Chapter 1 Introduction

Welcome to *Mathematica*! This tutorial manual is intended as a supplement to Rogawski's *Calculus* textbook and aimed at students looking to quickly learn *Mathematica* through examples. It also includes a brief summary of each calculus topic to emphasize important concepts. Students should refer to their textbook for a further explanation of each topic.

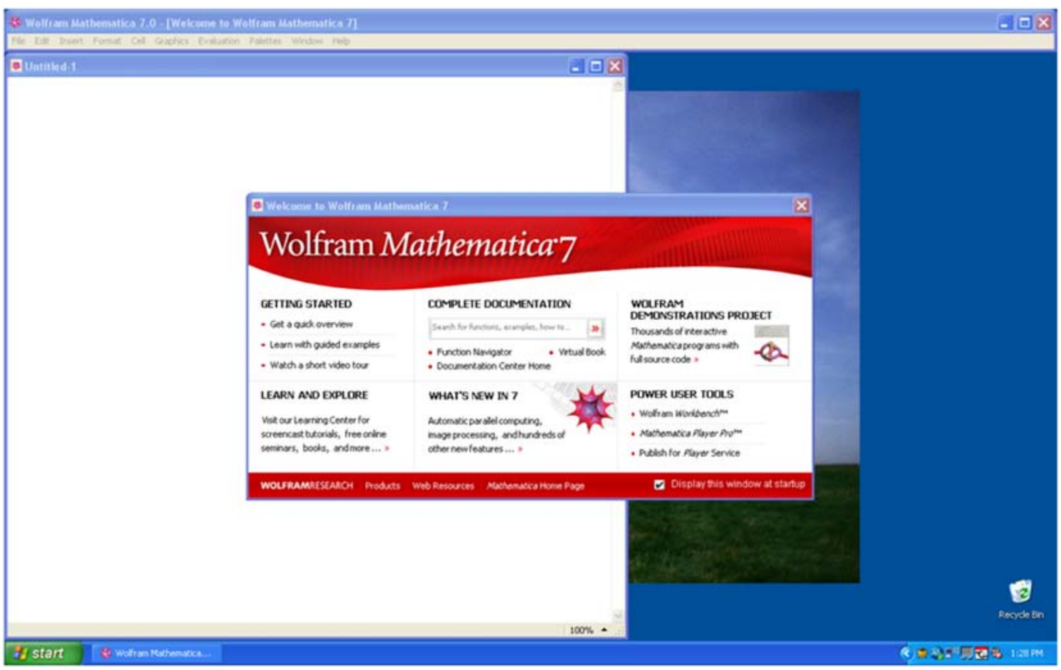
■ 1.1 Getting Started

Mathematica is a powerful computer algebra system (CAS) whose capabilities and features can be overwhelming for new users. Thus, to make your first experience in using *Mathematica* as easy as possible, we recommend that you read this introductory chapter very carefully. We will discuss basic syntax and frequently used commands.

NOTE: You may need to obtain a computer account on your school's computer network in order to access the *Mathematica* software package available on campus computers. Check with your instructor or your school's IT office.

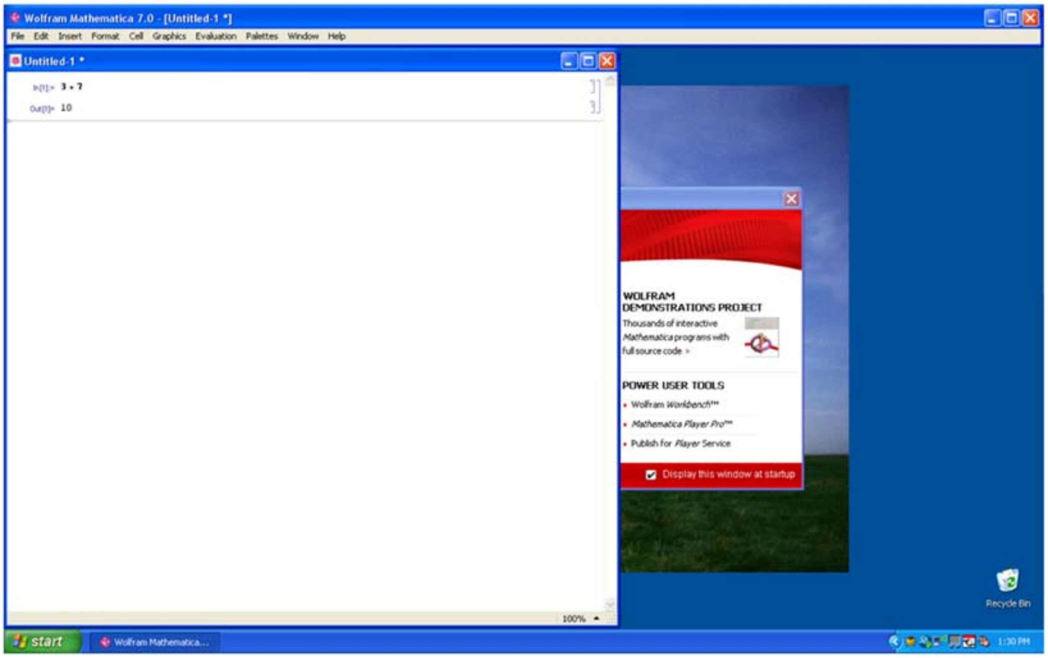
■ 1.1.1 First-Time Users of *Mathematica* 7

