Boundary Layer Flows

Modified by Robert P. Hesketh, Chemical Engineering, Rowan University Fall 2006

Laminar Boundary Layer flow

Boundary layer theory for laminar flow is developed in Geankoplis section 3.10 page 209 and the solution procedure is given in Cutlip and Shacham problem 5.18 page 204. Construct a Comsol simulation to obtain a numerical solution that can be compared with the approximate results presented in the above 2 texts. Use Geankoplis problem 3.10-1 as a basis for this problem. In this problem the plate has a length of 0.305 meters in the x-direction and water is flowing past the plate at 20°C at $v_{\infty} = 0.914$ m/s. The thickness of the boundary layer is defined as the velocity at $v \cong 0.99v_{\infty}$ The figure below is from Cutlip & Shacham page 204.



Figure 5–10 Laminar Boundary Layer for Flow Past a Flat Plate

Calculate the boundary layer thickness given as the result of Blasius's approximate solution

$$\delta = \frac{5.0x}{\sqrt{N_{\text{Re},x}}} = 5.0 \sqrt{\frac{\mu x}{\rho \upsilon_{\infty}}}$$

From this result you will see

Modeling using the Graphical User Interface

- 1. Start COMSOL
- 2. In the **Model Navigator**, click the **New** page
- 3. Select Chemical Engineering Module, Momentum Balance, Incompressible Navier-Stokes, Steady-state analysis
- 4. Click OK.



Options and Settings

Define the following constants in the **Constants** dialog box in the **Option** menu. Note that eta is COMSOL's notation for viscosity.

Geometry Modeling

- 1. Press the **Shift** key and click the **Rectangle/Square** button.
- 2. Type in values that will give the horizontal dimension of 0.05 m and the vertical dimension of 0.01m. Have the rectangle start at an x value of -0.005. We will use the section form x=-0.005 to x=0 to obtain a uniform velocity profile before encountering the horizontal plate.

OBJECT DIMENSIONS	EXPRESSION
Width	0.05
Height	0.01
x-position	-0.005
y-position	0

- 3. Click the **Zoom Extents** button in the Main toolbar.
- 4. Use the **Draw Point** button to place two points by clicking at (0, 0) and (0, 0.01).



5. Now specify the sub-domain settings using your constants defined above.

NAME	EXPRESSION
rho	$1e3 \text{ kg/m}^3$
eta	1e-3 kg/(m s)
v0	0.914 m/s

6. Next specify the boundary conditions. For the boundary defined by $-0.005 \le x \le 0$ specify a slip boundary condition as shown in the figure below:



- 7. Specify an inflow velocity and use the outflow pressure term. For the plate specify noslip and for far away from the plate we will assume slip.
- 8. Now solve this model after creating a grid. If you get the below error message then restart the problem from the current solution using Solver

Progress - Solve Problem	
Stopped	
Progress Log	
Error: Maximum number of Newton iterations reached. Returned solution has not converged.	
	>
Clear Log	
Close automatically	Close

Solver Manager	×
Initial Value Solve For Output Script	
Initial value	
O Initial value expression	
O Initial value expression evaluated using current solution	
Current solution	
O Initial value expression evaluated using stored solution	
◯ Stored solution	
Solution at time: Automatic 😪 Time: 0	
Value of variables not solved for and linearization point	
O Use setting from Initial value frame	
🔿 Zero	
O Current solution	
◯ Stored solution	
Solution at time: Automatic 😪 Time: 0	
Store Solution	
Solve OK Cancel Apply	

- 9. Look and see where the change in velocity is present!10. Now make the rectangle smaller and move PT2 to the border using Draw, Object Properties:

OBJECT DIMENSIONS	EXPRESSION
Width	0.025
Height	0.005
x-position	-0.005
y-position	0



11. Calculate the boundary layer thickness $\delta = \frac{5.0x}{\sqrt{N_{\text{Re},x}}} = 5.0\sqrt{\frac{\mu x}{\rho v_{\infty}}}$ where x = 0.02 m. Then





- 12. Solve and make set your values for the contours such that one of the values corresponds to the boundary layer thickness definition.
- 13. Make a cross-section plot at the x=0.02 m showing the velocity as a function of distance from the wall.
- 14. Add arrows to your contour plot showing the magnitude and direction of the velocity vectors.

15. Make a contour plot showing (default solution) and add arrows to your plot showing the magnitude and direction of the velocity vectors. (Go to Plot Parameters and select the Arrow tab). Next make a cross-section plot of the velocity. Using excel compare the COMSOL and analytical results by plotting them on the same graph.

Plot Parameters			X
Boundary Arrov General	v Streamline Surface	Max/Min D	eform Animate Contour
Contour plot			
Predefined quantities:	x-velocity	~	
Expression:	u		Smooth
Contour levels	of levels	Vector with isolev	/els
Levels: O 5		0.2, 0.4, 0.6, 0.8,	0.905
Predefined quantities: Expression:	Velocity field U_ns	~	
Color data Predefined quantities: Expression:	Velocity field U_ns	~	Range
Contour color Colormap: Uniform color:	Color Colors:	1024] Color scale
		ок	Cancel Apply

Thermal Boundary Layer

Now add an energy balance.

