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http://ciks.cbt.nist.gov/~garbocz/SP946/node8.htm

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6. Expressions for Describing Steady Shear Non-Newtonian Flow

The expressions shown here are used to characterize the *non-Newtonian* behavior of fluids under equilibrium, *steady shear flow* conditions. Many phenomenological and empirical models have been reported in the literature. Only those having a direct and significant implication for suspensions, gels and pastes have been included here. A brief description of each relationship is given with examples of the types of materials to which they typically are applied. In defining the number of parameters associated with a particular model, the term "parameter" in this case refers to adjustable (arbitrary) constants, and therefore excludes measured quantities. Some of these equations have alternative representations other than the one shown. More detailed descriptions and alternative expressions can be found in the sources listed in the bibliography.

Bingham

$$\sigma = \sigma_B + \eta_{pl}\dot{\gamma}$$
$$\dot{\gamma} = 0 \text{ for } \sigma < \sigma_B$$

Where $\dot{\gamma}$ is the shear rate (e.g. dv/dx). The Bingham relation is a two parameter model used for describing *viscoplastic* fluids exhibiting *a yield response*. The ideal Bingham material is an *elastic* solid at low *shear stress* values and a *Newtonian* fluid above a critical value called the Bingham *yield stress*, σ_B. The *plastic viscosity* region exhibits a linear relationship between shear stress and *shear rate*, with a constant *differential viscosity* equal to the plastic viscosity, η_{pl}.

Carreau-Yasuda (This is used in Comsol)

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \left[1 + (\lambda\dot{\gamma})^a\right]^{(n-1)/a}$$

A model that describes *pseudoplastic* flow with asymptotic viscosities at zero (η_0) and infinite (η_∞) *shear rates*, and with no *yield stress*. The parameter λ is a constant with units of time, where $1/\lambda$ is the critical shear rate at which viscosity begins to decrease. The *power-law* slope is (n-1) and the parameter *a* represents the width of the transition region between η_0 and the power-law region. If η_0 and η_∞ are not known independently from experiment, these quantities may be treated as additional adjustable parameters.

CARREAU MODEL: A mathematical expression describing the shear thinning behavior of polymers. It is more realistic than the power-law model because it fits the data very well at both high and low shear rates.

where: η_0, λ, n are curve fitting parameters and is the shear rate. Due to the mathematical complexities it is not possible to obtain analytical solutions with this model, but it is excellent for numerical simulations of flow processes.

Casson

$$\sigma^{1/2} = \sigma_y^{1/2} + \eta_1^{1/2}\dot{\gamma}^{1/2}$$
$$\dot{\gamma} = 0 \text{ for } \sigma < \sigma_y$$

A two parameter model for describing flow behavior in *viscoplastic* fluids exhibiting a *yield response*. The parameter σ_y is the *yield stress* and η_{pl} is the differential high shear (*plastic*) viscosity. This equation is of the same form as the *Bingham* relation, such that the exponent is ½ for a Casson plastic and 1 for a Bingham plastic.

Cross

$$\frac{\eta - \eta_\infty}{\eta_0 - \eta_\infty} = \frac{1}{\left(1 + \lambda\dot{\gamma}^m\right)}$$

Similar in form to the *Carreau-Yasuda* relation, this model describes *pseudoplastic* flow with asymptotic viscosities at zero (η_0) and infinite (η_∞) shear rates, and no *yield stress*. The parameter λ is a constant with units of time, and *m* is a dimensionless constant with a typical range from 2/3 to 1.

Ellis

$$\eta = \frac{\eta_0}{1+\left(\sigma/\sigma_2\right)^{\alpha-1}}$$

A two parameter model, written in terms of *shear stress*, used to represent a *pseudoplastic* material exhibiting a *power-law* relationship between shear stress and *shear rate*, with a low shear rate asymptotic viscosity. The parameter σ_2 can be roughly identified as the shear stress value at which η has fallen to half its final asymptotic value.

Herschel-Bulkley

$$\sigma = \sigma_y + k\dot{\gamma}^n$$

A three parameter model used to describe *viscoplastic* materials exhibiting a *yield response* with a *power-law* relationship between *shear stress* and *shear rate* above the *yield stress*, σ_y. A plot of *log* (σ - σ_y) versus *log* $\dot{\gamma}$ gives a slope *n* that differs from unity. The Herschel-Bulkley relation reduces to the equation for a *Bingham* plastic when *n*=1.

Krieger-Dougherty

$$\eta_r = \left(1-\Phi/\Phi_m\right)^{-[\eta]\Phi_m}$$

A model for describing the effect of particle self-crowding on suspension viscosity, where Φ is the particle volume fraction, Φ_m is a parameter representing the maximum packing fraction and [η] is the *intrinsic viscosity*. For ideal spherical particles [η]=2.5 (i.e. the Einstein coefficient). Non-spherical or highly charged particles will exhibit values for [η] exceeding 2.5. The value of [η] is also affected by the particle size distribution. The parameter Φ_m is a function of particle shape, particle size distribution and *shear rate*. Both [η] and Φ_m may be treated as adjustable model parameters.

The aggregate volume fraction (representing the effective volume occupied by particle aggregates, including entrapped fluid) can be determined using this equation if Φ_m is fixed at a reasonable value (e.g. 0.64 for random close packing or 0.74 for hexagonal close packing) and [η] is set to 2.5. In this case, Φ is the adjustable parameter and is equivalent to the aggregate volume fraction.