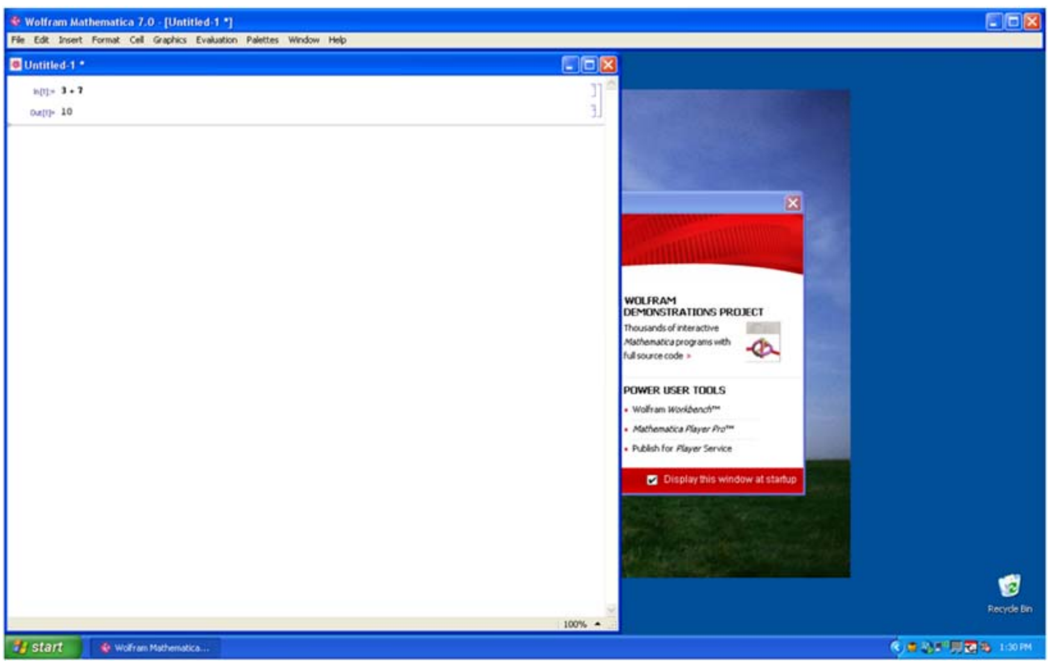
Launch the program *Mathematica* 7 on your computer. *Mathematica* will automatically create a new Notebook (see typical startup screen below).



■ 1.1.2 Entering and Evaluating Input Commands

Just start typing to input commands (a cell formatted as an input box will be automatically created). For example, type 3+7. To evaluate this command or any other command(s) contained inside an input box, simultaneously press SHIFT+ENTER, that is, the keys SHIFT and ENTER at the same time. Be sure your mouse's cursor is positioned inside the input box or else select the input box(es) that you want to evaluate. The kernel application, which does all the computations, will load at the first evaluation. This is a one-time procedure whenever *Mathematica* is launched and may take a few seconds depending on the speed of your computer, so be patient.

