

Modular Forms and Related Topics Conference
to Celebrate the 75th birthday of
Marvin Knapp

January 9-10, 2006

Temple University















MARVIN IS PRIME AT 73











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Connections between
Composition Operators and
Automorphic Functions

$$C_\varphi f = f \circ \varphi$$

and John Clifford

- Dearborn

2006

An operator is a linear map from a Hilbert Space of functions.

For each analytic $\varphi: U \rightarrow U$, we define

$$C_\varphi f = f \circ \varphi$$

C_φ is a Composition Operator.

φ is the inducing function for the composition operator C_φ .

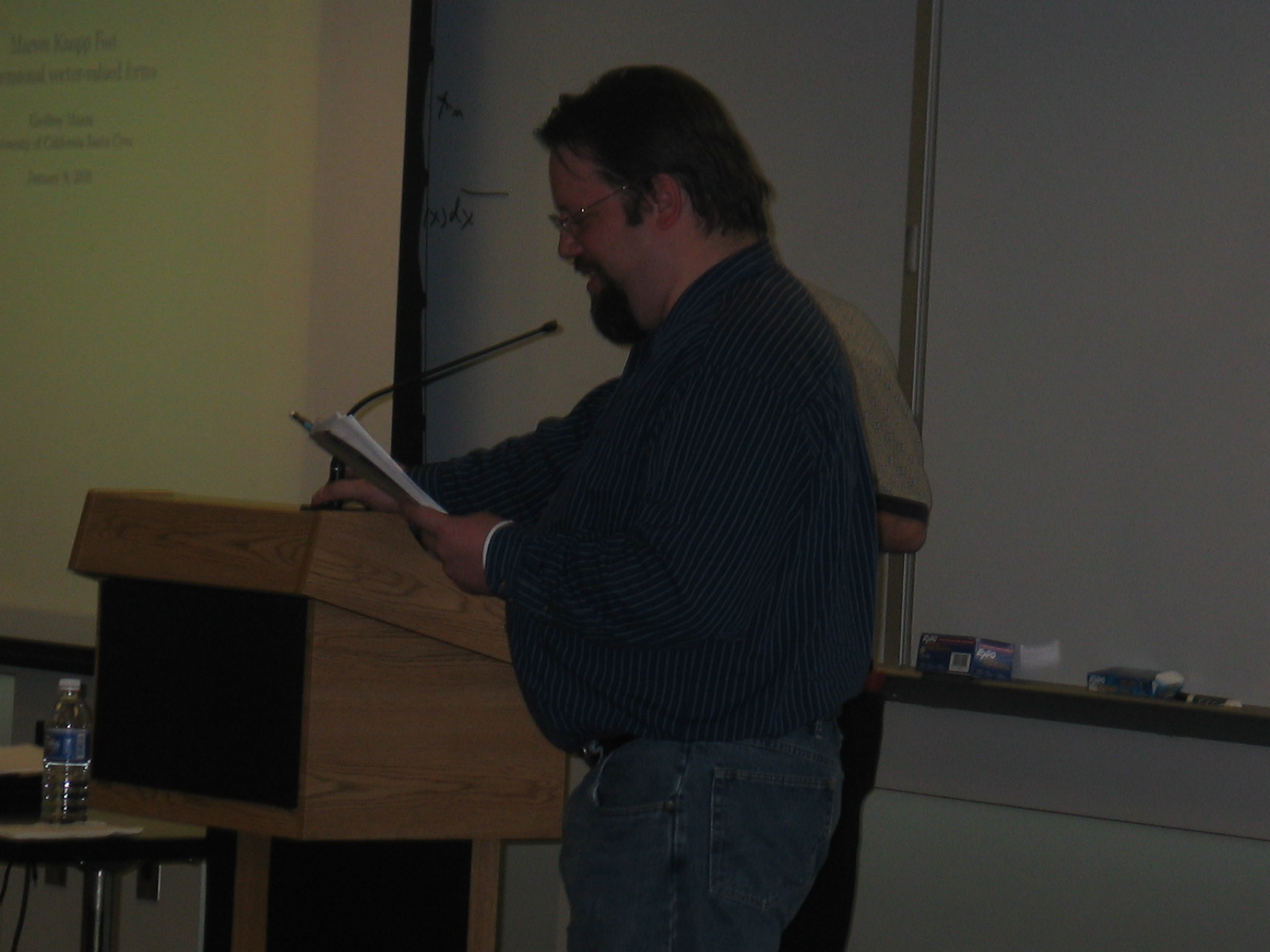
GOAL: To find and describe any connection between the function theoretic and the operator theoretic properties.