Meter

$$\eta = \eta_\infty + \frac{\eta_0 - \eta_\infty}{1 + (\sigma/\sigma_2)^{\alpha-1}}$$

Expressed in terms of *shear stress*, used to represent a *pseudoplastic* material exhibiting a *power-law* relationship between shear stress and *shear rate*, with both high (η_∞) and low (η_0) shear rate asymptotic viscosity limits. The parameter σ_2 can be roughly identified as the shear stress value at which η has fallen to half its final asymptotic value. The Meter and *Carreau-Yasuda* models give equivalent representations in terms of shear stress and shear rate, respectively. If η_0 and η_∞ are not known independently from experiment, these quantities may be treated as additional adjustable parameters.

Powell-Eyring

$$\eta = \eta_\infty + (\eta_0 - \eta_\infty)\,\frac{\sinh^{-1}(\tau\dot{\gamma})}{\tau\dot{\gamma}}$$

Derived from the theory of rate processes, this relation is relevant primarily to molecular fluids, but can be used in some cases to describe the *viscous* behavior of polymer solutions and *viscoelastic* suspensions over a wide range of *shear rates*. Here, η_∞ is the *infinite shear viscosity* η_0 is the *zero shear viscosity* and the fitting parameter τ represents a characteristic time of the measured system. If η_0 and η_∞ are not known independently from experiment, these quantities may be treated as additional adjustable parameters.

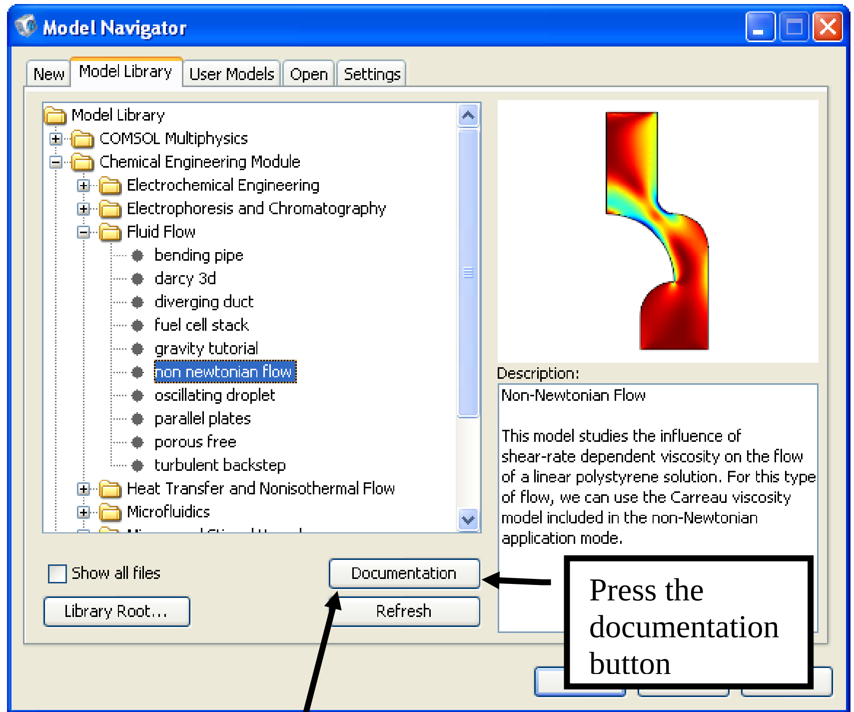
Power-law [Ostwald-de Waele]

$$\sigma = K\dot{\gamma}^{\,n}$$

A two parameter model for describing *pseudoplastic* or *shear-thickening* behavior in materials that show a negligible *yield response* and a varying *differential viscosity*. A log-log plot of σ versus $\dot{\gamma}$ gives a slope n (the *power-law* exponent), where n<1 indicates pseudoplastic behavior and n>1 indicates shear-thickening behavior.

Computer Laboratory Exercises:

1. Open the Documentation for the Non-Newtonian Fluids Module. To get to the documentation go to Model Library, Chemical Engineering Module, Fluid Flow, non Newtonian flow. Then press the Documentation button. You must allow blocked content to see the documentation.
2. Next follow the steps given in the documentation. The remainder of this pdf file gives help or hints for selected steps in the documentation from COMSOL.
3. You will simulate a 3-d object using axial symmetry.