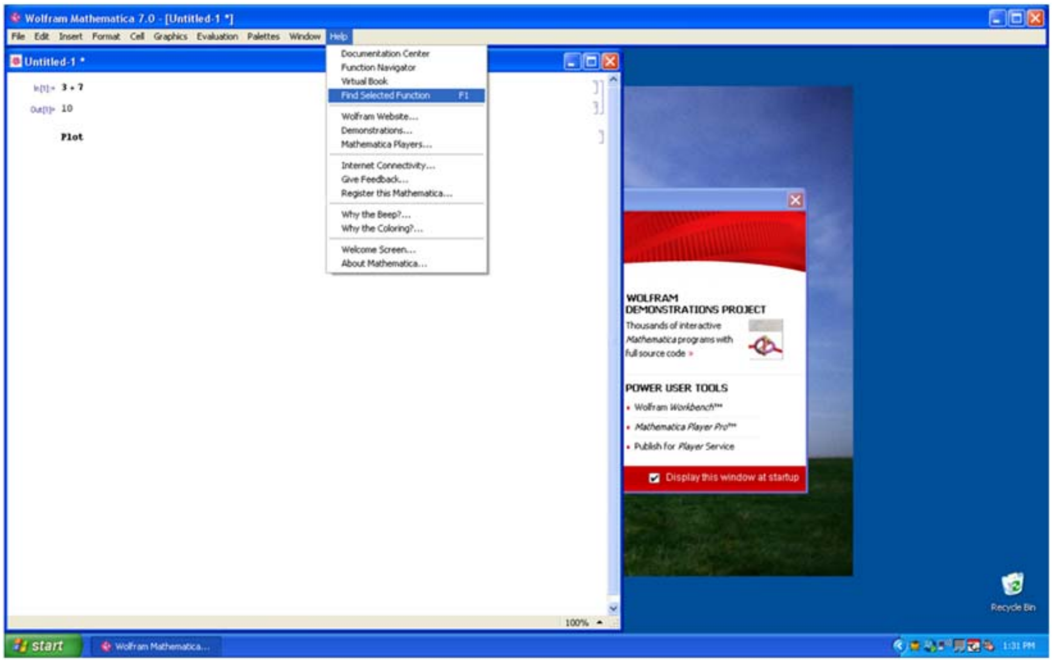




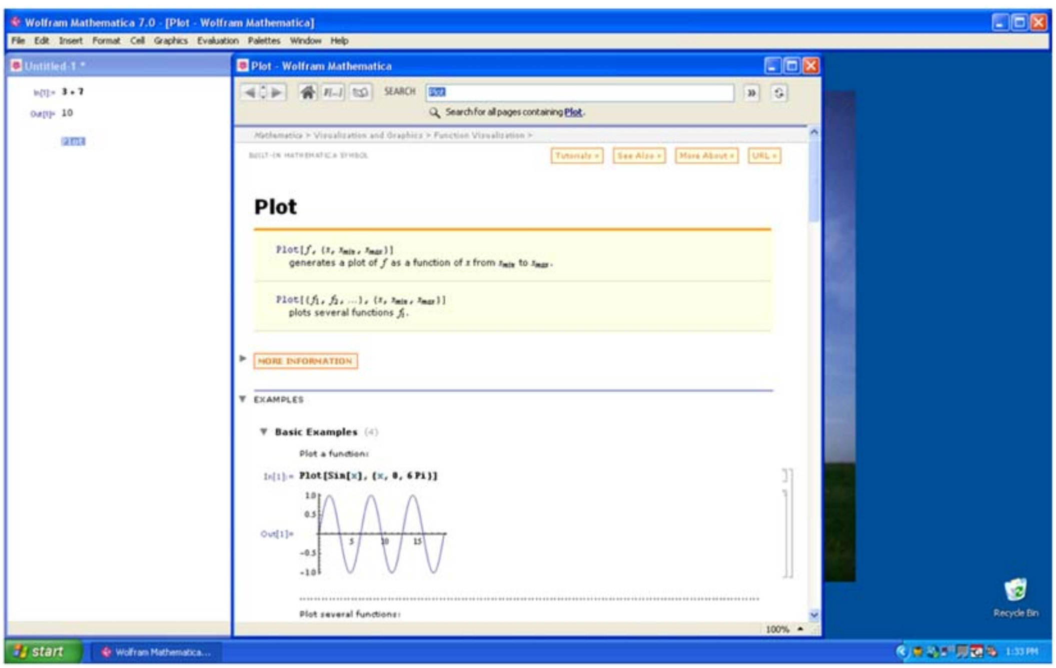
As can be seen from the screen shot above, a cell formatted as an output box and containing the value 10 is generated as a result of the evaluation. To create another input box (cell), just start typing again and an input box will be inserted at the position of the cursor (use the mouse to position the cursor where you would like to insert the new input box).

■ 1.1.3 Documentation Center (Help Menu)

Mathematica provides an online help menu to answer many of your questions about the program. One can search for a particular command expression in the Documentation Center under this menu or else just position the cursor next to the expression (for example, **Plot**) and select **Find Selected Function** (F1) under the Help menu (see screen shot that follows).



Mathematica will then display a description of **Plot**, including examples on how to use it (see screen shot below).



For only a brief description of **Plot** (or any other expression **expr**), just evaluate **?Plot** (or **?expr**).

```
?Plot
```

Plot[f, {x, x_{min}, x_{max}}] generates a plot of f as a function of x from x_{min} to x_{max}.
Plot[{f_1, f_2, …}, {x, x_{min}, x_{max}}] plots several functions f_i. ≫

1.2 *Mathematica*'s Conventions for Inputting Commands

1.2.1 Naming

Built-in *Mathematica* commands, functions, constants, and other expressions begin with capital letters and are (for the most part) one or more full-length English words (each word is capitalized). Furthermore, *Mathematica* is case sensitive; a common cause of error is the failure to capitalize command names. For example, **Plot**, **Integrate**, and **FindRoot** are valid function names. **Sin**, **Exp**, **Det**, **GCD**, and **Max** are some of the standard mathematical abbreviations that are exceptions to the full-length English word(s) rule.

User-defined functions and variables can be any mixture of uppercase and lowercase letters and numbers. However, a name cannot begin with a number. User-defined functions may begin with a lowercase letter, but this is not required. For example, **f**, **g1**, **myPlot**, **r12**, **sOLution**, and **Method1** are permissible function names.

