Real Analysis I: Problems for Chapters 4, 5, and 6

1. Use the definition of the derivative to find $f'(x)$ for the following functions.

(a) $f(x) = 1/x$ for $x \neq 0$.

(b) $f(x) = \sqrt{x}$ for $x > 0$. Does $f'(0)$ exist?

2. Let $f(x) = \begin{cases} 3x + 1, & \text{if } x < 1 \\ x^2 + 2x + 1, & \text{if } x \geq 1 \end{cases}$ Is f continuous at $x = 1$? Does $f'(1)$ exist? In each case prove your assertions.

3. Let $f(x) = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 1$. Find all points where the tangent line is parallel to the line $y = 6x$.

4. Let $f(x) = x^3 - x$. What does Rolles Theorem guarantee on $[0, 1]$? Find such a point.

5. Let $f(x) = x^3 + 2x - 1$. What does the Mean Value Theorem guarantee on $[-1, 2]$? Can you find such a point?

6. Determine whether the given series converges or diverges. Indicate the test you used to reach your conclusion.

a) $\sum_{k=1}^{\infty} \frac{(-1)^k}{2^{k-1}}$

b) $\sum_{k=1}^{\infty} \frac{3^k - 2^k}{6^k}$

c) $\sum_{k=1}^{\infty} \frac{k+1}{k^2}$

d) $\sum_{k=1}^{\infty} \frac{(k}{k+200}$

e) $\sum_{k=1}^{\infty} \frac{1}{k^2 + 2k}$

f) $\sum_{k=1}^{\infty} \frac{\sin k}{k^2}$

g) $\sum_{k=1}^{\infty} \frac{(k}{e^k}$

h) $\sum_{k=1}^{\infty} \frac{k^2 2^{k+1}}{3^k}$

i) $\sum_{k=1}^{\infty} \frac{\sin k}{k^2}$

7. Show that $\sum_{k=1}^{\infty} \frac{1}{4k^2 - 1} = \frac{1}{2}$.

8. Find all values of x for which the series converges.

a) $\sum_{k=1}^{\infty} \frac{(x)^k}{2^k}$

b) $\sum_{k=1}^{\infty} 3(x - 1)^k$

c) $\sum_{k=1}^{\infty} \frac{k^2(x+3)^k}{k!}$

9. Section 5.1 Exercises 1, 5, 6, 7,

10. Section 6.1 Exercises 1, 2