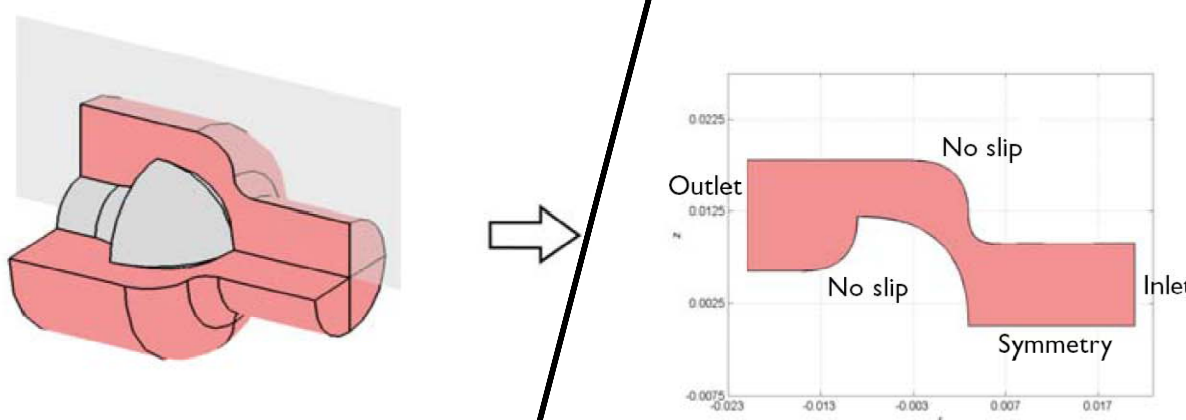
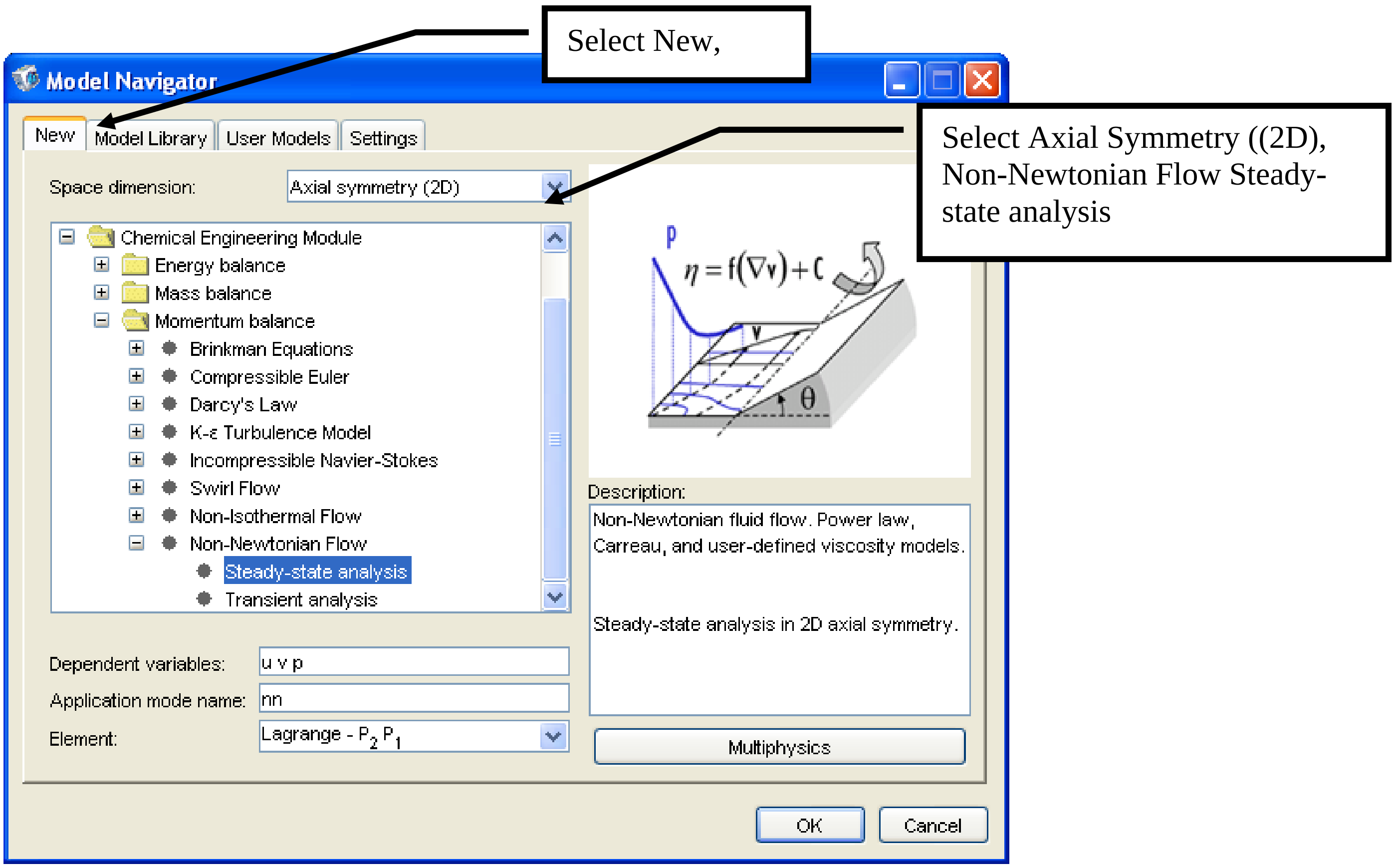


Figure 2-11: Model domain. The geometry can be simplified assuming axisymmetry.