1.2.2. Parenthesis, Brackets, and Braces

Mathematica interprets various types of delimiters (brackets) differently. Using an incorrect type of delimiter is another common source of error. *Mathematica*'s bracketing conventions are as follows:

1) Parentheses, **( )**, are used only for grouping expressions. For example, **(x-y)^2**, **1/(a+b)** and **(x^3-y)/(x+3y^2)** demonstrate proper use of parentheses. Users should realize that *Mathematica* understands **f(2)** as f multiplied with 2 and not as the function $f(x)$ evaluated at $x = 2$.

2) Square brackets, **[ ]**, are used to enclose function arguments. For example, **Sqrt[346]**, **Sin[Pi]**, and **Simplify[(x^3-y^3)/(x-y)]** are valid uses of square brackets. Therefore, to evaluate a function $f(x)$ at $x = 2$, we can type **f[2]**.

3) Braces or curly brackets, **{ }**, are used for defining lists, ranges and iterators. In all cases, list elements are separated by commas. Here are some typical uses of braces:
{1, 4, 9, 16, 25, 36}: This lists the square of the first six positive integers.
Plot[f[x],{x,-5,5}]: The list **{x,-5,5}** here specifies the range of values for x in plotting f.
Table[m^3,{m,1,100}]: The list **{m,1,100}** here specifies the values of the iterator m in generating a table of cube powers of the first 100 whole numbers.

1.2.3. Lists

A *list* (or string) of elements can be defined in *Mathematica* as **List[e_1, e_2,...,e_n]** or **{e_1, e_2,...,e_n}**. For example, the following command defines $S = \{1, 3, 5, 7, 9\}$ to be the list (set) of the first five odd positive integers.

```
S = List[1, 3, 5, 7, 9]
```

```
{1, 3, 5, 7, 9}
```

To refer to the kth element in a list named **expr**, just evaluate **expr[[k]]**. For example, to refer to the fourth element in S, we evaluate

```
S[[4]]
```

```
7
```

It is also possible to define nested lists whose elements are themselves lists, called *sublists*. Each sublist contains *subelements*. For example, the list $T = \{\{1, 3, 5, 7, 9\}, \{2, 4, 6, 8, 10\}\}$ contains two elements, each of which is a list (first five odd and even positive integers).

```
T = {{1, 3, 5, 7, 9}, {2, 4, 6, 8, 10}}
```

```
{{1, 3, 5, 7, 9}, {2, 4, 6, 8, 10}}
```

To refer to the *k*th subelement in the *j*th sublist of **expr**, just evaluate **expr[[j,k]]**. For example, to refer to the third subelement in the second sublist of *T* (or 6), we evaluate

```
T[[2, 3]]
```

```
6
```

A detailed description of how to manipulate lists (e.g., to append elements to a list or delete elements from a list) can be found in *Mathematica*'s Documentation Center (under the Help menu). Search for the entry **List**.

■ 1.2.4. Equal Signs

Here are *Mathematica*'s rules regarding the use of equal signs:

1) A single equal sign (=) assigns a value to a variable. Thus, entering **q = 3** means that *q* will be assigned the value 3.

```
q = 3
```

```
3
```

If we then evaluate **10+q^3**, *Mathematica* will return 37.

```
10 + q^3
```

```
37
```

As another example, suppose the expression **y = x^2-x-1** is entered.

```
y = x^2 - x - 1
```

$-1 - x + x^2$

If we then assign a value for x, say $x = 3$, then in any future input containing y, *Mathematica* will use this value of x to calculate y, which would be 5 in our case.

```
x = 3
y
```

```
3
```

```
5
```

2) A colon-equal sign (**:=**) creates a delayed statement for an expression and can be used to define a function. For example, typing **f[x_]: = x^2-x-1** tells *Mathematica* to delay the assignnment of $f(x)$ as a function until f is evaluated at a particular value of x.

```
f[x_] := x^2 - x - 1
f[3]
```

```
5
```

We will say more about defining functions in section 1.3 below.

3) A double-equal sign (= =) is a test of equality between two expressions. Since we previously set **x = 3**, then evaluating **x = = 3** returns True, whereas evaluating **x = = -3** returns False.

```
x == 3
x == -3
```

```
True
```

```
False
```

Another common usage of the double equal sign (**= =**) is to solve equations, such as the command **Solve[x^2-x-1= = 0, x]** (see Section 1.5). Be sure to clear the variable *x* beforehand.

```
Clear[x]
Solve[x^2 - x - 1 == 0, x]
```

$$\left\{\left\{x \to \frac{1}{2}\left(1-\sqrt{5}\right)\right\}, \left\{x \to \frac{1}{2}\left(1+\sqrt{5}\right)\right\}\right\}$$

■ 1.2.5. Referring to Previous Results

Mathematica saves all input and output in a session. Type **In[k]** (or **Out[k]**) to refer to input (or output) line numbered *k*. One can also refer to previous output by using the percent sign **%**. A single **%** refers to *Mathematica*'s last output, **%%** refers to the next-to-last ouput, and so forth. The command **%k** refers to the output line numbered *k*. For example, **%12** refers to output line number 12.

```
Out[12]
```

```
5
```