Now look at the temperature boundary layer by specifying a temperature of the plate as 40° C and the water temperature of 20° C. The thermal conductivity of water can be approximated at 0.6 W/(m K) and the heat capacity is 4,184 J/kg K.

Model Navigator			
Space dimension:	2D	Multiphy	sics
Application Modes			Add Remove
		📄 Geo	om1 (2D)
🖃 🔄 Chemical Engineering	g Module	•	Incompressible Navier-Stokes (ns
Energy balance	and Conduction		
 Steady- 	state analysis		
 Transier 	nt analγsis		
😑 🔶 Conduction	• Conduction		
Steady-	state analysis		
Transier	nt analysis	<	
🗄 🔝 Mass balance		Depende	ent variables: u v p
🗄 🔛 Momentum balan	ice	A	pplication Mode Properties
I Pseudo 3D			
			Add Geometry
Dependent variables: T		Ruling app	olication mode:
Application mode name: ht		Incompre	ssible Navier-Stokes (ns) 🛛 💙
Element: Lag	range - Quadratic	✓	Multiphysics
			OK Cancel
Subdomain Settings	- Heat Transfer	by Conductio	n (ht)
Subdomain Settings	- Heat Transfer	by Conductio	n (ht) 🔰
Subdomain Settings	- Heat Transfer	by Conductio	n (ht) 👂
Subdomain Settings Equation $-\nabla \bullet (k \nabla T) = Q + h_{trans}(T_{ext}-T)$	- Heat Transfer + C _{trans} (T _{ambtrans} ⁴ - T ⁴	by Conductio), T= temperature	n (ht) 🔰
Subdomain Settings Equation $-\nabla \bullet (k \nabla T) = Q + h_{trans}(T_{ext}-T)$ Subdomain selection	- Heat Transfer + C _{trans} (T _{ambtrans} ⁴ - T ⁴ Physics Init Elemer	by Conductio), T= temperature tt	n (ht) 👂
Subdomain Settings Equation $-\nabla \cdot (k \nabla T) = Q + h_{trans}(T_{ext}-T)$ Subdomain selection	- Heat Transfer + C _{trans} (T _{ambtrans} ⁴ - T ⁴ Physics Init Elemer Thermal properties a	by Conductio), T= temperature tt nd heat sources/sinks	n (ht)
Subdomain Settings Equation $-\nabla \bullet (k \nabla T) = Q + h_{trans}(T_{ext}-T)$ Subdomain selection	- Heat Transfer + C _{trans} (T _{ambtrans} ⁴ - T ⁴ Physics Init Elemer Thermal properties a Library material	by Conductio	n (ht)
Subdomain Settings Equation $-\nabla \cdot (k \nabla T) = Q + h_{trans}(T_{ext}-T)$ Subdomain selection	- Heat Transfer + C _{trans} (T _{ambtrans} ⁴ - T ⁴ Physics Init Elemer Thermal properties a Library material:	by Conductio), T= temperature nt nd heat sources/sinks	n (ht)
Subdomain Settings Equation $-\nabla \cdot (k\nabla T) = Q + h_{trans}(T_{ext}-T)$ Subdomain selection	- Heat Transfer + C _{trans} (T _{ambtrans} ⁴ - T ⁴ Physics Init Elemen Thermal properties a Library material: Quantity	by Conductio	n (ht)
Subdomain Settings Equation $-\nabla \cdot (k \nabla T) = Q + h_{trans}(T_{ext} - T)$ Subdomain selection	- Heat Transfer + C _{trans} (T _{ambtrans} ⁴ - T ⁴ Physics Init Elemer Thermal properties a Library material: Quantity Č _{ts}	by Conductio	n (ht)
Subdomain Settings Equation $-\nabla \cdot (k\nabla T) = Q + h_{trans}(T_{ext}-T)$ Subdomain selection	 Heat Transfer + C_{trans}(T_{ambtrans}⁴ - T⁴ Physics Init Element Thermal properties a Library material: Quantity õ_{ts} k (isotropic) 	by Conductio	n (ht) s ad Description Time-scaling coefficient Thermal conductivity
Subdomain Settings Equation -⊽•(k⊽T) = Q + h _{trans} (T _{ext} -T) Subdomain selection	 Heat Transfer + C_{trans}(T_{ambtrans}⁴ - T⁴ Physics Init Elemer Thermal properties a Library material: Quantity õ_{ts} k (isotropic) k (anisotropic) 	by Conductio	n (ht) s ad Description Time-scaling coefficient Thermal conductivity Thermal conductivity
Subdomain Settings Equation $-\nabla \cdot (k\nabla T) = Q + h_{trans}(T_{ext}-T)$ Subdomain selection	 Heat Transfer + C_{trans}(T_{ambtrans}⁴ - T⁴ Physics Init Elemer Thermal properties a Library material: Quantity ^δts k (isotropic) k (anisotropic) ρ 	by Conductio	n (ht) s ad Description Time-scaling coefficient Thermal conductivity Thermal conductivity Density