4. READ the COMSOL documentation in you web browser. It will go over the model etc

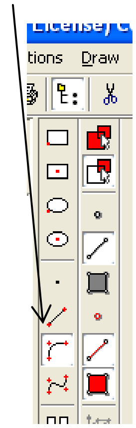
Press the documentation button

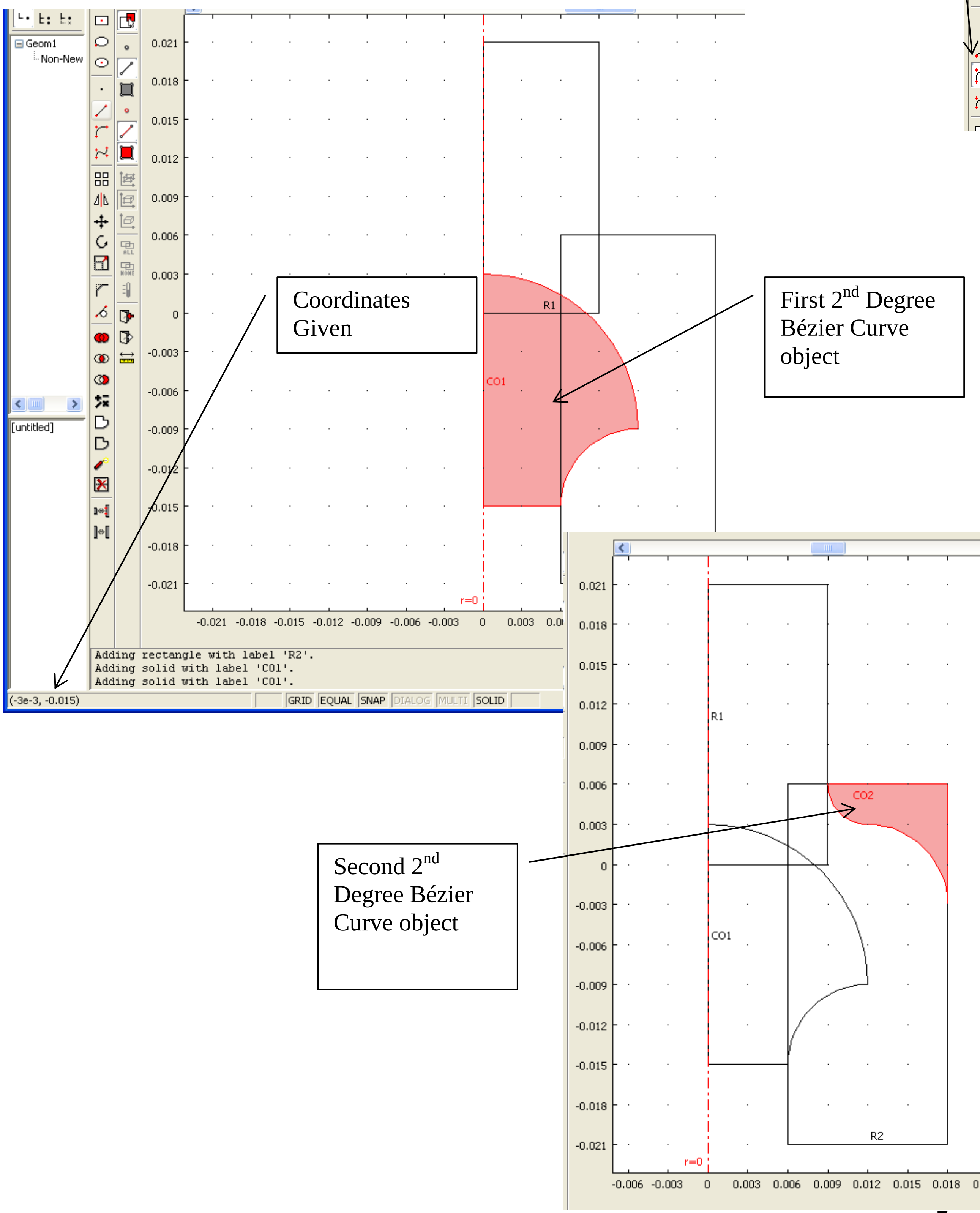
5. MAKE sure you select Axial Symmetry ((2D), Non-Newtonian Flow Steady-state analysis
6. Using the Graphical User Interface construct and complete this module. Conduct a parametric study of pressure to reproduce Figure 2-14.

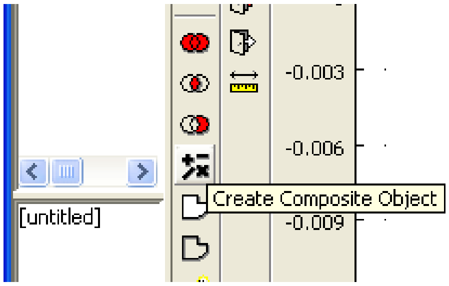


Geometry Hints:

Click on the numbers with the 2nd Degree Bézier Curve tool and then select the Line tool and click on the last coordinate. Only when you right click does the geometry get produced. Notice that in the lower left hand corner the r and z coordinates are given.