Mathematica saves all input and output in a session. Type **In[k]** (or **Out[k]**) to refer to input (or output) line numbered *k*. One can also refer to previous output by using the percent sign **%**. A single **%** refers to *Mathematica*'s last output, **%%** refers to the next-to-last ouput, and so forth. The command **%k** refers to the output line numbered *k*. For example, **%12** refers to output line number 12.

```
%12
```

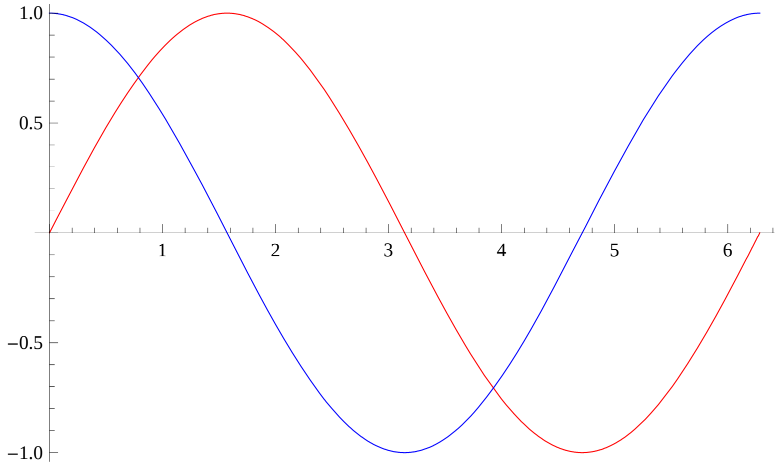
```
5
```

NOTE: CTRL+L reproduces the last input and CTRL+SHIFT+L reproduces the last output.

■ 1.2.6. Commenting

One can insert comments on any input line. The comments should be enclosed between the delimiters (* and *). For example,

```
(* This command plots the graph of two functions in different colors. *)
Plot[{Sin[x], Cos[x]}, {x, 0, 2 Pi}, PlotStyle -> {Red, Blue}]
```



NOTE: One can also insert comments by creating a text box. First, create an input box. Then select it and format it as Text using the drop-down window menu.

■ 1.3 Basic Calculator Operations

Mathematica uses the standard symbols **+**, **-**, *****, /, ^, **!** for addition, subtraction, multiplication, division, raising powers (exponents), and factorials, respectively. Multiplication can also be performed by leaving a blank space between factors. Powers can also be entered by using the palette menu to generate a superscript box (or else press CTRL+6) and fractions can be entered by generating a fraction box (from palette menu or pressing CTRL+/).

To generate numerical output in decimal form, use the command **N[expr]** or **N[expr,d]**. In most cases, **N[expr]** returns six digits of **expr** by default and may be in the form *n*.abcde $* 10^m$ (scientific notation), whereas **N[expr,d]** attempts to return *d* digits of expr.

NOTE: *Mathematica* can perform calculations to arbitrary precision and handle numbers that are arbitrarily large or small.

Here are some examples:

```
Pi
```

π

```
N[Pi]
```

3.14159

```
N[Pi, 200]
```

3.1415926535897932384626433832795028841971693993751058209749445923078164062862089986280348253421170679821480865132823066470938446095505822317253594081284811174502841027019385211055596446229489549303820

6^{5^4}

```
2 210 708 544 304 025 665 789 890 545 869 282 983 189 550 730 342 026 817 054 484 706 923 451
 925 215 263 872 221 875 601 412 877 526 055 033 568 150 952 983 731 997 599 172 762 855 409 042
 386 638 455 130 114 567 918 179 610 415 056 135 043 685 865 981 465 821 197 678 998 054 981 600
 364 232 459 680 450 883 986 513 397 952 866 100 532 961 319 277 446 513 221 836 325 497 685 382
 494 082 501 890 188 075 860 096 650 899 943 982 604 939 901 346 570 765 022 869 199 395 889 789
 728 382 946 141 484 842 179 531 904 056 612 897 175 359 078 633 987 736 867 003 878 781 857 613
 656 893 578 474 392 372 463 398 376 238 316 805 554 810 164 724 551 909 376
```

```
1 / 300 !
```

```
1 /
 306 057 512 216 440 636 035 370 461 297 268 629 388 588 804 173 576 999 416 776 741 259 476 533
  176 716 867 465 515 291 422 477 573 349 939 147 888 701 726 368 864 263 907 759 003 154 226
  842 927 906 974 559 841 225 476 930 271 954 604 008 012 215 776 252 176 854 255 965 356 903
  506 788 725 264 321 896 264 299 365 204 576 448 830 388 909 753 943 489 625 436 053 225 980
  776 521 270 822 437 639 449 120 128 678 675 368 305 712 293 681 943 649 956 460 498 166 450
  227 716 500 185 176 546 469 340 112 226 034 729 724 066 333 258 583 506 870 150 169 794 168
  850 353 752 137 554 910 289 126 407 157 154 830 282 284 937 952 636 580 145 235 233 156 936
  482 233 436 799 254 594 095 276 820 608 062 232 812 387 383 880 817 049 600 000 000 000 000
  000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000 000
```

```
(* This command returns a decimal answer of the last output *)
N[%]
```

$3.267359761105326 \times 10^{-615}$

Example 1.1. How close is $e^{\sqrt{163}\,\pi}$ to being an integer?

