{-1}{x(\ln x)^2}$

39. Find the derivative of the function $f(x) = x^2 \ln x$.　**Answer:** $x + 2x \ln x$

40. Find the derivative of the function $f(x) = \ln\left(\dfrac{2}{3x^5}\right)$.　　　**Answer:** $\dfrac{-5}{x}$

41. Find the derivative of the function $f(x) = e^{2x} \ln(x+2)$. **Answer:** $\dfrac{e^{2x}}{x+2} + 2e^{2x} \ln(x+2)$

42. Use logarithmic differentiation to find the derivative of the function
$y = (2x+1)^2 (x+3)^3$.　　　**Answer:** $5(2x+3)(2x+1)(x+3)^2$

43. Use logarithmic differentiation to find the derivative of the function
$y = \sqrt[3]{x+4}(2x+3)^3$.　　　**Answer:** $\sqrt[3]{x+4}(2x+3)^3 \left[\dfrac{1}{3(x+4)} + \dfrac{6}{2x+3}\right]$

44. Find the interval(s) on which $f(x) = x - \ln x$ is increasing and the interval(s) on which it is decreasing.　　**Answer: Increasing:** $(1, \infty)$; **decreasing:** $(0, 1)$

45. Let $f(x) = xe^{2x}$.
　(a) Find the interval(s) on which $f(x)$ is concave upward.　　**Answer:** $(-1, \infty)$
　(b) Find the interval(s) on which $f(x)$ is concave downward.　　**Answer:** $(-\infty, -1)$
　(c) Find the x-coordinate(s) of any point(s) of inflection of f.　　**Answer:** $x = -1$