Math for Engineering Analysis I Problems for Chapter 12

1. Evaluate $\int_0^1 \int_1^2 x^2\, y\, dx\, dy$.

2. Consider the double integral $\int\int_R x\, y^2\, dA$ where R is the region bounded by $y = x^2$ and $y = 2x$.

a) Write an iterated integral with the order $dy\, dx$ that is equal to the above integral.

b) Write an iterated integral with the order $dx\, dy$ that is equal to the above integral.

3. Evaluate the integral $\int_0^1 \int_{\sqrt{y}}^1 e^{x^3}\, dx\, dy$ by reversing the order of integration.

4. Find the area of the region that is bounded by the graphs of $x = y^2$ and $y = x - 2$.

5. Evaluate $\int_0^2 \int_0^{\sqrt{4-x^2}} x\, dy\, dx$ by converting it to an equivalent double integral in polar coordinates.

6. Consider the solid in the first octant bounded above by the plane $z = 4 - x - y$ and below by the $xy-$plane. Find the volume of the solid.

7. Find the surface area of the portion of the surface $z = \sqrt{9 - y^2}$ that is above the rectangle $R = \{(x, y) : 0 \le x \le 2;\ 1 \le y \le 3\}$.

8. Consider the solid bounded above by the plane $z = 4y$ and below by the region R in the $xy-$plane given by: $x^2 + y^2 \le 16$ and $y \ge 0$.

a) Write an iterated double integral in rectangular coordinates to calculate the volume of the solid.

b) Write an iterated double integral in polar coordinates to calculate the volume of the solid.

9. Evaluate the triple integral $\int_0^1 \int_0^x \int_0^{xy} x\, dz\, dy\, dx$.

10. Consider the portion of the solid $x^2 + y + z = 1$ in the first octant. Find the volume of the solid.

11. Find the volume of the solid Q that is bounded by the $xy-$plane, the plane $z = 4y$ and the cylinder $x^2 + y^2 = 16$.

12. What is the surface area of the portion of the plane $x + y + z = 3$ that is above the triangular region on the xy-plane that is bounded by $x = 0$, $y = 0$ and $y = 3 - x$?

13. Evaluate $\int\int_R r\, \cos\theta\, dA$ where R is the region in the second quadrant that lies inside the circle $r = 2$ and outside the circle $r = 1$.

14. Evaluate the triple integral $\int_0^1 \int_0^x \int_0^{xy} x\, dz\, dy\, dx$.

15. Evaluate the double integral $\int\int_R \sqrt{1 - x^2}\, dA$ where R is the region on the $xy-$plane that is bounded by $y = 0,\ y = x$ and $x = 1$.

16. Use a triple integral to find the volume of the ellipsoidal solid $\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$.

Math for Engineering Analysis I Problems for ODE

17. Use the method of separation of variables to solve the following differential equations.

a) $\frac{dy}{dx} = \frac{1-x^2}{y^2}$ b) $x\frac{dy}{dx} = \frac{1-4y^2}{3y}$ c) $\frac{dy}{dx} = y\sin x, \quad y(\pi) = -3.$

18. Determine if the differential equation is exact. If it is, solve it.

a) $(3x^2 + 2y^2)dx + (4xy + 6y^2)dy = 0.$ b) $(1 + ye^{xy})dx + (2y + xe^{xy})dy = 0.$

c) $(\frac{1}{x} + 2y^2x)dx + (2yx^2 - \cos y)dy = 0\ , \quad y(1) = \pi.$

19. Solve the following linear differenatil equations.

a) $xy' + 2y = 3x$ b) $(1 + x)y' + y = \cos x$ c) $y' + y - x^2 = 0.$

20. **(Mixing Problems)** Suppose a brine containing 2 kg of salt per liter runs into a tank initially filled with 500 L of water containing 50 kg of salt. The brine enters the tank at rate of 5 L/min. The mixture, kept uniform by strring, is flowing ouot at the rate of 5 L/min.

a) Find the concentratio of salt in the tank after 10 minutes.

b) After 10 minutes, a leak develops in the tank and an additional liter per minute of the mixture flows out of the tank. What will be the concentration of the salt in the tank 20 minutes after the leak develops.