Solution:

```
E^(Pi * Sqrt[163])
```

$e^{\sqrt{163}\,\pi}$

```
N[%, 40]
```

$2.625374126407687439999999999992500725972 \times 10^{17}$

We can rewrite this output in non-scientific notation by moving the decimal point 17 places to the right. This shows that $e^{\sqrt{163}\,\pi}$ is very close to being an integer. Another option is to use the command **Mod[n,m]**, which returns the remainder of n when divided by m, to obtain the fractional part of $e^{\sqrt{163}\,\pi}$:

```
Mod[%, 1]
```

```
0.999999999992500725972
```

```
1 - %
```

$7.499274028 \times 10^{-13}$

■ 1.4 Functions

There are two different ways to represent functions in *Mathematica*, depending on how they are to be used. Consider the following example:

Example 1.2. Enter the function $f(x) = \frac{x^2+x+2}{x+1}$ into *Mathematica*.

Solution:

Method 1: Simply assign f the expression $\frac{x^2+x+2}{x+1}$, for example,

```
Clear[f, x] (* This clears the arguments f and x *)
f = (x^2 + x + 2) / (x + 1)
```

$$\frac{2 + x + x^2}{1 + x}$$

To evaluate $f(x)$ at $x = 10$, we use the substitution command **/.** (slash-period) as follows:

```
f /. x -> 10
```

$$\frac{112}{11}$$

Warning: Recall that *Mathematica* reads *f(x)* as *f* multiplied by *x*; commas are considered delimiters.

```
f (10)
```

$$\frac{10\,(2 + x + x^2)}{1 + x}$$

Method 2: An alternative way to explicitly represent *f* as a function of the argument *x* is to enter

```
Clear[f]
f[x_] := (x^2 + x + 2) / (x + 1)
```

Evaluating the command **f[10]** now tells *Mathematica* to compute f at $x = 10$.

```
f[10]
```

$$\frac{112}{11}$$

More generally, the command **f[{a,b,c,...}]** evaluates $f(x)$ for every value of x in the list **{a,b,c,...}**:

```
f[{-3, -2, -1, 0, 1, 2, 3}]
```

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

$$\left\{-4, -4, \text{ComplexInfinity}, 2, 2, \frac{8}{3}, \frac{7}{2}\right\}$$

Here, *Mathematica* is warning us that it has encountered the undefined expression $\frac{1}{0}$ in evaluating $f(-1)$ by returning the message ComplexInfinity.

Remark: If there is no need to attach a label to the expression $\frac{x^2+x+2}{x+1}$, then we can directly enter this expression into *Mathematica*:

$$\frac{x^2+x+2}{x+1} \text{ /. x -> 10}$$

$$\frac{112}{11}$$

$$\frac{x^2+x+2}{x+1} \text{ /. x -> \{-3, -2, -1, 0, 1, 2, 3\}}$$

Power::infy : Infinite expression $\frac{1}{0}$ encountered. ≫

$$\left\{-4, -4, \text{ComplexInfinity}, 2, 2, \frac{8}{3}, \frac{7}{2}\right\}$$

Piece-wise functions can be defined using the command **If[cond, *p*, *q*]**, which evaluates **p** if **cond** is true; otherwise, **q** is evaluated.

Example 1.3. Enter the following piece-wise function into *Mathematica:*

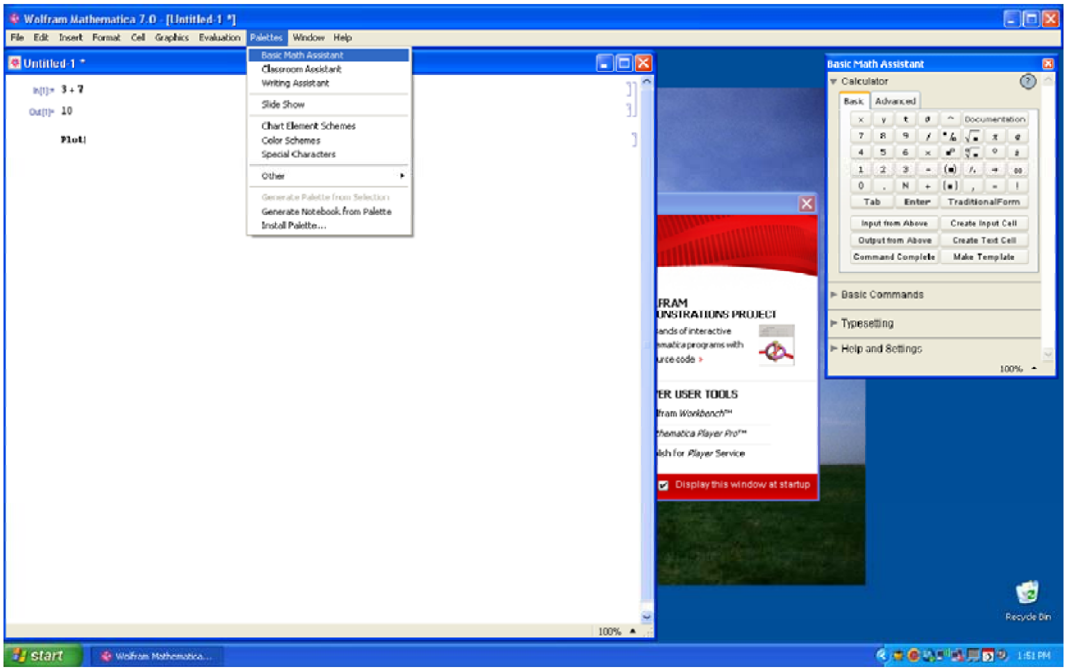
$$f(x) = \begin{cases} \tan\left(\frac{\pi x}{4}\right), & \text{if } |x| < 1; \\ x, & \text{if } |x| \geq 1. \end{cases}$$