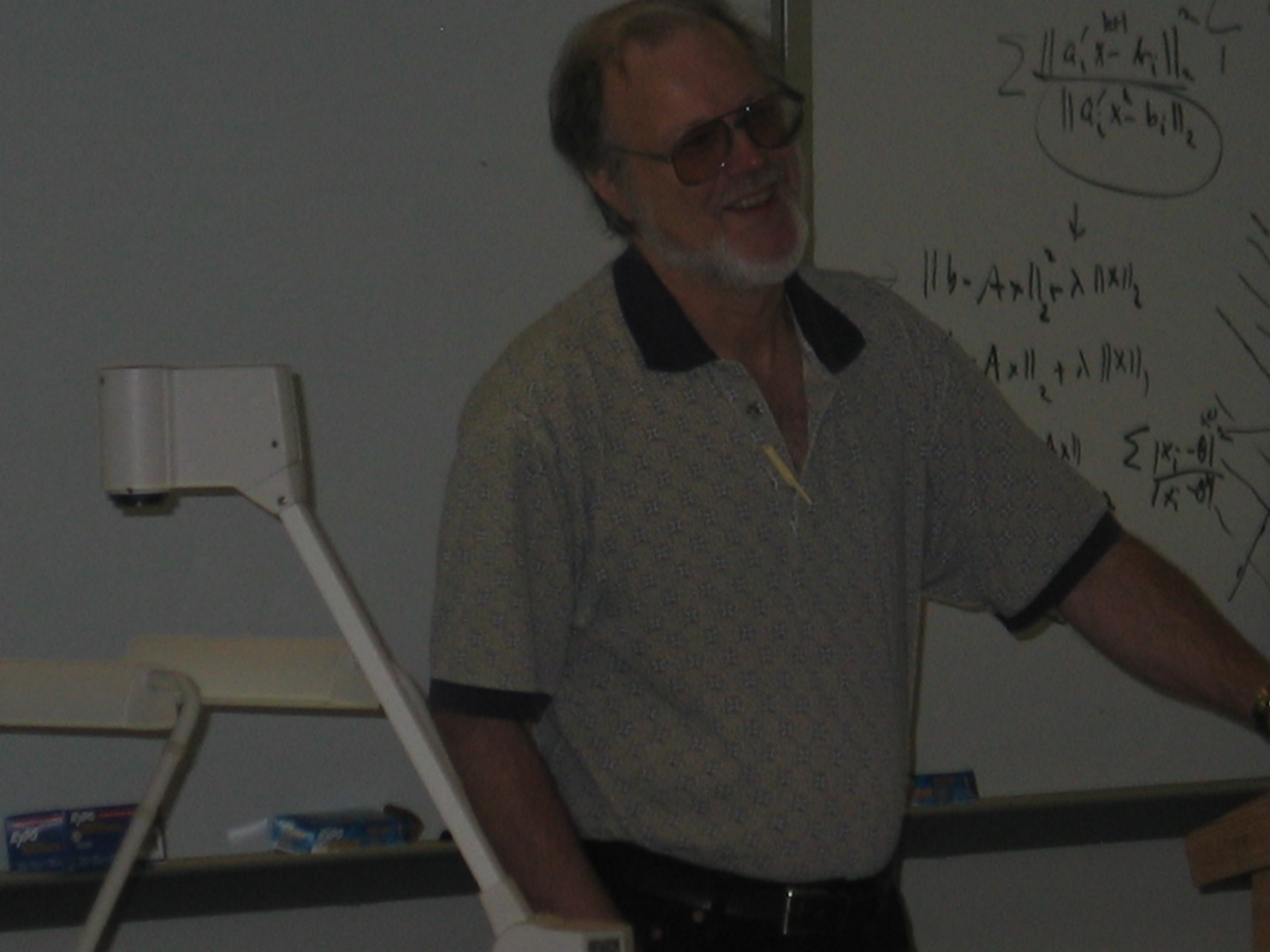
$$\int f(x) dx$$

Marvin Knapp Post
terminal vector-valued forms

Calvin M. ...
University of California Santa Cruz
January 1, 2008

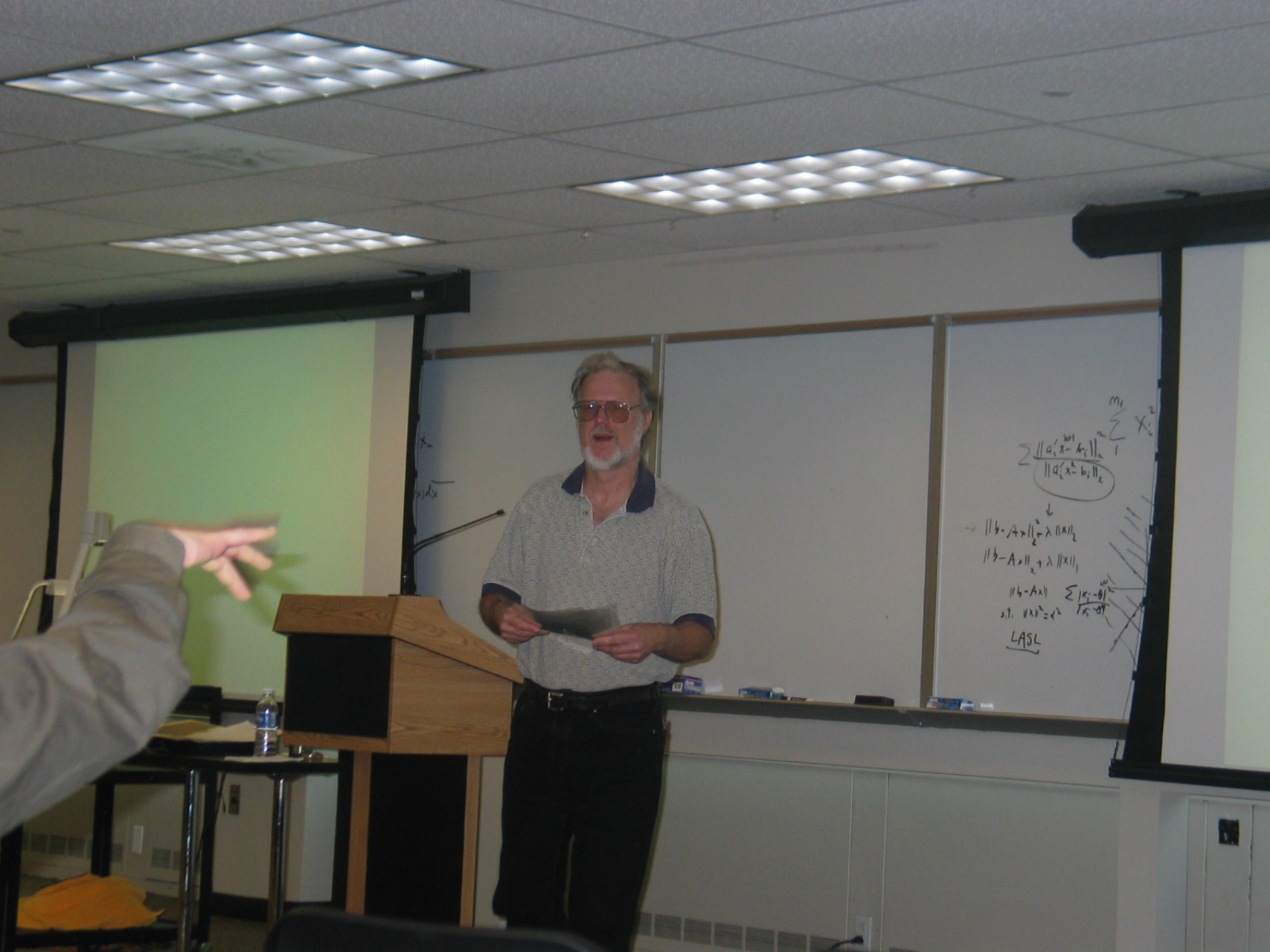
$$(x)dx$$











$$\sum \frac{\|a_i'x - b_i\|_2}{\|a_i'x - b_i\|_2} \sum_{i=1}^{m_1} x_i^2$$

↓

$$\rightarrow \|b - Ax\|_2 + \lambda \|x\|_2$$
$$\|b - Ax\|_2 + \lambda \|x\|_2$$
$$\|b - Ax\|_2 = \sum_{i=1}^m |r_i - \theta_i|$$

LAST



MARVIN IS PRIME

MOUNT HOLYOKE • WILLEST •
BRYAN MARIN • VASSAR •







FIRE

① Sign change of lower coefficients

Theorem ('well-known') Let $f(x) = \sum_{n=0}^{\infty} a(n) x^n$

Let a - non-zero

Guest









$$\alpha(u) = \lambda(u) u^{\frac{b-1}{k}}$$

$$S(\lambda) = \sum_{u \in V} \log^k \left(\frac{\lambda(u)}{\alpha(u)} \right) = \sum_{u \in V} \lambda(u)^{1/k} \left(\frac{\lambda(u)}{\alpha(u)} \right)^{k-1}$$