Create Composite Object Button

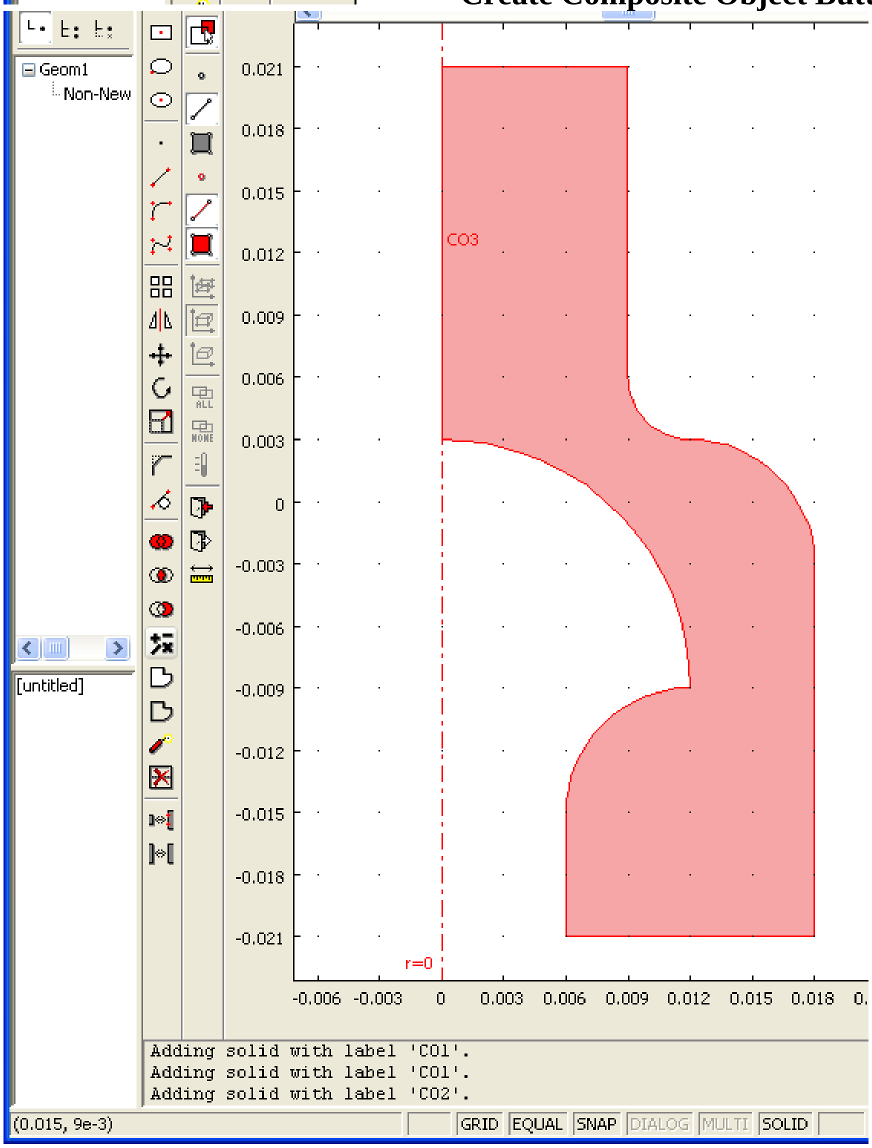


Figure 1: Result of Create Composite Object

MESH Generation

3. From the **Point selection** list, select Points 2, 3, 8, 9, and 11. (Press and hold the Ctrl button while selecting points)