Solution:

```
f[x_] := If[Abs[x] < 1, Tan[Pi * x / 4], x]
```

■ 1.5 Palettes

Mathematica allows us to enter commonly used mathematical expressions and commands from six different palettes. Palettes are calculator pads containing buttons that can be clicked on to insert the desired expression or command into a command line. These palettes can be found under the Palettes menu. If the Basic Math Assistant Palette does not appear by default, then click on Palettes from the menu and select it. One can also select more advanced math typesetting palettes such as the Basic Math Input and Algebraic Manipulate Palettes.



Example 1.4. Enter $\sqrt{\dfrac{3}{\pi^4}}$ into a notebook.

Solution:

Here is one set of instructions for entering this expression using the Basic Math Assistant Palette:

a) Click on the palette button $\sqrt{\square}$.

b) Click on $\frac{\square}{\square}$.

c) Enter the number 3 into the highlighted top placeholder.

$$\sqrt{\frac{3}{\square}}$$

d) Press the TAB key to move the cursor to the bottom placeholder.

e) Click on $\square^{\square}$.

f) To insert π into the base position, click on the palette button for π.

$$\sqrt{\frac{3}{\pi^{\square}}}$$

g) Press the TAB key to move the cursor to the superscript placeholder.

h) Enter the number 4.

$$\sqrt{\frac{3}{\pi^4}}$$

■ 1.6 Solving Equations

Mathematica has a host of built-in commands to help the user solve equations and manipulate expressions. The command **Solve[lhs == rhs, var]** solves the equation **lhs == rhs** (recall *Mathematica*'s use of the double-equal sign) for the variable **var**. For example, the command below solves the quadratic equation $x^2 - 4 = 0$ for x.

```
Solve[x^2 - 4 == 0, x]
```

```
{{x → -2}, {x → 2}}
```

A system of m equations in n unknowns can also be solved with using the same command, but formatted as **Solve[{lhs$_1$ == rhs$_1$, lhs$_2$ == rhs$_2$, ..., lhs$_m$ == rhs$_m$}, {x_1, x_2, ..., x_n}]**. In situations where exact solutions cannot be obtained (e.g., certain polynomial equations of degree 5 or higher), numerical approximations can be obtained through the command **NSolve[lhs == rhs, var]**. Here are two examples:

```
Clear[x, y]
Solve[{2 x - y == 3, x + 4 y == -2}, {x, y}]
```

$$\left\{\left\{x \to \frac{10}{9},\ y \to -\frac{7}{9}\right\}\right\}$$

```
NSolve[x^5 - x + 1 == 0, x]
```

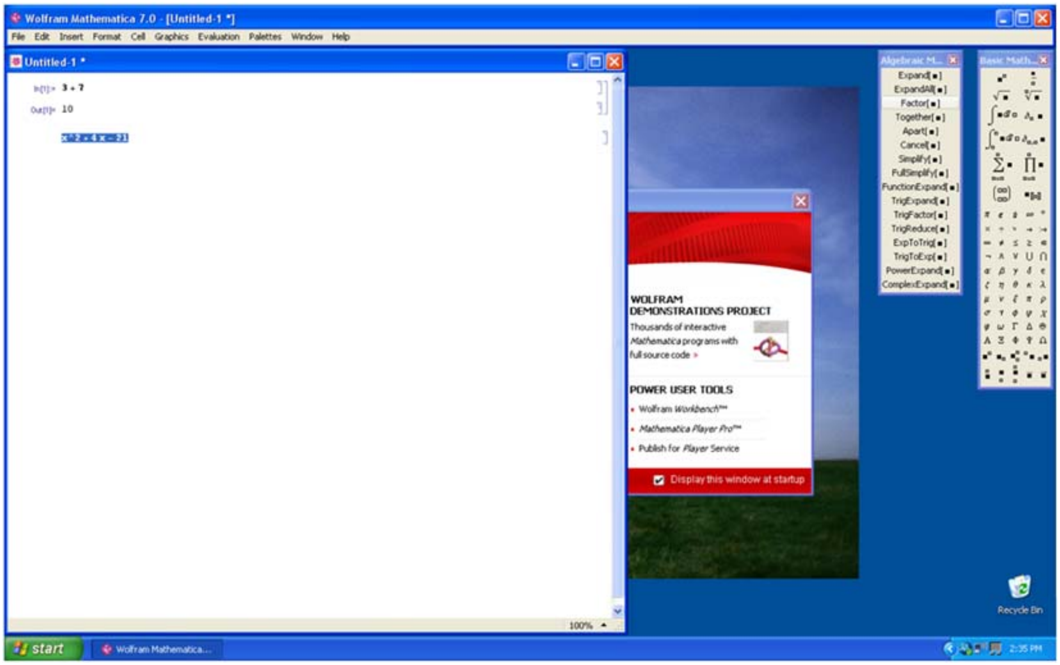
```
{{x → -1.1673}, {x → -0.181232 - 1.08395 i}, {x → -0.181232 + 1.08395 i},
 {x → 0.764884 - 0.352472 i}, {x → 0.764884 + 0.352472 i}}
```

There are many commands to algebraically manipulate expressions: **Expand**, **Factor**, **Together**, **Apart**, **Cancel**, **Simplify**, **FullSimplify**, **TrigExpand**, **TrigFactor**, **TrigReduce**, **ExpToTrig**, **PowerExpand**, and **ComplexExpand**.

```
Factor[x^2 + 4 x - 21]
```

```
(-3 + x) (7 + x)
```

NOTE: These commands can also be entered from the Algebraic Manipulation Palette; highlight the expression to be manipulated and click on the button corresponding to the command to be inserted. The screen shot below demonstrates how to select the **Factor** command from the Algebraic Manipulate Palette to factor the highlighted expression $x^2 + 4x - 21$.



■ Exercises

In Exercises 1 through 5, evaluate the expressions:

1. 103.41+20*76 2. $\frac{5^2+\pi}{1+\pi}$ 3. $\frac{1}{1+\frac{1}{1+\frac{1}{4!}}}$ 4. $\frac{2.06*10^9}{0.99*10^{-8}}$ 5. What is the remainder of 1998 divided by 13?

In Exercises 6 through 8, enter the functions into *Mathematica* and evaluate each at $x = 1$:

6. $f(x) = 2x^3 - 6x^2 + x - 5$ 7. $g(x) = \frac{x^2-1}{x^2+1}$ 8. $h(x) = \left|\sqrt{x} - 3\right|$

In Exercises 9 through 11, **e**valuate the functions at the given point using *Mathematica*:

9. $f(x) = 1001 + x^4$ at $x = 25$ 10. $1 + \sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}$ at $x = \pi$ 11. $1 + \frac{1}{2+\frac{(2x+1)^2}{2+\frac{(4x+1)^2}{2}}}$ at $x = 1$

In Exercises 12 through 17, enter the expressions into *Mathematica*:

12. $\sqrt[3]{80}$ 13. $\frac{\sqrt[5]{1024}}{2^{-3}}$ 14. $\sqrt[3]{\sqrt{125}}$

15. $\sqrt{\sqrt[3]{10\,a^7\,b}}$ 16. $\left(\frac{x^{-3}\,y^4}{5}\right)^{-3}$ 17. $\left(\frac{3\,m^{\frac{1}{6}}\,n^{\frac{1}{3}}}{4\,n^{-\frac{2}{3}}}\right)^2$

In Exercises 18 through 21, expand the expressions:

18. $(x+1)(x-1)$ 19. $(x+y-2)(2x-3)$ 20. $\left(x^2+x+1\right)(x-1)$ 21. $\left(x^3+x^2+x+1\right)(x-1)$

In Exercises 22 through 25, factor the expressions:

22. $x^3 - 2x^2 - 3x$ 23. $4x^{2/3} + 8x^{1/3} + 3.6$ 24. $6 + 2x - 3x^3 - x^4$ 25. $x^5 - 1$

In Exercises 26 through 29, **s**implify the expressions using both of the commands **Simplify** and **FullSimplify** (the latter uses a wider variety of methods to simplify expressions).

26. $\frac{x^2+4x-12}{3x-6}$ 27. $\frac{\left(\frac{2}{x}-3\right)}{1-\frac{1}{x-1}}$ 28. $(x(1-2x))^{-3/2}+(1-2x)^{-1/2}$ 29. $\frac{x^5-1}{x-1}$

In Exercises 30 through 33, **s**olve the equations for x (compare outputs using both the **Solve** and **NSolve** commands):

30. $x^2-x+1=0$ 31. $x(1-2x)^{-3/2}+(1-2x)^{-1/2}=0$ 32. $x^3-1=0$ 33. $\sqrt{1+\sqrt{x+\sqrt{x^2}}}=5$