$$\lambda(u) \geq 0$$
$$\lambda(u) = 1$$



$$\sigma(u) = \lambda(u) u^{\frac{u-1}{2}}$$
$$St_{out} \sum_{u \leq v} \lambda(u)^2 \binom{v}{u, 1} = \sum_{u \leq v} \lambda(u)^{1/2} \log$$
$$\left(\sum_{u \leq v} \lambda(u)^2 \binom{v}{u, 1} \right)^{1/2} \left(\sum_{u \leq v} \lambda(u)^2 \binom{v}{u, 1} \right)^{1/2}$$



$$\sigma(u) = \lambda(u) u^{\frac{b-1}{2}}$$
$$St(u) = \sum_{\substack{v \leq u \\ (v, N)=1}} \lambda\left(\frac{v}{u}\right) = \sum_{v \leq u} \lambda(v) \left(\frac{u}{v}\right)^{\frac{1}{2}} \left(\sum_{\substack{w \leq u/v \\ (w, N)=1}} \lambda(w)\right)$$
$$\lambda(u) \geq 0$$





Overview

Weyl group multiple Dirichlet series are multiple Dirichlet series in several variables whose coefficients involve Gauss sums. These functions and their residues unify and generalize a number of examples which have been previously treated individually, often with applications to analytic number theory. They are also expected to be connected to metaplectic Eisenstein series.



$$\frac{\sqrt{3} \sqrt{n-k}}{k} \approx \frac{\sqrt{3} \sqrt{n-k}}{k} \log \left(\left(\frac{\sqrt{3} \sqrt{n-k}}{k} \right)^k \right)$$

$$k \leq 1 \text{ di logarithm}$$



Proposition (see [6])

$$p_k(n) \leq \frac{\pi e^{\pi \sqrt{2/3} \sqrt{n-k}}}{\sqrt{6(n-k)} + \pi} e^{-\frac{\sqrt{6(n-k)}}{\pi}}$$

$$L_2(z) = \sum_{m=1}^{\infty} \frac{z^m}{m^2} \quad |z| \leq 1 \text{ di logarithm}$$





















































Automorphic forms and
L-functions in the Group $GL(n, \mathbb{R})$

to appear C.U. June 2006.