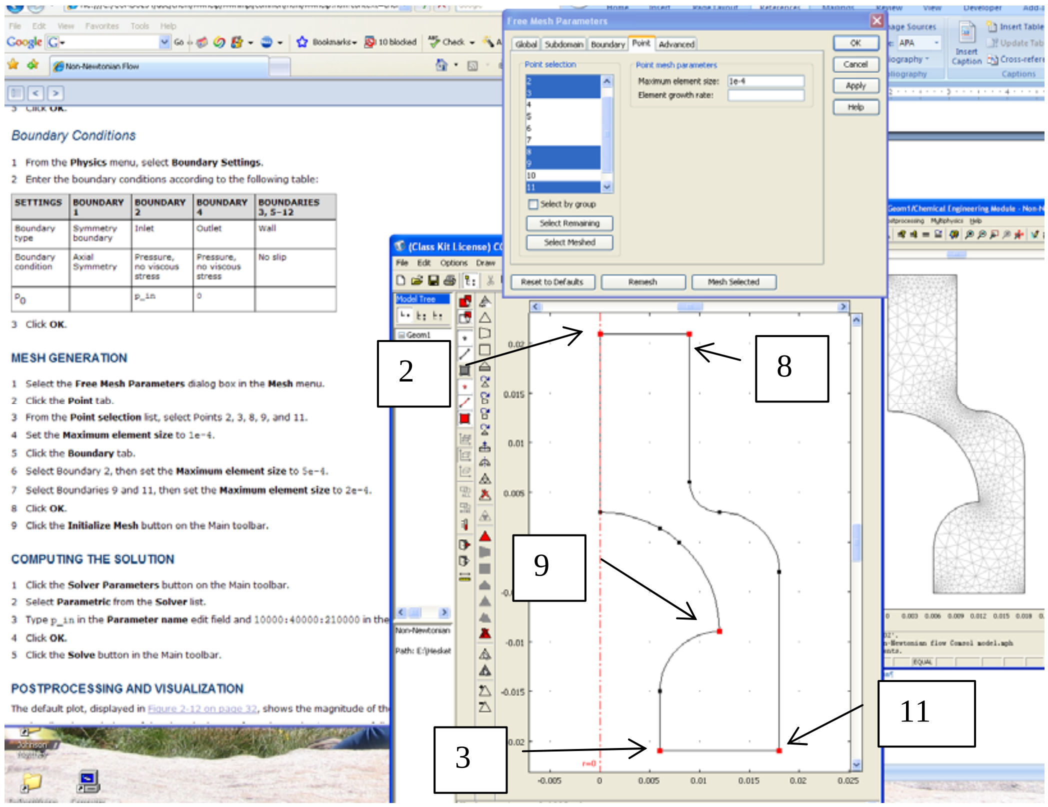


Figure 2: Result of Point Selection for Maximum element size

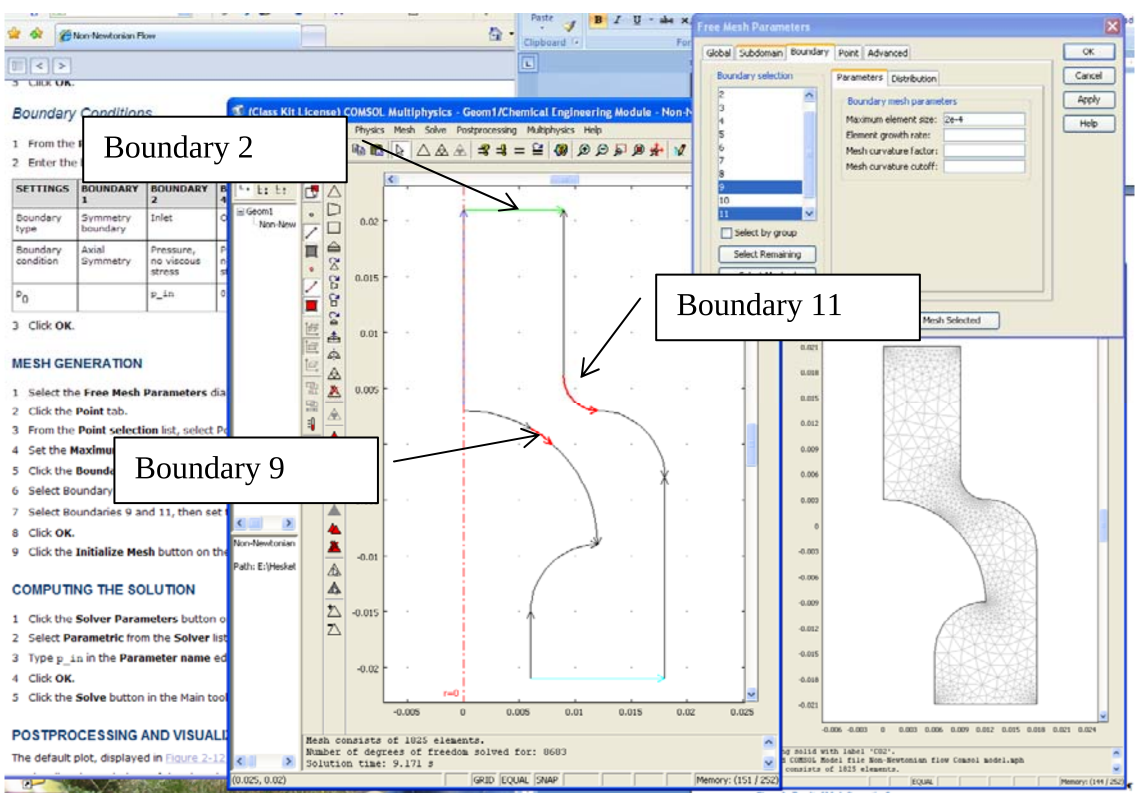


Figure 3: Results of specifing maximum element size for boundaries meshing

Why were these points and boundaries chosen to have a maximum mesh size?