Subdomain Settings Equation $-\nabla \cdot (k\nabla T) = Q + h_{trans}(T_{ext}-T)$ Subdomain selection	 Heat Transfer + C_{trans}(T_{ambtrans}⁴ - T⁴ Physics Init Element Thermal properties at Library material: Quantity õ_{ts} k (isotropic) p C_p 	by Conductio	n (ht) s ad Description Time-scaling coefficient Thermal conductivity Thermal conductivity Density Heat capacity
Subdomain Settings Equation $-\nabla \bullet (k \nabla T) = Q + h_{trans}(T_{ext}-T)$ Subdomain selection	 Heat Transfer + C_{trans}(T_{ambtrans}⁴ - T⁴ Physics Init Elemer Thermal properties a Library material: Quantity 	by Conductio	n (ht) Description Time-scaling coefficient Thermal conductivity Density Heat capacity Heat source
Subdomain Settings Equation $-\nabla \cdot (k\nabla T) = Q + h_{trans}(T_{ext}-T)$ Subdomain selection	- Heat Transfer + C _{trans} (T _{ambtrans} ⁴ - T ⁴ Physics Init Elemen Thermal properties a Library material: Quantity δ _{ts} • k (isotropic) • k (anisotropic) • C _p · Q · h _{trans}	by Conductio	n (ht) s ad Description Time-scaling coefficient Thermal conductivity Thermal conductivity Density Heat capacity Heat source Convective heat transfer coefficient
Subdomain Settings Equation -⊽•(k⊽T) = Q + h _{trans} (T _{ext} -T) Subdomain selection	 Heat Transfer + C_{trans}(T_{ambtrans}⁴ - T⁴ Physics Init Elemer Thermal properties a Library material: Quantity 	by Conductio), T= temperature tt nd heat sources/sinks Value/Expression 1 0.6 400 0 0 400 rho 4184 0 0 0	n (ht) Description Time-scaling coefficient Thermal conductivity Thermal conductivity Density Heat capacity Heat source Convective heat transfer coefficient External temperature
Subdomain Settings Equation -∇•(k∇T) = Q + h _{trans} (T _{ext} -T) Subdomain selection 1 1 Subdomain selection	 Heat Transfer + C_{trans}(T_{ambtrans}⁴ - T⁴ Physics Init Elemer Thermal properties a Library material: Quantity δ_{ts} k (isotropic) k (anisotropic) p C_p Q h_{trans} T_{ext} C_{trans} 	by Conductio), T= temperature It Ind heat sources/sinks Value/Expression 1 0.6 400 0 0 400 rho 4184 0 0 0 0	n (ht)
Subdomain Settings Equation -∇•(k⊽T) = Q + h _{trans} (T _{ext} -T) Subdomain selection	- Heat Transfer + C _{trans} (T _{ambtrans} ⁴ - T ⁴ Physics Init Elemer Thermal properties a Library material: Quantity δ _{ts} () k (anisotropic) P C _p Q h _{trans} T _{ext} C _{trans} T _{ambtrans}	by Conductio), T= temperature tt nd heat sources/sinks Image: Conduction of the sources/sinks Image: Conduction of th	n (ht) s ad Description Time-scaling coefficient Thermal conductivity Thermal conductivity Density Heat capacity Heat source Convective heat transfer coefficient External temperature User-defined constant Ambient temperature
Subdomain Settings Equation -∇•(k∇T) = Q + h _{trans} (T _{ext} -T) Subdomain selection Image: Subdomain selection Image: Subdomain selection Image: Subdomain selection Image: Select by group Image: Select by group Image: Active in this domain	 Heat Transfer + C_{trans}(T_{ambtrans}⁴ - T⁴ Physics Init Elemer Thermal properties a Library material: Quantity δ_{ts} k (isotropic) k (anisotropic) p C_p Q h_{trans} T_{ext} C_{trans} T_{ambtrans} 	by Conductio	n (ht)

Subdomain Settings	- Heat Tra	ansfer by Conduction	(ht)	X
Equation -⊽•(k⊽T) = Q + h _{trans} (T _{ext} -T)	+ C _{trans} (T _{ambt}	⁴ - T ⁴), T= temperature		
Subdomain selection	Physics Ini	Element		
1 🔼	Initial value			
	Variable	Initial value	Description	
	T(t ₀)	20	Temperature	
~				
Select by group				
Active in this domain				
			OK Cancel Apply	

Submit:

1. Boundary Layer thickness calculation

- 2. Contour Plots (Basic Solution) Add arrows on top of plot to signify velocity 2.1. large rectangle 2.2. small rectangle
- 3. Cross Section Plots:
 - 3.1. large rectangle
 - 3.2. small rectangle