MATH. FOR ENG. ANAL. I REVIEW PROBLEMS

1. Let $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2-4}}$

a) Find $f(3,1)$. [**Answer:** $\frac{\sqrt{6}}{6}$]

b) Sketch and shade the region where f is continuous.

2. Evaluate $\displaystyle\lim_{(x,y)\to(1,2)} (x^2+3y)$. [**Answer:** 7]

3. Show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2+y^2}$ does not exist. [**Answer:** Along $x=0$ the limit is 0, whereas along $y=0$ the limit equals 1.]

4. Sketch the graph of the surface $f(x,y) = x^2+y^2$.

5. Which of the following is the same as f_{xyy}? [**Answer:** b]

a) $\dfrac{\partial^3 f}{\partial x\, \partial^2 y}$ or b) $\dfrac{\partial^3 f}{\partial^2 y\, \partial x}$

6. Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2}$. [**Answer:** 0]

7. If $z = x^3+4xy-y^2$, then find

a) $\dfrac{\partial z}{\partial x}$ [**Answer:** $3x^2+4y$] b) $\dfrac{\partial z}{\partial y}(0,2)$ [**Answer:** -4]

8. Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ if $x^2+y^2+yz^2=5$. [**Answer:** $\dfrac{-x}{yz}$]

9. If $f(x,y) = x^3y^5-2x^2y+x$, then find f_{xxy}. [**Answer:** $30xy^4-4$]

10. Let $W = e^{x^2+4y^2-z^2}$. Find $\dfrac{\partial w}{\partial y}(3,2,-5)$. [**Answer:** 16]

Typeset by $\mathcal{AMS}$-TEX

11. If $f(x,y) = \dfrac{xy}{\sqrt{x+y}}$ for $x + y > 0$, then find f_x. [**Answer:** $\dfrac{xy+2y^2}{2(x+y)^{3/2}}$]

12. Let $z = f(x,y)$ where f has continuous partial derivatives. If $x = v - u$ and $y = 2v - u$, then which of the following is equal to $\dfrac{\partial z}{\partial u} + \dfrac{\partial z}{\partial v}$? [**Answer:** c]

a) $-\dfrac{\partial z}{\partial y}$ *b*) $\dfrac{\partial z}{\partial x}$ *c*) $\dfrac{\partial z}{\partial y}$ *d*) $\dfrac{\partial z}{\partial x} - \dfrac{\partial z}{\partial y}$

13. a) Find an equation of the plane tangent to the surface $z = x^2 + y^2x - 2$ at the point (1, 1, 0). [**Answer:** $3x + 2y - z - 5 = 0$]

b) Find an equation of the normal line to the surface $z = x^2 + y^2x - 2$ at the point (1, 1, 0). [**Answer:** $x = 1 + 3t,\ y = 1 + 2t,\ z = -t$]

14. What is the directional derivative of $f(x,y) = \dfrac{x}{x+y}$ at the point P(1,0) in the direction of the vector $a = -3i + 4j$. [**Answer:** $-\dfrac{4}{5}$]

15. If y is a differentiable function of x such that $yx + x^2y^2 - 2 = 0$, then find $\dfrac{dy}{dx}$. [**Answer:** $-\dfrac{y+2xy^2}{x+2x^2y}$]

16. Suppose $F(x,y) = xy$, $x = x(t)$ and $y = y(t)$. If $x(1) = 1$, $y(1) = 3$, $x'(1) = 2$ and $y'(1) = -3$, then find $\dfrac{dF}{dt}|_{t=1}$. [**Answer:** 3]

17. Find a unit vector that is normal to the surface $z = \dfrac{1}{xy^2}$ at the point (1, -1, 1). [**Answer:** $-\dfrac{\sqrt{6}}{6}\mathbf{i} + \dfrac{\sqrt{6}}{3}\mathbf{j} - \dfrac{\sqrt{6}}{6}\mathbf{k}$]

18. If $f(x,y) = x^3 + xy + y^2,\ x = 2u + v,$ and $y = u - v$, then use the chain rule to find $\dfrac{\partial f}{\partial u}$. [**Answer:** $24u^2 + 24uv + 6u - 3v + 6v^2$]

19. Use the chain rule to find $\dfrac{dz}{dt}$ if $z = 3x^2y^3$, $x = t^4$ and $y = t^2$. [**Answer:** $42t^{13}$]

20. Let $f(x, y) = x^3y^3 - xy$.

a) Find the gradient of f, $\nabla f(x, y)$. [**Answer:** $(3x^2y^3 - y)\mathbf{i} + (3x^3y^2 - x)\mathbf{j}$]

b) Find a unit vector u for which the directional derivative of f at (1,-1) is zero. i.e. Find u such that $D_u f(1, -1) = 0$. [**Answer:** $\frac{\sqrt{2}}{2}\mathbf{i} + \frac{\sqrt{2}}{2}\mathbf{j}$]

21. Use the chain rule to find $\dfrac{dz}{dy}$ if $z = xy$ and $x = y \cos y$. [**Answer:** $-y^2 \sin y + 2y \cos y$]

22. The temperature, T, at a point (x, y) on a semicircular plate is given by $T = 3x^2y - y^3 + 273$ degrees Celsius.

a) Find the temperature at (1,2). [**Answer:** 271]

b) Find the rate of change of temperature at (1,2) in the direction of $a = i - 2j$. [**Answer:** $6\sqrt{5}$]

c) Find a unit vector in the direction in which the temperature increases most rapidly at (1,2). [**Answer:** $\dfrac{4}{5}\mathbf{i} - \dfrac{3}{5}\mathbf{j}$]

d) Find the maximum rate of increase in temperature at the point (1,2). [**Answer:** 15]

23. Find the maximum and minimum value of $f(x, y) = x^2y^2$ subject to the constraint $4x^2 + y^2 = 8$. Solve the above problem by using two methods:

Method 1: Use the constraint to eliminate a variable.

Method 2: Use the method of Lagrange multipliers.

[**Answer:** Maximum $= f(-1, -2) = f(1, -2) = f(1, 2) = f(-1, 2) = 4$, Minimum $= f(0, -\sqrt{8}) = f(0, \sqrt{8}) = f(-\sqrt{2}, 0) = f(\sqrt{2}, 0) = 0$.]

24. Let $f(x, y) = x^3 + y^3 - 3x - 3y$.

a) Find the critical points of f . [**Answer:** $(-1, -1)$, $(-1, 1)$, $(1, -1)$, $(1, 1)$]

b) Find the point(s) at which f has a saddle point. [**Answer:** (1, -1), (-1,1)]

c) What are the point(s) at which f has a relative maximum? [**Answer:** (-1,-1)]

d) Find the point(s) at which f has a relative minimum. [**Answer:** (1,1)]

25. Find an equation of the tangent plane to the surface $xyz+2x+3y+3z+2=0$ at the point $P(1,2,-2)$. [**Answer:** $-2x+y+5z+10=0$]

26. A company manufactures two products. The total revenue from x_1 units of product 1 and x_2 units of product 2 is

$$R=-2x_1^2-3x_2^2+3x_1\,x_2+100x_2+1600x_1.$$

Find x_1 and x_2 so as to maximize the revenue.
[**Answer:** $x_1=660$ and $x_2=\frac{1040}{3}\approx 346.67$]

27. Let $f(x,y)=x^2-3y^2-2x+6y$ and R be the closed square region with vertices (0, 0), (0, 2), (2, 2) and (2, 0).

a) Find all critical point(s) of f that lie in the interior of R. [**Answer:** (1,1)]

b) Find the absolute maximum value of f on R. [**Answer:** Absolute maximum $=f(2,1)=f(0,1)=3$],

c) Find the absolute minimum value of f on R. [**Answer:** Absolute minimum $=f(1,0)=f(1,2)=-1$]

28. Suppose that $x^2-3xyz^2+yz-2=0$ defines z implicitly as a function of x and y. Find $\dfrac{\partial z}{\partial x}$. [**Answer:** $\dfrac{2x-3yz^2}{6xyz-y}$]

29. Find the least squares regression line for the points (0,0), (1,1), (3,4), (4,2) and (5,5). [**Answer:** $y=\dfrac{37}{43}x+\dfrac{7}{43}$]

30. Let C be the curve of intersection of the cylinder $x^2+y^2=5$ and the plane $z=2x$. Find a symmetric equation of the tangent line to C at the point (1,-2,2). [**Answer:** $\dfrac{x-1}{4}=\dfrac{y+2}{2}=\dfrac{z-2}{8}$]

31. Use Lagrange multipliers to find the maximum of $f(x,y)=2xy$ subject to the constraint $\dfrac{x^2}{4}+\dfrac{y^2}{2}=1$. [**Answer:** Maximum $=f(\sqrt{2},1)=f(-\sqrt{2},-1)=2\sqrt{2}$]