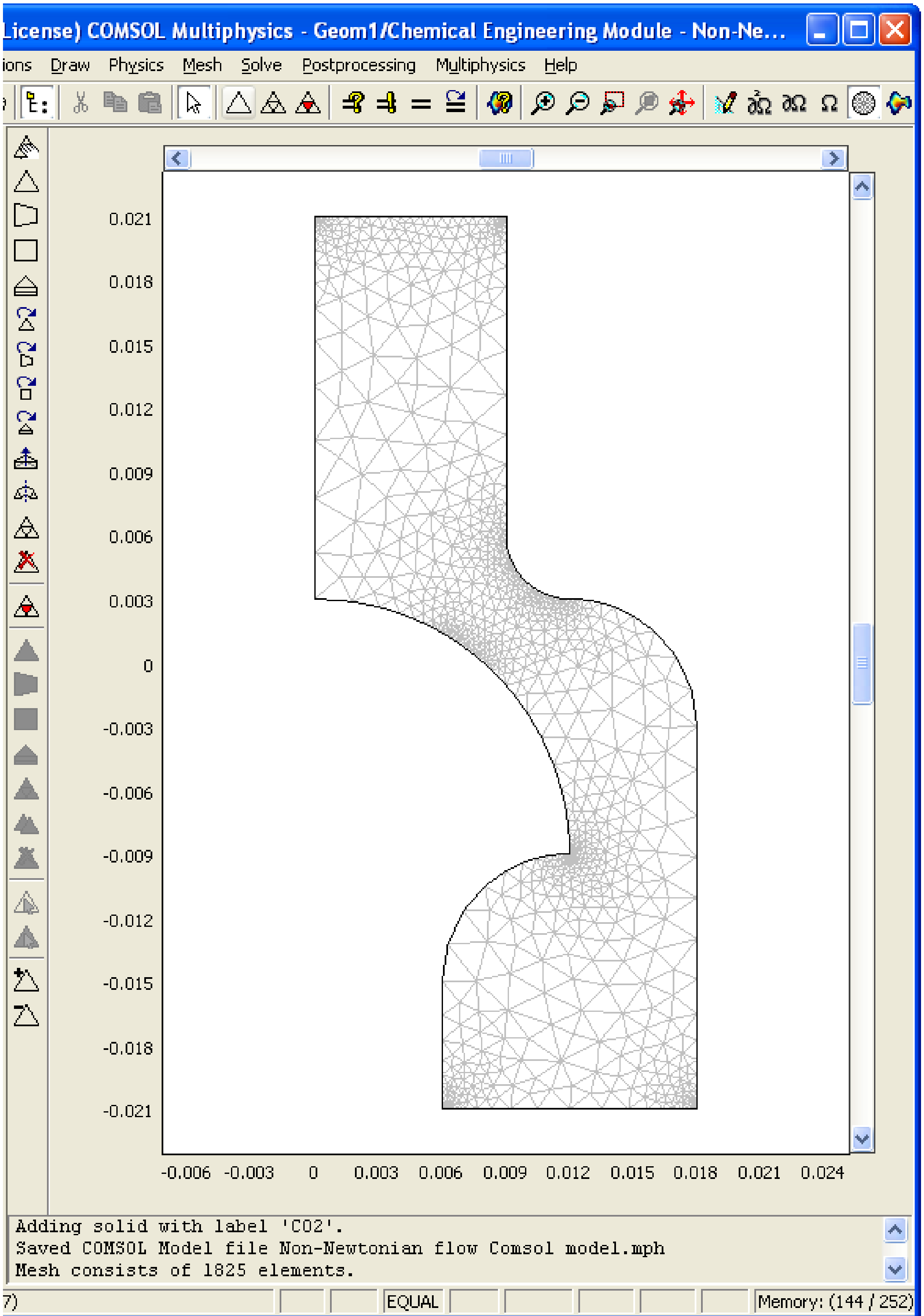


Figure 4: Result of Mesh Generation

Computing the Solution

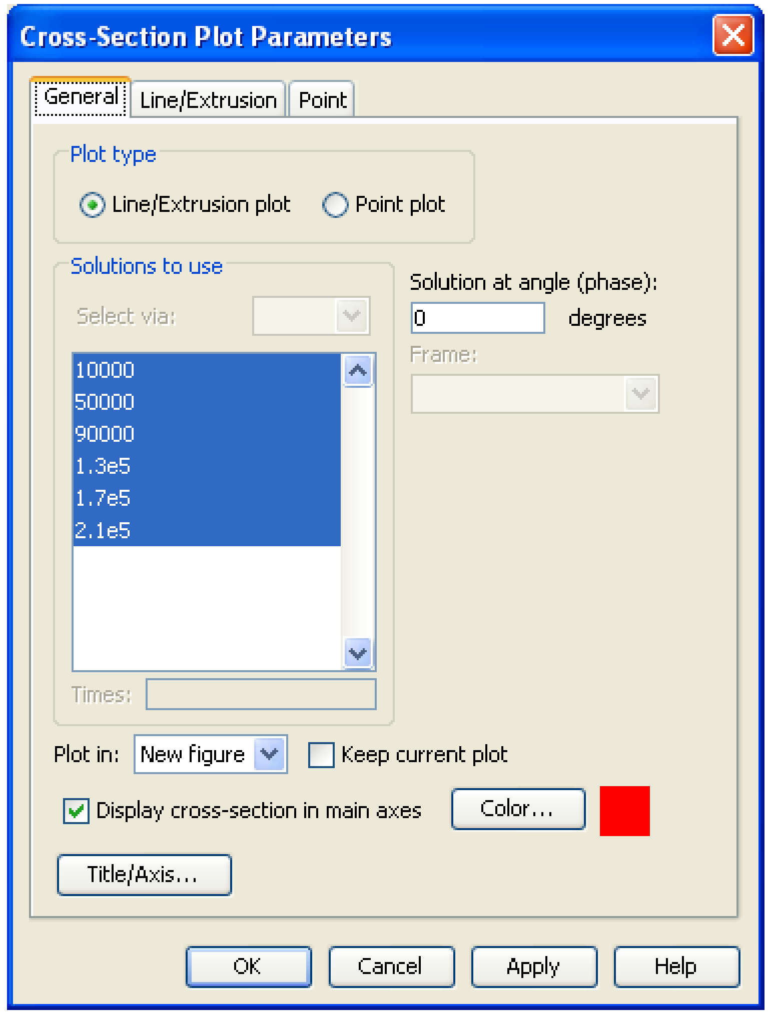
1 Click the Solver Parameters button on the Main toolbar.

2 Select Parametric from the Solver list.

3 Type p_in in the Parameter name edit field and 10000:40000:210000 in the Parameter values edit field. The solution examines p_in values from 10,000 Pa to 2.1×10^5Pa in increments of 40,000 Pa.