$GL(2)$

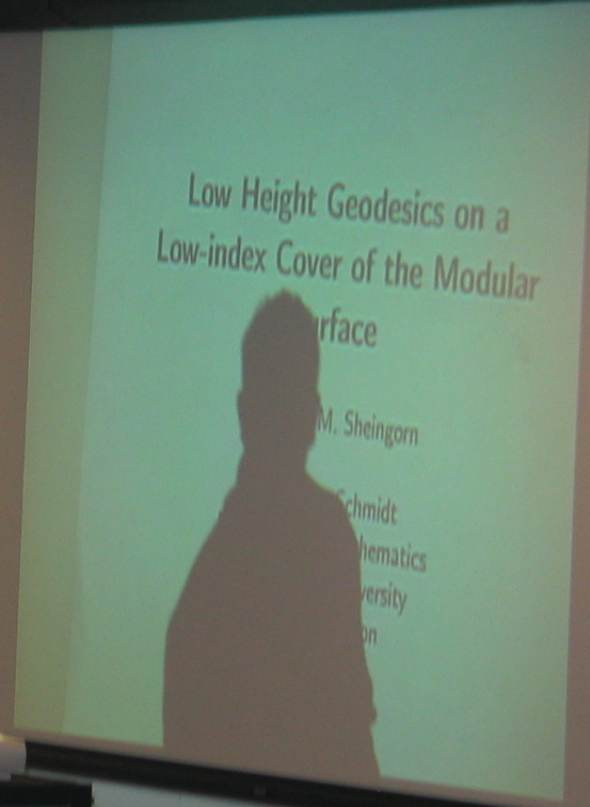
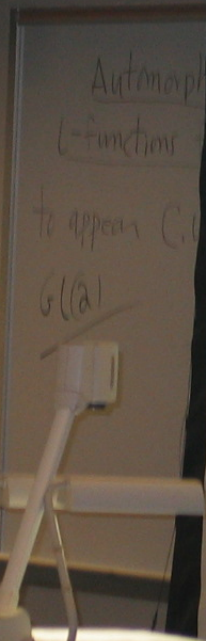
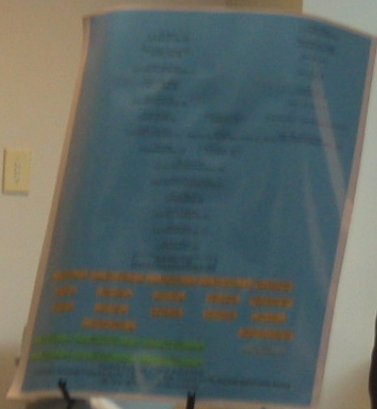
Voronoi formulas

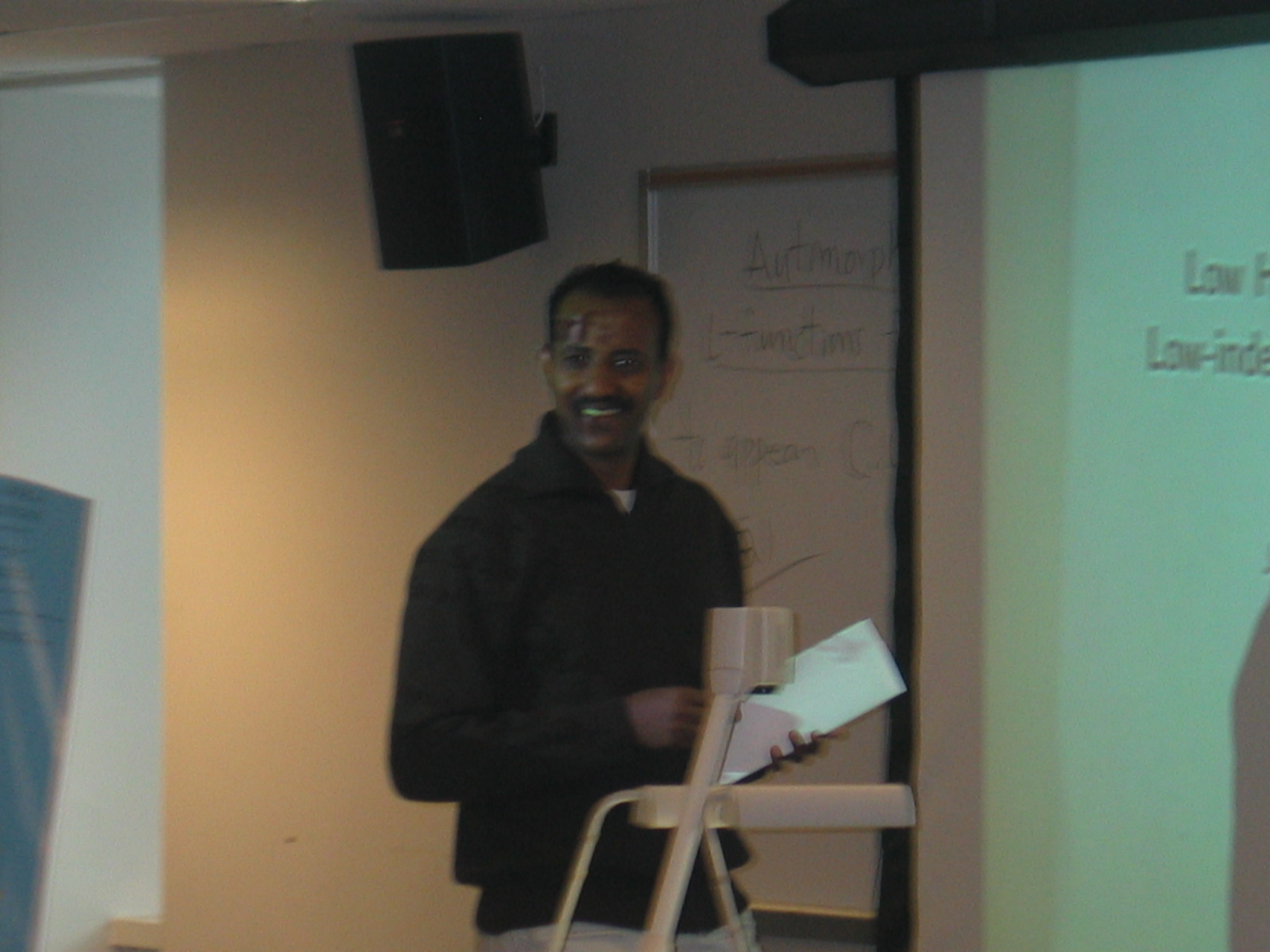
$$\sum f(m) = \sum \hat{f}(m)$$

$$\sum A(m) f(m) = \sum \tilde{A}(m) F(m)$$

Voronoi formula in $GL(n)$

$\phi(x)$





Automorph

L-functions

to appear C

Low F
Low-inde

$$n \Gamma(n)$$

$$\frac{\tilde{\phi}(s) \prod_1^n \Gamma\left(\frac{1-s-\overline{\lambda_i(v)}}{2}\right)}{\prod \Gamma\left(\frac{s-\lambda(v)}{2}\right)}$$

$$\cdot x^s ds$$

$$K_n(m, q) = \sum e^{\frac{2\pi i}{q} (x_1 + \dots + x_m)}$$

$$m \equiv m \pmod{q}$$

$$\sum_m$$









Random Graphs

Definition:

$$G(n, p) = (V, E)$$

where:

$$V = \{1, 2, \dots, n\}$$

$$E = \{e_1, e_2, \dots, e_m\}$$

$$e_i = (v_i, w_i)$$

$$v_i, w_i \in V$$

$$p = \Pr(e_i \in E)$$

$$p = \Pr(v_i, w_i \in E)$$

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Modular Setting

One such is:

$$\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}, B = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

the co... (2, R).

and its product
dividing by one
equation.

Automorphic
L-functions for
to appear C.U.
 $G(2)$

convergence formula on $t(L_h)$

$$\Phi(x) = \frac{1}{2\pi i} \int_{-2-i\infty}^{-2+i\infty} \frac{q\pi^{\frac{\alpha}{2}}}{2} \tilde{\Phi}(s) \frac{\prod_{j=1}^n \Gamma\left(\frac{1-s-\lambda_j(v)}{2}\right)}{\pi \Gamma\left(\frac{s-\lambda(v)}{2}\right)} x^s ds$$

$K_n(m, q)$

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$$

$$\langle A, B \rangle = \Gamma',$$





Automorphic
L-functions for

appear C.U.P.

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Automorphic
L-functions

appear C.U.P.

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Automorphic
L-functions for
to be a C.U.P.





joint work with Kumar Maruy



joint work with Kumar Murty

Lang-Trotter Conjecture

$$E: y^2 = x^3 + ax + b \quad a, b \in \mathbb{Z}$$

$$\#E(\mathbb{F}_p) = p - a_p(E)$$

Question: Can one say $a_p(E) \neq 0$?

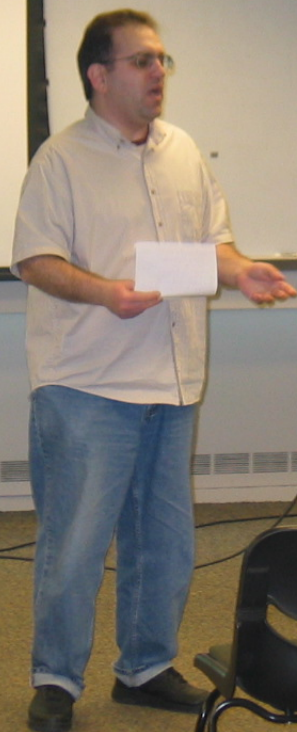




Hypergeometric Zeta Functions

Abdul Hassen (13th Student of Marvin)
(Joint work with Hieu D. Nguyen)
Rowan University, Glassboro, NJ

Conference on Modular Forms and Related Topics
in Honor of Marvin Knopp's 70th Birthday
January 7 and 18, 2006
Philadelphia, PA





Hypergeometric Zeta Functions

$$\zeta(a,b) = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{1}{n! m!} \int_0^1 \int_0^1 x^{a-1} y^{b-1} (1-x)^{n-1} (1-y)^{m-1} dx dy$$

$$= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{1}{n! m!} \int_0^1 \int_0^1 x^{a-1} y^{b-1} (1-x)^{n-1} (1-y)^{m-1} dx dy$$



Hypergeometric Zeta Functions

$$\zeta_N(s) = \frac{1}{(1+N^{-s})} \sum_{n=0}^{\infty} \frac{N^{ns}}{1+N^{-n}} \quad N > 1$$

$$\lim_{N \rightarrow \infty} \zeta_N(s) = \sum_{n=0}^{\infty} \frac{1}{1+N^{-n}} = \sum_{n=0}^{\infty} \frac{1}{1+e^{-n}}$$

Hypergeometric Zeta Functions

$$\zeta_N(s) = \frac{1}{(1+N^{-s})} \sum_{n=0}^{\infty} \frac{N^{ns}}{1+N^{-n}} \quad N > 1$$
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Geometric Zeta Functions

$$\frac{1}{\Gamma(s+N-1)} \int_0^\infty \frac{x^{s+N-1}}{e^x - T_{N-1}(x)} dx \quad N \geq 1$$

$$\zeta_2(s) = \frac{1}{2} \frac{x}{x^2} = 1 + \frac{x}{2} + \frac{x^2}{2^2} + \frac{x^3}{2^3} + \dots$$

$$N=1: \zeta(s) = \zeta(s) = \frac{1}{\Gamma(s)} \int_0^\infty \frac{x^{s-1}}{e^x - 1} dx$$

$$N=2: \zeta(s) = \frac{1}{\Gamma(s+1)} \int_0^\infty \frac{x^s}{e^x - 1 - x} dx$$

$$N=3: \zeta(s) = \frac{1}{\Gamma(s+2)} \int_0^\infty \frac{x^{s+1}}{e^x - 1 - x - x^2/2} dx$$



Function?

$$+ x)^{n-1} dx$$

$$^{n-1} dx$$

$$^{n-1} dx$$

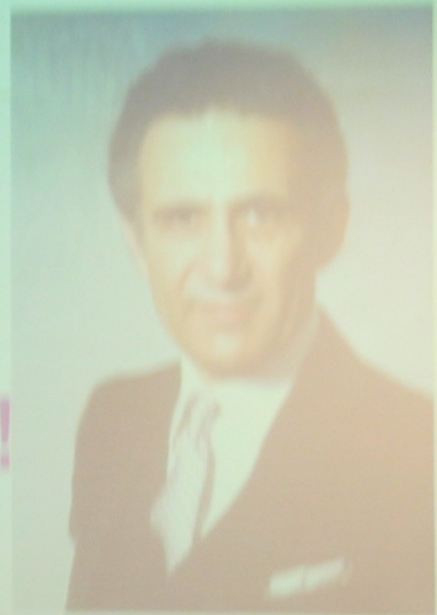




$$e^{\frac{1}{2}} = 1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{24} + \dots$$

On the other hand

HAPPY BIRTHDAY!





Automatic Control and
Reliability of
Robert. Wenzel

Robert. Wenzel



Arithmetic Genus and
Rationality of
Hilbert Modular Varieties

Helen G. Grundman
Bryn Mawr College



The Compactification

$X_k = \mathbb{P}^1/k$ consists of a finite number of points.
(The class number of k .)

The resulting compact variety
(for $a > 1$).

→ cusp singularities

→ fixed points of f_1







AND

ER









Derivatives of Modular Forms
of Negative Weight

D. Bump and V. Choe

Derivati
of



Derivatives of Modular Forms
of Negative Weight
D. Bump and Y. Chao

Derivatives of Modular Forms
of Negative Weight
D. Bump and Y. Chao

Let $\mathcal{H}$ be the usual upper half plane and Γ be a discrete subgroup of $SL(2, \mathbb{R})$.

Let S be a finite set of points in $\Gamma \backslash \mathcal{H}$, where a modular form is allowed to have poles. Let $\mathcal{H}_S$ be the set of all $z \in \mathcal{H}$ such that the image of z in $\Gamma \backslash \mathcal{H}$ is not in S .

Let $M_k^{\infty}(\Gamma)$ be the space of the smooth functions $f: \mathcal{H}_S \rightarrow \mathbb{C}$ such that

$$f(z) = (cz + d)^{-k} f\left(\frac{az + b}{cz + d}\right), \forall \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma.$$

$$f(z) = \frac{d}{dz} f(z)$$

$$(cz+d)^{-k} f\left(\frac{az+b}{cz+d}\right), \forall \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma.$$

entity-1949: f

functions in $\mathcal{M}_{-r}^S$

$$(2iyD)^j$$

1 then

$$\frac{\Gamma(1)}{j-r} (2iyD)^j$$







A

FIN

DR