4 Click OK.

5 Click the Solve button in the Main toolbar.



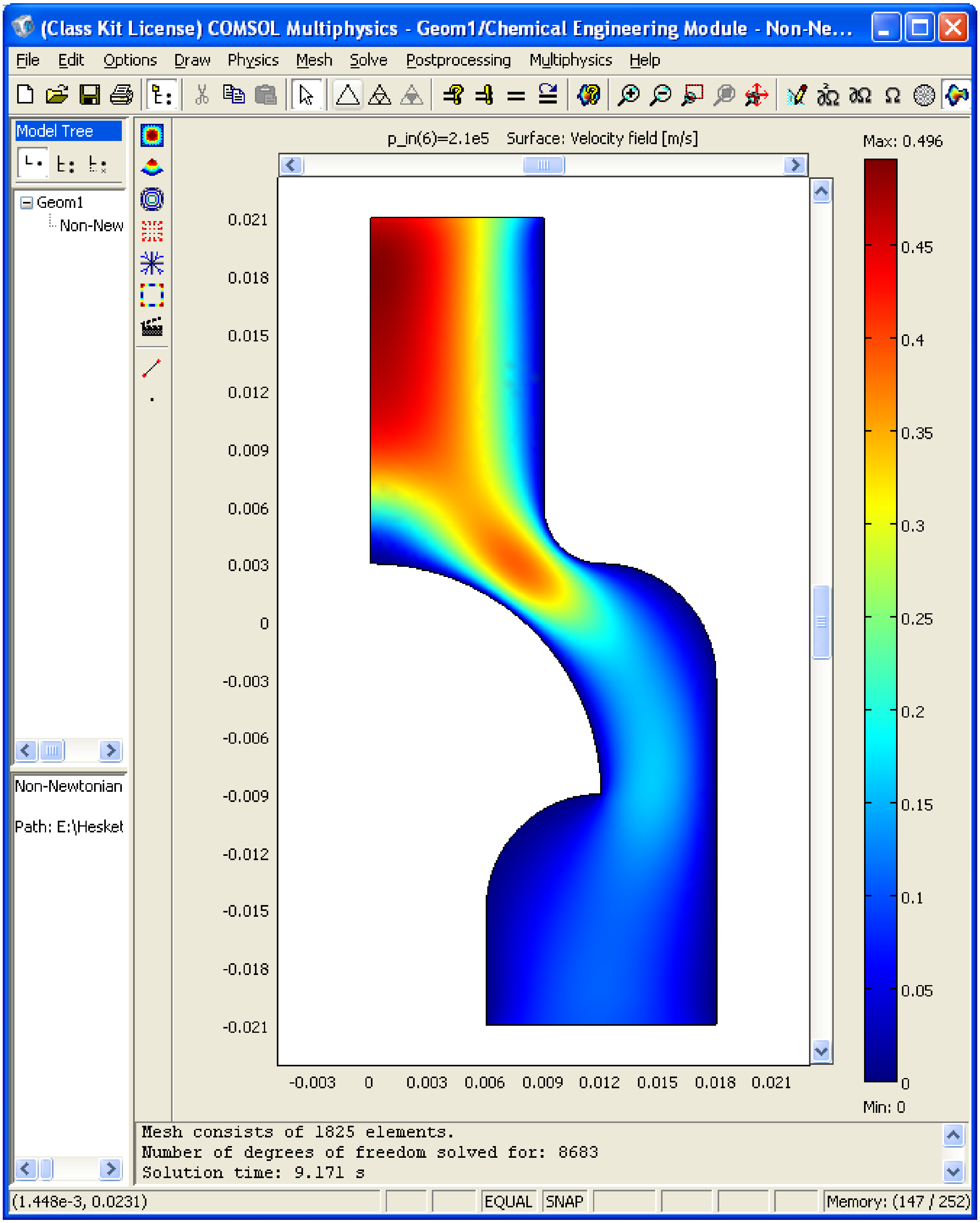
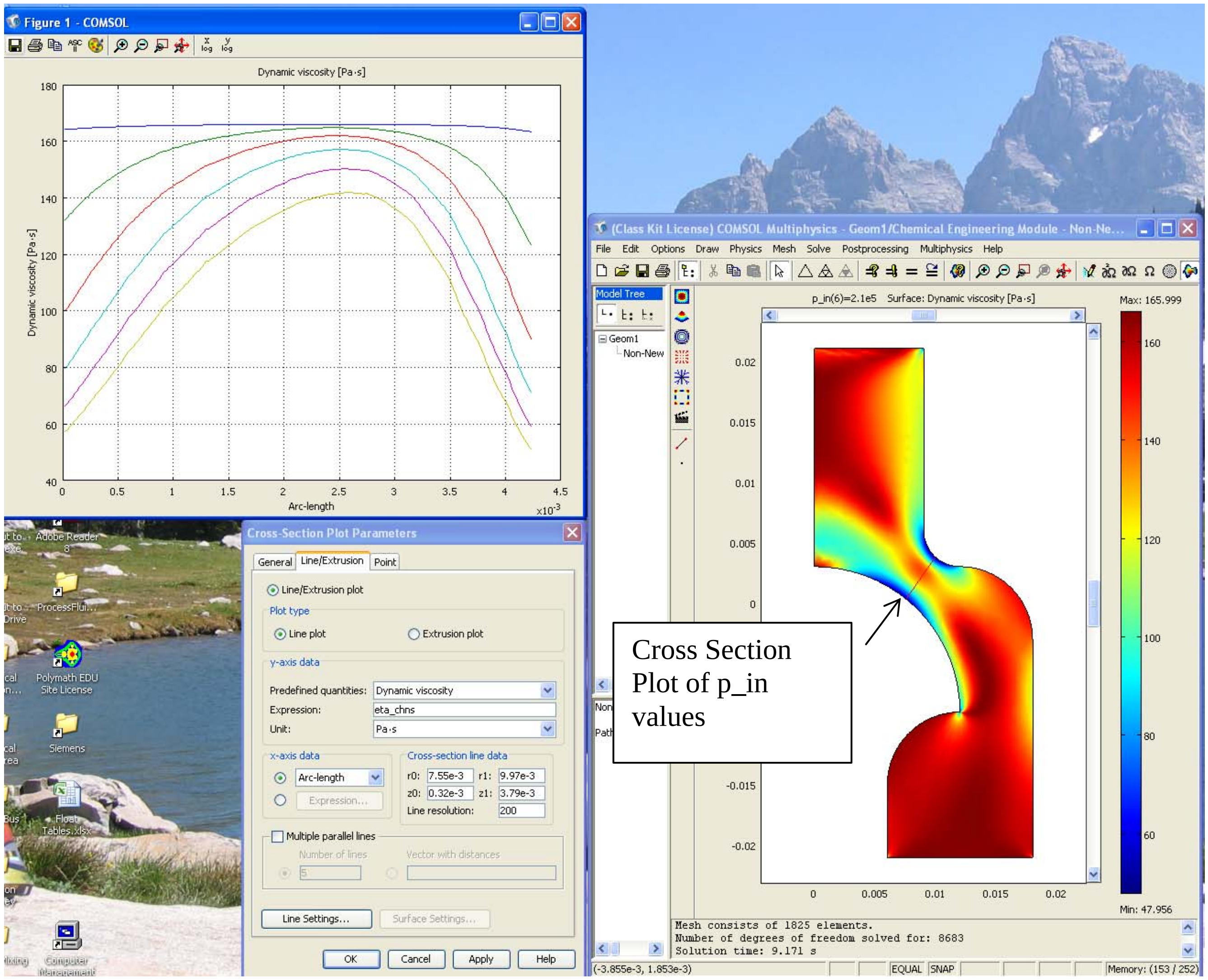


Figure 5: First Solution Result - for p_in=2.1e5



Submit at the end of the period (or unless otherwise instructed) in a MS word document:

1. Explain why certain regions were meshed to a finer element size?
2. Contour/Surface Plot of velocity
3. Contour/Surface Plot of viscosity
4. Explain the relationship of velocity with viscosity between these 2 plots.
5. Figure 2-14: Parametric study of the process, sweeping the inlet pressure from 10 kPa to 210 kPa, while investigating a Cross-sectional viscosity plot. The greater the inlet pressure (and pressure differential) the less the viscosity and more varied its distribution through the cross-section.
6. Submit this through assignments in blackboard.

Hints for C&S 8.4c (C&S 1st ed. 5.4c) for power law fluid with n>1 (n=2)

From COMSOL: The problem with power law models with an exponent >1 is that the viscosity is zero at zero shear rate. In the center of the channel where vr=0, the shear rate is zero. This means that without artificial stabilization, there is no hope for convergence.

After applying anisotropic diffusion, pressure stabilization and also splitting the domain into two subdomain's (this provides access to the mesh parameters on the center line) and meshing a bit more agressive toward the center of the channel we obtained a maximum of 0.641 compared to the theoretical value of 0.644.

Create 2 domains and mesh as shown. Then select the refine mesh button once

Now create subdomain and boundary conditions appropriate to this problem.
Finally add the following Artificial Diffusion conditions recommended by COMSOL for this problem: