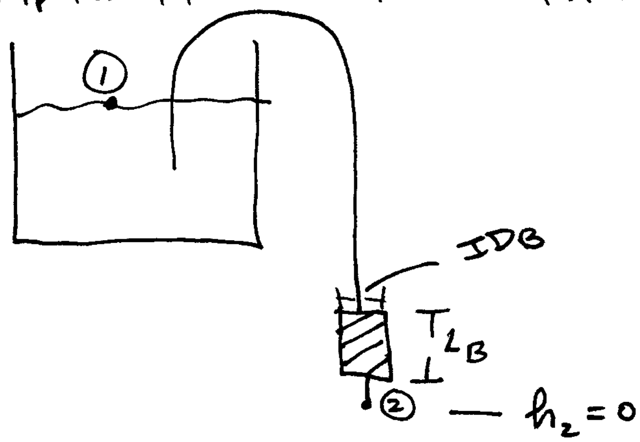
Chapter 11 Flow through Porous Media



$$\rho g h_1 + \underbrace{\cancel{\tfrac{1}{2}\rho v_1^2}}_{\text{Small}} + \underbrace{\cancel{P_1}}_{0} = \underbrace{\cancel{\rho g h_2}}_{0} + \tfrac{1}{2}\rho v_2^2 + \underbrace{\cancel{P_2}}_{0} + \rho \mathcal{F}$$

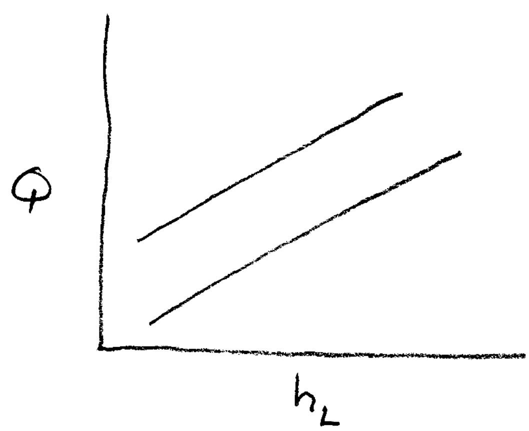
$$\underbrace{\rho g h_1}_{\text{pushing fluid through}} \underbrace{- \tfrac{1}{2}\rho v_2^2 - \rho \mathcal{F}}_{\text{losses}} = 0$$

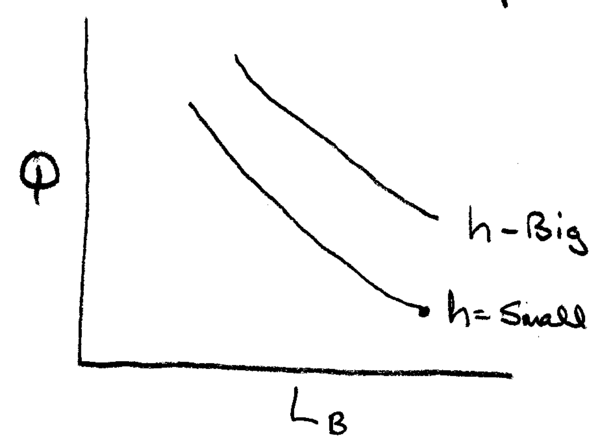
losses in tubing?
bends?
entrance?

Packed Bed?

$$\rho g h_1 = \frac{150\, v_s\, \mu\, (1-\epsilon)^2 L_B}{D_p^2\, \varepsilon^3}$$

Section: Data





PFR

① Show transparency — Chemical Engineering Science (1985)

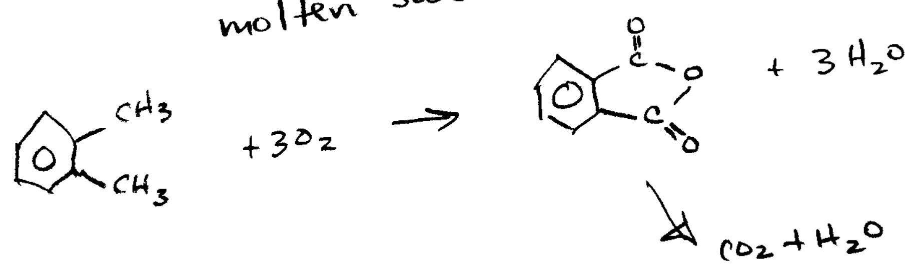
Phthalic Anhydride manufacture → exothermic

Vertical Shell & tube heat exchanger

catalyst in tubes (solid spheres, rings etc.

molten salt in shell side { Na-K & nitrates

$\Delta H_{rxn} = -265 \frac{kcal}{gmol}$



45,000 metric ton/yr → ~20,000 tubes with ID ~ 2.5 cm

tube length 3.65 m

Previous work — keep mixture below flammable limits

current reactors — working above the flammability limits }

V_2O_5 catalyst with TiO_2 + promotors } 10 wt%

non-porous carborundum }

$d_p = 6\text{ mm}$

$6/26 = \frac{d_P}{ID_{tube}} = 0.23$

$\rho_c \cong 1{,}780 \frac{kg_c}{m^3_{cat}}$

assume $\phi = 0.45$

Perry's Figure 5-70

Cylindrical Rings — $\phi = 0.47$

Spheres — $\phi \cong 0.45$

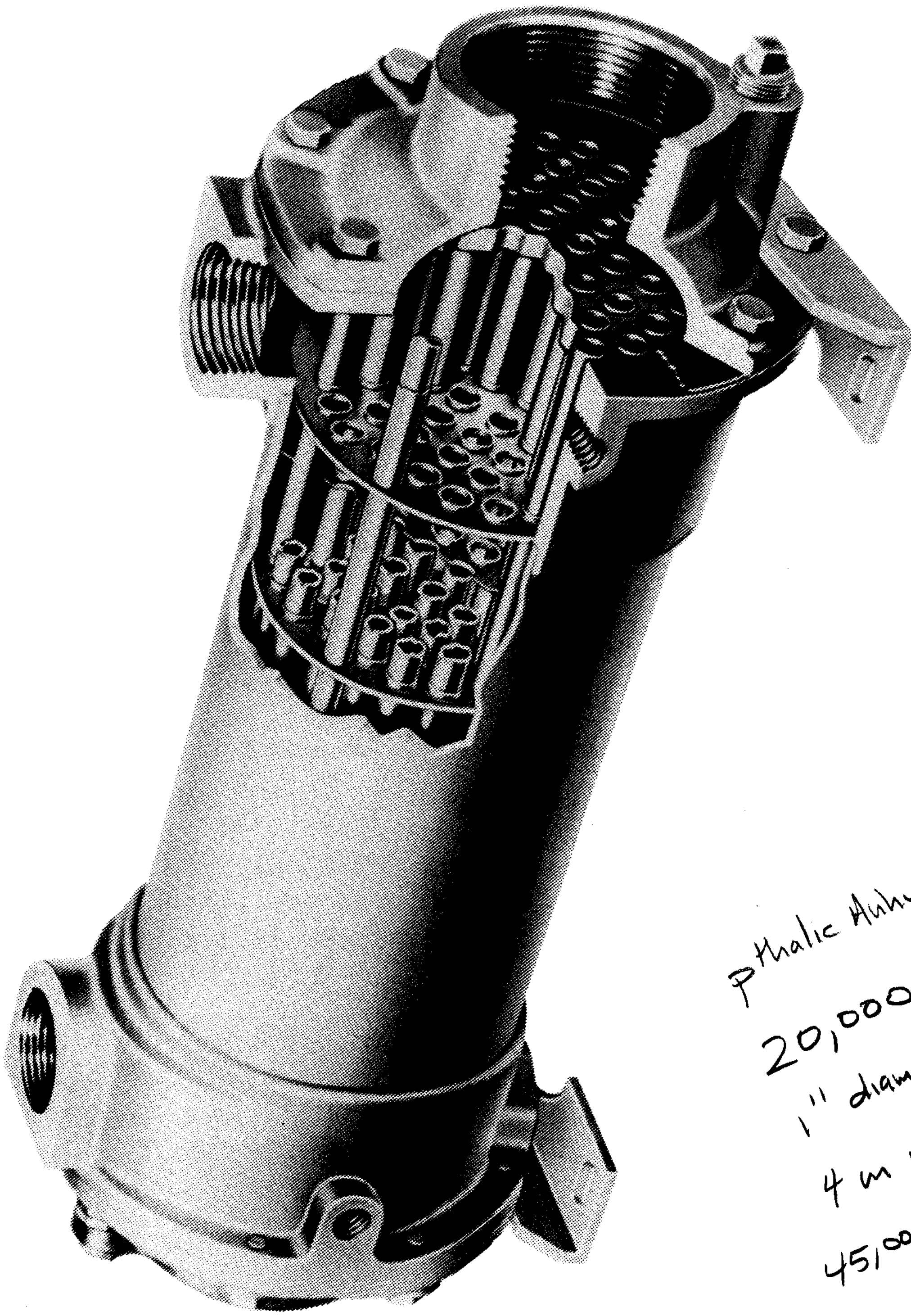
Raschig Ring — $\phi = 0.6$

$\rho_{bulk} = \rho_c(1-\phi)$

$\frac{kg_{cat}}{m^3_{cat}} \quad \frac{m^3_{cat}}{m^3_{Bed}}$

$\rho_{bulk} = 980\, kg/m^3_{Bed}$

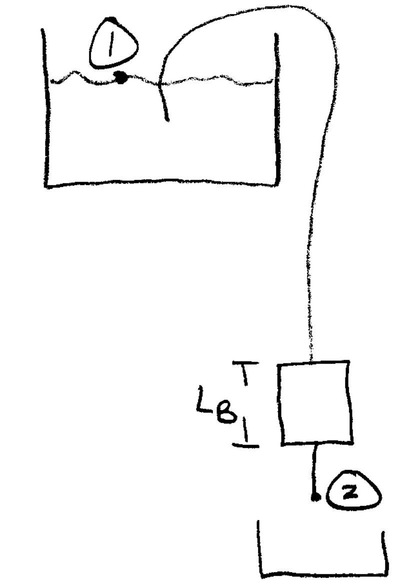
(m^3_{cat}/m^3_{Bed})



pthalic Anhydride
20,000 tubes
1" diameter
4 m in length
45,000 metric ton/yr

When thermal efects are important, shell and tube heat exchangers are often used as plug-flow reactors. In some processes such as methanol synthesis or phthalic anhydride production, tubes are packed with catalyst pellets forming a number of small-diameter packed-bed reactors in parallel. The shell is filled with a heat-transfer medium. (ITT Fluid Technology Corporation, by permission.)

Chapter 11 Lab Experiment

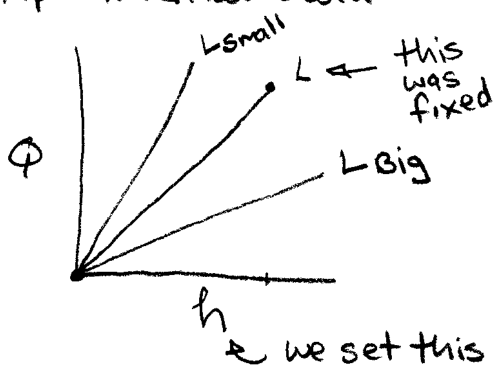


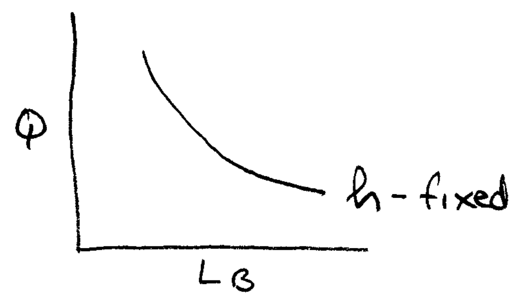
Apply Engineering Bernoulli

$$\cancel{P_1}^{\,0} + \rho g h_1 + \tfrac{1}{2}\rho \cancel{v_1^2}^{\,0} = \cancel{P_2}^{\,0} + \rho g \cancel{h_2}^{\,0} + \tfrac{1}{2}\rho v_2^2 + \rho F$$

$$\rho g h_1 = \tfrac{1}{2}\rho v_2^2 + \rho F$$

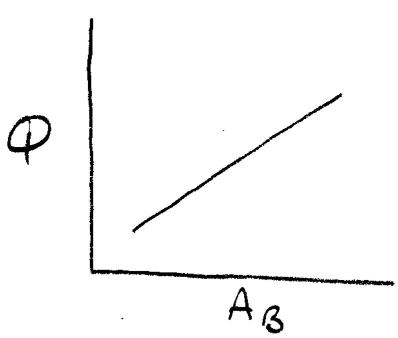
Experimental Data – what do you think it looks like





$$Q \propto \frac{h}{L_B}$$

What about A_B & dp ?



How do we model this?

Book (de Nevers) section 11.1

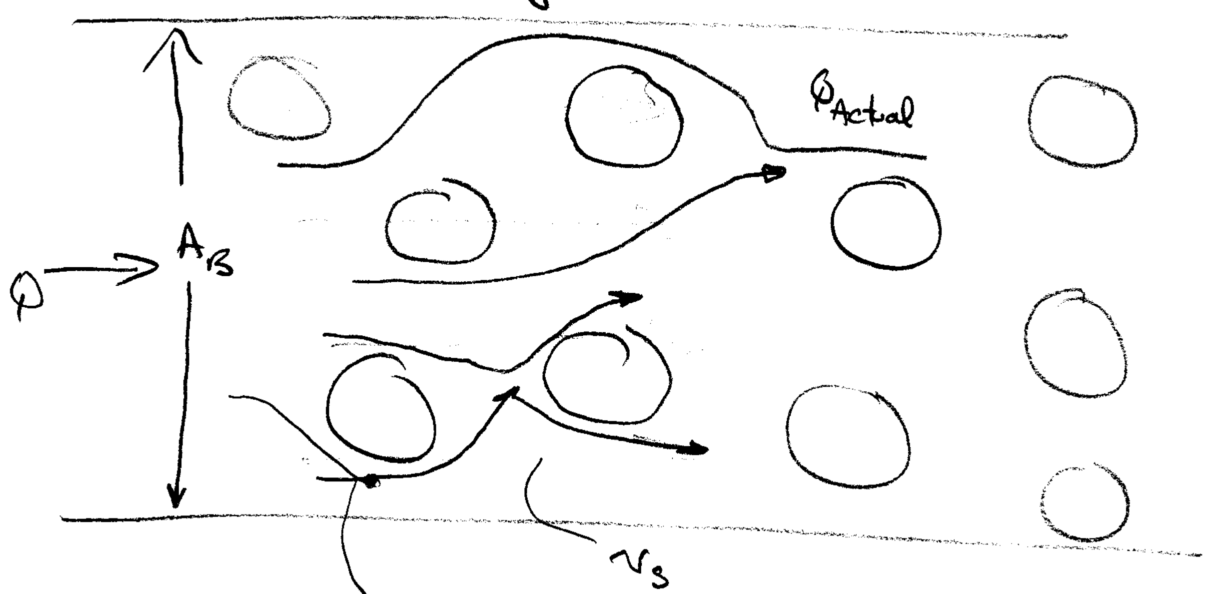
Definitions: (use picture below)

superficial Velocity $v_s = \frac{Q}{A}$ ← cross sectional area of pipe or tube

This can be calculated for any type of equipment - without needing to know about its internals. Pack

- Packed Bed Reactors
  Distillation column
  absorber
  Heat exchanger - for one of the flows

Interstitial velocity



what is the actual velocity if measured?

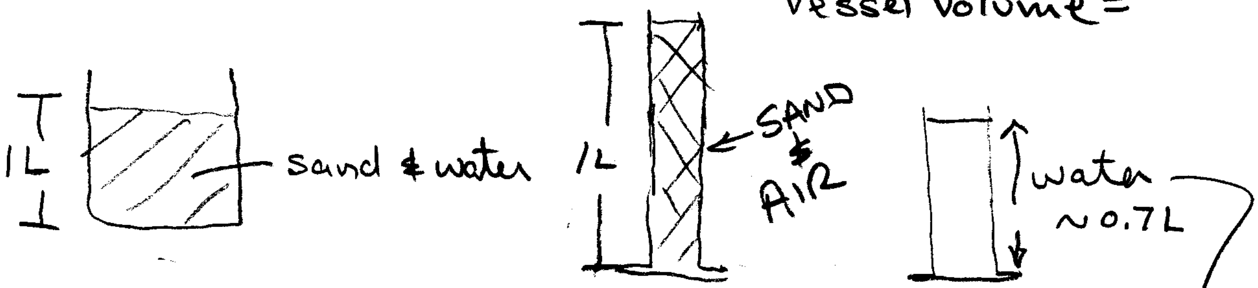
Void fraction (volume) (Porosity usually for solids with a pore network)

$$\varepsilon \equiv \frac{\text{Volume not solids}}{\text{total volume}} \quad (\text{gas or liquid})$$

$$= \frac{\text{total volume} - \text{volume solids}}{\text{total volume}}$$

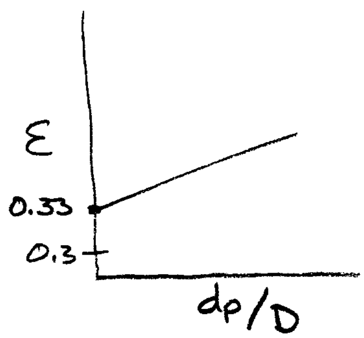
Show using beakers

Void volume =
Solid volume =
Vessel volume =

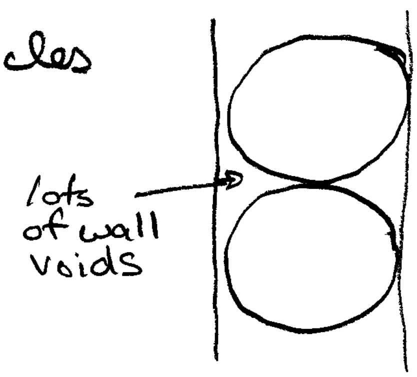


this is called "VOID"

Show curves from Perry's 5th Edition



what about large particles



Void Fraction Curve for spheres
Taken from Perry's 5th Edition
Figure 5-70

Scale x 0.003671
Scale y 0.003636

Dp/Dtube x	void y	Dp/Dtube	void
0	7.86	0.000	0.329
4.21	9.56	0.015	0.335
4.71	9.86	0.017	0.336
17.21	14.86	0.063	0.354
26.71	18.96	0.098	0.369
40.21	24.86	0.148	0.390
41.71	27.36	0.153	0.399
45.71	27.16	0.168	0.399
54.21	31.86	0.199	0.416
54.21	30.86	0.199	0.412
59.11	**32.96**	**0.217**	**0.420**
70.31	37.86	0.258	0.438
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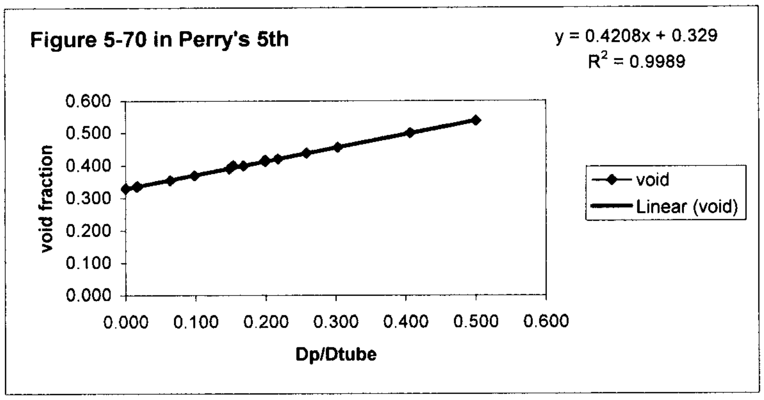


linear fit to spherical data
$y = 0.4208x + 0.329$

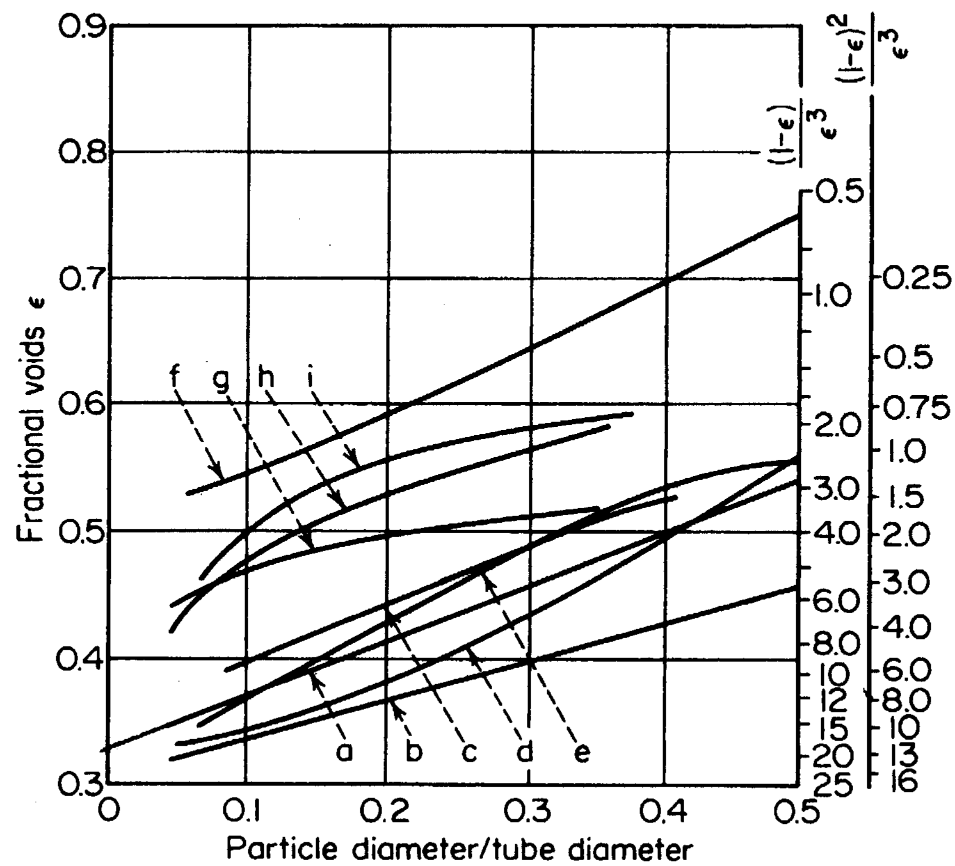
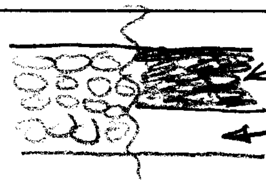


FIG. 5-70. Voidage in packed beds. Spherical: *a*, smooth, uniform; *b*, smooth, mixed; *c*, clay. Cylindrical: *d*, smooth, uniform; *e*, alundum, uniform; *f*, clay Raschig rings. Granules: *g*, fused magnetite (synthetic ammonia catalyst); *h*, fused alundum; *i*, Aloxite. (*Leva, "Fluidization," p. 54, McGraw-Hill, New York, 1959.*)

Back to interstitial velocity



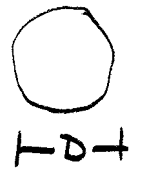
$$v_I = \frac{Q}{(\text{Volume Void})} = \frac{Q}{\varepsilon A_{pipe}} \;[=]\; \frac{\frac{m^3_{fluid}}{s}}{\frac{m^3_{fluid}}{m^3_{pipe}} m^2_{pipe}} = \frac{m_{pipe}}{s}$$

this again is a true velocity

From Chapter 6 for Non circular cross-sections

$$D_H = \frac{4(\text{Area Cross Section})}{\text{Wetted Perimeter}}$$

Hydraulic Diameter



$$\frac{4(\pi D^2/4)}{\pi D} = D$$

for circular cross section $D_H = D$

good check

$$\text{Hydraulic Radius} \equiv \frac{\text{Area Cross Section}}{\text{wetted perimeter}}$$

$$R_H = \underbrace{\frac{\pi R^2}{2\pi R}}_{\text{for circle}} = \frac{R}{2}$$

$$D_H = 4(R_H) = 4(R/2) = 2\underbrace{R}_{\text{circle } R}$$

for non-circular ducts such as rectangular ducts for home heating/air conditioning use R_H in ΔP calculation (fig 6.10) p. 186

$$\mathcal{F} = 4\underbrace{f_F}_{\text{fanning friction factor}} \frac{L}{D}\frac{v^2}{2} \quad (6.17)\ p185$$

Plug in D_H

$$\frac{\Delta P}{L} = \frac{\rho 4 f_F v^2}{2D} = \frac{\rho 4 f_F v^2}{2 D_H} = \frac{\rho}{2}\frac{f_F v^2}{} \frac{\cancel{4}}{(\cancel{4}R_H)}$$

de Nevers 6.35 modified

and $f_F = f(\mathcal{R}, \varepsilon/D_H)$

$\&\ \mathcal{R} = f(D_H) \quad \because Re = \mathcal{R} = \frac{\rho v D_H}{\mu}$

what is R_H for a bed of sand?

$$R_H = \left(\frac{\text{Cross sectional area}}{\text{wetted perimeter}}\right)\frac{L}{L} = \frac{\text{Volume open to flow}}{\text{wetted area}}$$

$$R_H\Big|_{\substack{\text{packed}\\ \text{bed}}} = \frac{V_B\,\varepsilon}{(\#\text{particles})\,\frac{A_p}{1\,\text{particle}}} \;[=]\; \frac{m^3_B\;\frac{m^3_{void}}{m^3_{Bed}}}{\#\;\; m^2_{\substack{area\\ particle}}}$$

$$\frac{A_p}{1\,\text{particle}} = \pi D_p^2 \quad \text{for sphere}$$

$$\underbrace{V_p}_{\frac{\pi D_p^3}{6}}(\#\text{particles}) = \underline{V_B(1-\varepsilon)} \;[=]\; \underline{m^3_{Bed}\;(m^3_{solid}/m^3_{Bed})} = m^3_{solid}$$

$$\therefore \quad \#\text{particles} = \frac{V_B(1-\varepsilon)}{\frac{\pi D_p^3}{6}}$$

plug in

$$R_H = \frac{\cancel{V_B}\,\varepsilon}{\underbrace{\frac{6\cancel{V_B}(1-\varepsilon)}{\cancel{\pi} D_p^3}}_{\#}\underbrace{(\cancel{\pi} D_p^2)}_{\substack{\text{Area}\\ \text{Surface}}}} = \frac{D_p}{6}\frac{\varepsilon}{(1-\varepsilon)} \qquad \begin{matrix}11.6\\ \text{for}\\ \text{spheres}\end{matrix}$$

Now go back to f_F ΔP eqn

$$\frac{\rho F}{L} = \frac{\Delta P}{L} = \frac{\rho}{2} f_F' v^2 \frac{1}{R_H} = \frac{\rho}{2} f_F v^2 \frac{6(1-\varepsilon)}{D_p\,\varepsilon}$$

$$f_F = f'_{pM_{\substack{\text{porous}\\ \text{medium}}}} = \frac{2F}{v^2}\,\frac{D_p\,\varepsilon}{6(1-\varepsilon)}\,\frac{1}{L}$$

what is v_I? if we use $v = v_I$

$$f_F' = \frac{2\mathcal{F} D_p}{L} \frac{\varepsilon}{6(1-\varepsilon)} \frac{1}{v_I^2} \qquad v_I = \frac{Q}{\varepsilon A} = \frac{v_S}{\varepsilon}$$

$$f_F' = \frac{2\mathcal{F} D_p}{L} \frac{\varepsilon}{6(1-\varepsilon)} \frac{\varepsilon^2}{v_s^2}$$

$$= \frac{2\mathcal{F} D_p}{L} \frac{\varepsilon^3}{6(1-\varepsilon)} \frac{1}{v_s^2}$$

$$Re' = \mathcal{R}' = \frac{\rho v_I D_H}{\mu} = \frac{\rho}{\mu} \frac{v_s}{\varepsilon} 4R_H = \frac{\rho}{\mu} \frac{v_s}{\cancel{\varepsilon}} \frac{4 D_p \cancel{\varepsilon}}{6(1-\varepsilon)}$$

$$\mathcal{R}' = \frac{2}{3} \frac{D_p v_s}{1-\varepsilon} \frac{\rho}{\mu}$$

Book Drops constants
$$\boxed{\left\{ \begin{aligned} \mathcal{R}_{pm} &= \frac{D_p v_s}{(1-\varepsilon)} \frac{\rho}{\mu} \\ f_{pm} &= \mathcal{F} \frac{D_p}{L} \frac{\varepsilon^3}{(1-\varepsilon)} \frac{1}{v_s^2} \end{aligned} \right\}}$$

$$\frac{\cancel{2} \cdot \cancel{8}/6\ 3}{\cancel{2}}$$

$$\text{for } f_{\substack{\text{pipe}\\\text{flow}}} = \frac{16}{\mathcal{R}'} = \frac{16}{\frac{2}{3} \frac{\rho}{\mu} \frac{v_s D_p}{(1-\varepsilon)}} = \frac{24}{\frac{\rho}{\mu} \frac{v_s D_p}{(1-\varepsilon)}}$$

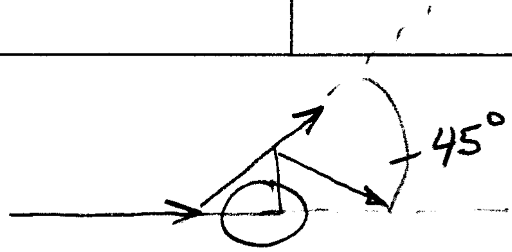
$$\begin{array}{r} 24 \\ \times\ 3 \\ \hline 12 \\ 60 \\ \hline 72 \end{array}$$

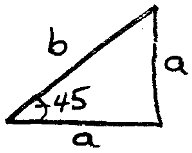
$$f_F' = \frac{2}{6} \frac{\mathcal{F} D_p}{L} \frac{\varepsilon^3}{(1-\varepsilon)} \frac{1}{v_s^2}$$

$$\frac{1}{3} \underbrace{\left(\frac{\mathcal{F} D_p}{L} \frac{\varepsilon^3}{(1-\varepsilon)} \frac{1}{v_s^2} \right)}_{f_{pm}'} = \frac{24}{\underbrace{\frac{\rho}{\mu} \frac{v_s D_p}{(1-\varepsilon)}}_{\mathcal{R}_{pm}}}$$

$$f_{pm}'' = \frac{72}{\mathcal{R}_{pm}}$$

<u>One more correction</u>





$$\sin(45) = a/b \qquad b = \frac{a}{\sin(45)} =$$

$$0.707 = a/b$$

$$b = 1/0.707 = \sqrt{2}$$

Additional $\quad L_{actual} = \sqrt{2} L$

$$v_I\Big|_{again} = \sqrt{2}\, v_I\Big|_{old}$$

$$\frac{\mathcal{F} D_p}{\cancel{\sqrt{2}} L} \frac{\varepsilon^3}{(1-\varepsilon)} \frac{1}{2(v_s)^2} = \frac{72}{\frac{\rho}{\mu} \frac{v_s \cancel{\sqrt{2}} D_p}{(1-\varepsilon)}}$$

$$\underbrace{\frac{\mathcal{F} D_p}{L} \frac{\varepsilon^3}{(1-\varepsilon)} \frac{1}{v_s^2}}_{f_{pm}} = \frac{144}{\frac{\rho}{\mu} \frac{v_s D_p}{1-\varepsilon}} \qquad \mathcal{F} = \frac{\Delta P}{\rho}$$

$$f_{pm} = \frac{144}{Re_{pm}} \quad \text{where } f_{pm} = \frac{\mathcal{F} D_p}{L} \frac{\varepsilon^3}{(1-\varepsilon)} \frac{1}{v_s^2}$$

the actual value is 150 from experiments 4% difference between experiment and theory

$$f_{pm} = \frac{150}{Re_{pm}}$$

porous media

$$\mathcal{F} = 150 \frac{v_s \mu L}{D_p^2 \rho} \frac{(1-\varepsilon)^2}{\varepsilon^3}$$

∴ 

$$P_1 + \tfrac{1}{2}\cancel{\rho v_1^2} + \cancel{\rho g h_1} = P_2 + \tfrac{1}{2}\cancel{\rho v_2^2} + \cancel{\rho g h_2} + \rho F$$

$$(P_1 - P_2) = \rho F_{pm} = \rho \; 150 \, v_s \frac{\mu}{\rho} \frac{L}{D_p^2} \frac{(1-\varepsilon)^2}{\varepsilon^3}$$

Blake Kozeny or Kozeny – Carmen equation

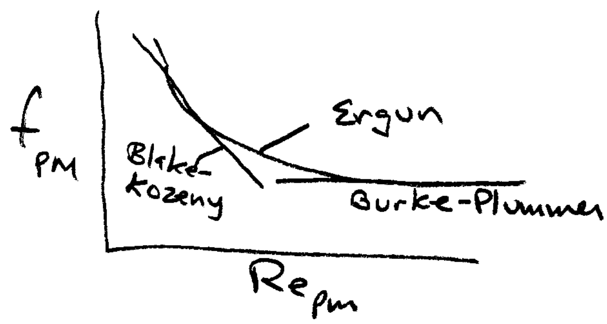
this equation is only valid for $Re_{pm} \leq 10$

for turblent flow from experiments

$f_{pm} = 1.75$ turbulent $Re_{pm} \geq 1000$

Ergun put these 2 together

$$f_{pm} = 1.75 + \frac{150}{Re_{pm}} = \underbrace{F \frac{D_p}{L} \frac{1}{v_s^2} \frac{\varepsilon^3}{(1-\varepsilon)}}$$



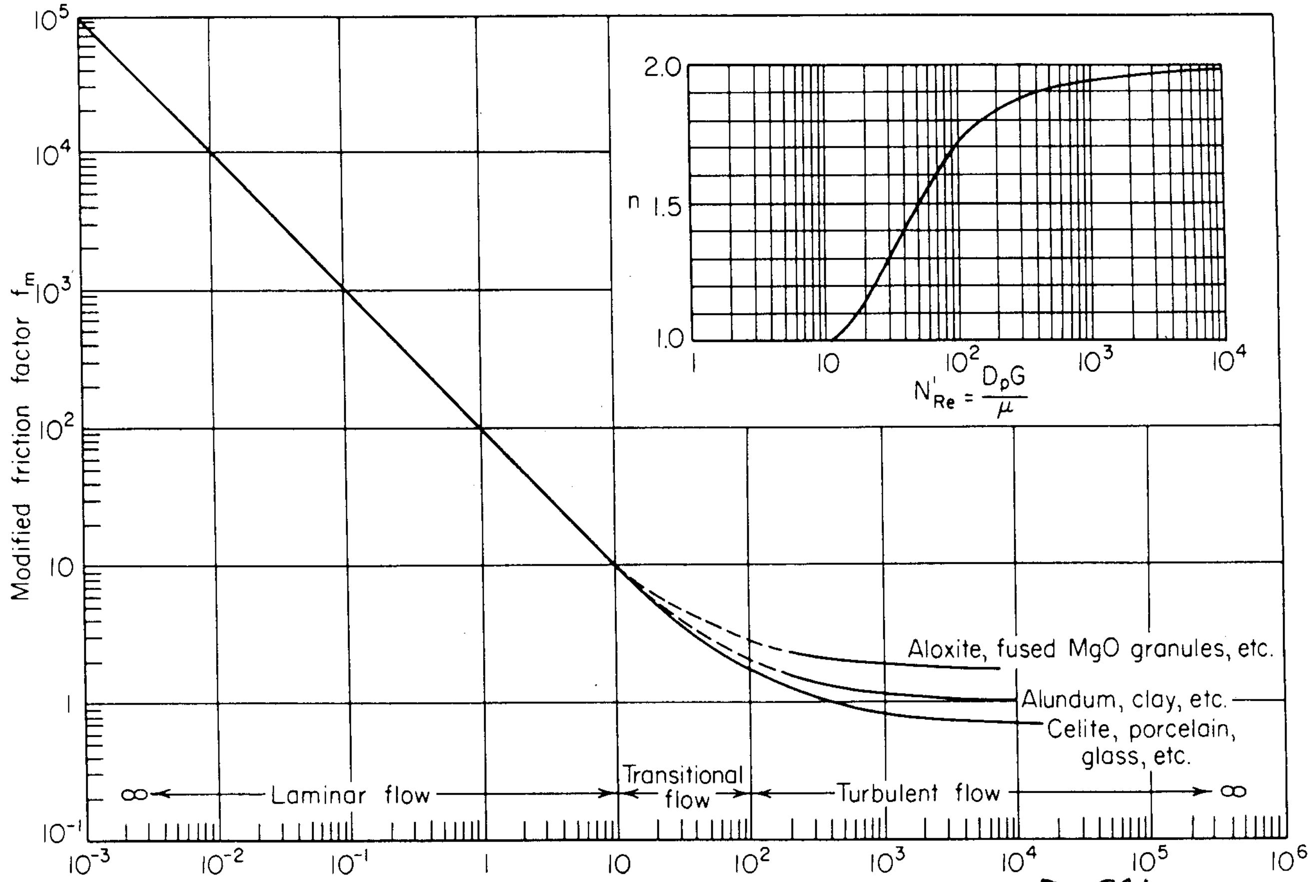
show graph

$$F = 1.75 \frac{L}{D_p} v_s^2 \frac{(1-\varepsilon)}{\varepsilon^3} + 150 \frac{\mu(1-\varepsilon)}{\rho \cancel{v_s} D_p} \frac{L}{D_p} v_s^2 \frac{(1-\varepsilon)}{\varepsilon^3}$$

$$F = 1.75 \frac{L v_s^2 (1-\varepsilon)}{D_p \;\; \varepsilon^3} + 150 \frac{\mu}{\rho} \frac{L}{D_p^2} v_s \frac{(1-\varepsilon)^2}{\varepsilon^3}$$ Ergun 11.18

$f_M = \frac{150}{Re_p} + 1.75$

$$f_M = \frac{\Delta P}{\rho} \frac{D_p}{L} \frac{1}{v_s^2} \frac{\varepsilon^3}{(1-\varepsilon)}$$



$= \frac{D_p \rho v_s}{\mu}$ $\quad G = \rho v_s \; [=] \; \frac{kg}{s\,m^2}$

FIG. 5-69. Friction factor for beds of solids. (*Leva, "Fluidization," p. 49, McGraw-Hill, New York, 1959.*)

Void Fraction Curve for spheres
Taken from Perry's 5th Edition
Figure 5-70

Scale x 0.003671
Scale y 0.003636

Dp/Dtube	void			
x	y	Dp/Dtube	void	
0	7.86	0.000	0.329	
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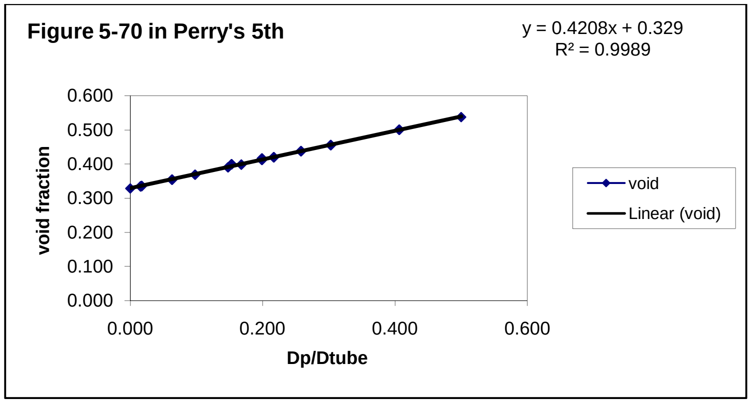


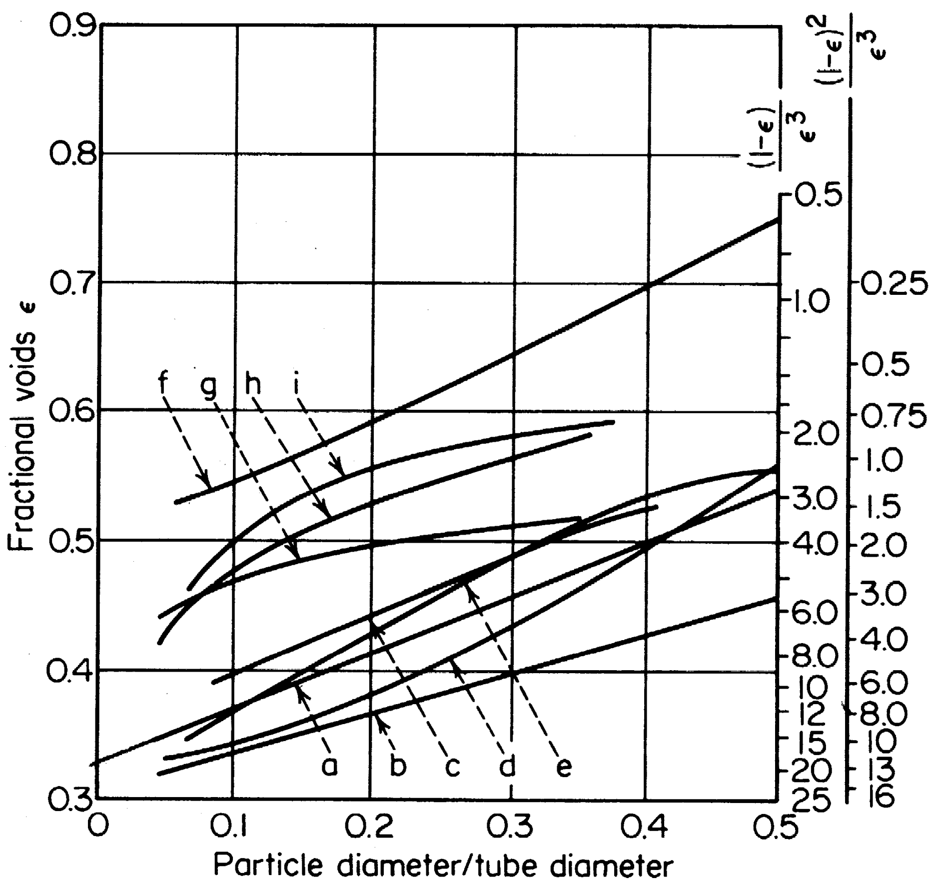


FIG. 5-70. Voidage in packed beds. Spherical: *a*, smooth, uniform; *b*, smooth, mixed; *c*, clay. Cylindrical: *d*, smooth, uniform; *e*, alundum, uniform; *f*, clay Raschig rings. Granules: *g*, fused magnetite (synthetic ammonia catalyst); *h*, fused alundum; *i*, Aloxite. (*Leva, "Fluidization," p. 54, McGraw-Hill, New York, 1959.*)

Chemical Process Equipment: Selection & Design
Stanley M. Walas, Butterworth-Heinemann (1990)

Direct application of these equations in Example 6.14 is not successful, but if E_2 is taken as the reciprocal of the given expression, a plausible result is obtained.

6.9. GRANULAR AND PACKED BEDS

Flow through granular and packed beds occurs in reactors with solid catalysts, adsorbers, ion exchangers, filters, and mass transfer equipment. The particles may be more or less rounded or may be shaped into rings, saddles, or other structures that provide a desirable ratio of surface and void volume.

Natural porous media may be consolidated (solids with holes in them), or they may consist of unconsolidated, discrete particles. Passages through the beds may be characterized by the properties of porosity, permeability, tortuosity, and connectivity. The flow of underground water and the production of natural gas and crude oil, for example, are affected by these characteristics. The theory and properties of such structures is described, for instance, in the book of Dullien (*Porous Media, Fluid Transport and Pore Structure*, Academic, New York, 1979). A few examples of porosity and permeability are in Table 6.9. Permeability is the proportionality constant k in the flow equation $u = (k/\mu)\, dP/dL$.

Although consolidated porous media are of importance in chemical engineering, only unconsolidated porous media are incorporated in process equipment, so that further attention will be restricted to them.

Granular beds may consist of mixtures of particles of several sizes. In flow problems, the mean surface diameter is the appropriate mean, given in terms of the weight fraction distribution, x_i, by

$$D_p = 1/\left(\sum x_i/D_i\right). \tag{6.106}$$

When a particle is not spherical, its characteristic diameter is taken as that of a sphere with the same volume, so that

$$D_p = (6V_p/\pi)^{1/3}. \tag{6.107}$$

SINGLE PHASE FLUIDS

Extensive measurements of flow in and other properties of beds of particles of various shapes, sizes and compositions are reported by Leva et al. (1951). Differences in voidage are pronounced as Figure 6.8(c) shows.

A long-established correlation of the friction factor is that of Ergun (*Chem. Eng. Prog.* **48**, 89–94, 1952). The average deviation from his line is said to be ±20%. The friction factor is

$$f_p = \frac{g_c D_p \varepsilon^3}{u^2(1-\varepsilon)}\left(\frac{\Delta P}{L}\right) \tag{6.108}$$

$$= 150/\mathrm{Re}_p + 1.75 \tag{6.109}$$

with

$$\mathrm{Re}_p = D_p G/\mu(1-\varepsilon). \tag{6.110}$$

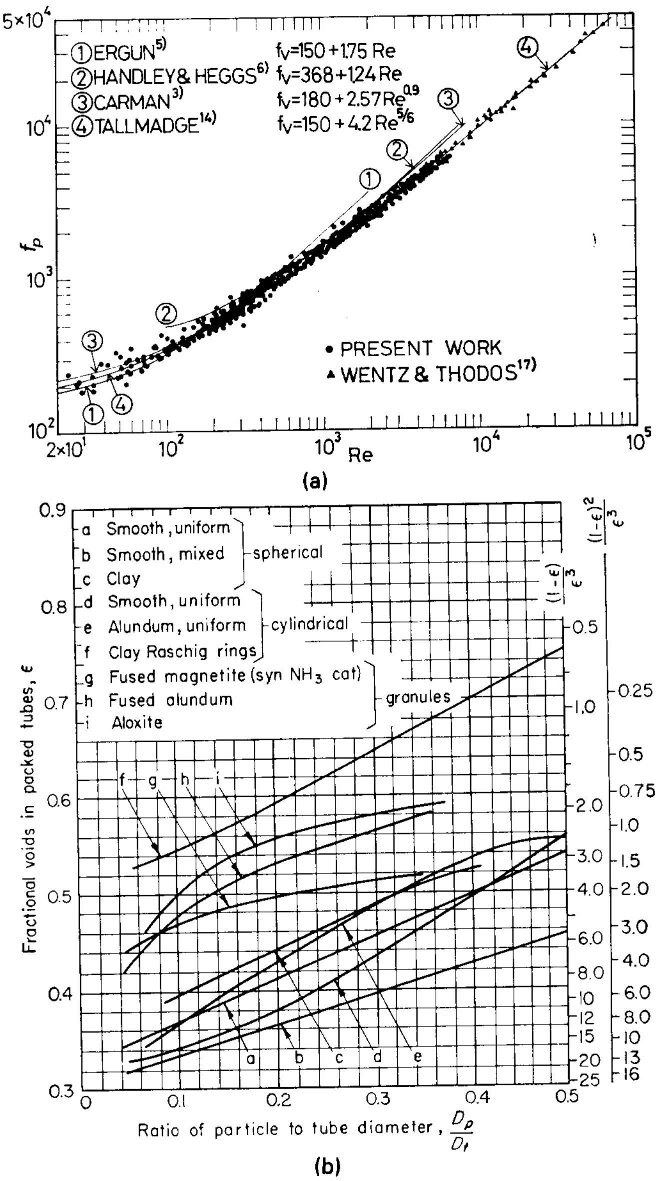


Figure 6.8. Friction factors and void fractions in flow of single phase fluids in granular beds. (a) Correlation of the friction factor, $\mathrm{Re} = D_p G/(1-\varepsilon)\mu$ and $f_p = [g_c D_p \varepsilon^3/\rho u^2(1-\varepsilon)](\Delta P/L = 150/\mathrm{Re} + 4.2/(\mathrm{Re})^{1/6}$ [*Sato et al.*, J. Chem. Eng. Jpn. *6, 147–152 (1973)*]. (b) Void fraction in granular beds as a function of the ratio of particle and tube diameters [*Leva, Weintraub, Grummer, Pollchik, and Storch*, U.S. Bur. Mines Bull. *504* (*1951*)].

TABLE 6.9. Porosity and Permeability of Several Unconsolidated and Consolidated Porous Media

Media	Porosity (%)	Permeability (cm^2)
Berl saddles	68–83	1.3×10^{-3}–3.9×10^{-3}
Wire crimps	68–76	3.8×10^{-5}–1.0×10^{-4}
Black slate powder	57–66	4.9×10^{-10}–1.2×10^{-9}
Silica powder	37–49	1.3×10^{-10}–5.1×10^{-10}
Sand (loose beds)	37–50	2.0×10^{-7}–1.8×10^{-6}
Soil	43–54	2.9×10^{-9}–1.4×10^{-7}
Sandstone (oil sand)	8–38	5.0×10^{-12}–3.0×10^{-8}
Limestone, dolomite	4–10	2.0×10^{-11}–4.5×10^{-10}
Brick	12–34	4.8×10^{-11}–2.2×10^{-9}
Concrete	2–7	1.0×10^{-9}–2.3×10^{-7}
Leather	56–59	9.5×10^{-10}–1.2×10^{-9}
Cork board	—	3.3×10^{-6}–1.5×10^{-5}
Hair felt	—	8.3×10^{-6}–1.2×10^{-5}
Fiberglass	88–93	2.4×10^{-7}–5.1×10^{-7}
Cigarette filters	17–49	1.1×10^{-5}
Agar-agar	—	2.0×10^{-10}–4.4×10^{-9}

(A.E. Scheidegger, *Physics of Flow through Porous Media*, University of Toronto Press, Toronto, Canada, 1974).

The pressure gradient accordingly is given by

$$\frac{\Delta P}{L}=\frac{G^2(1-\varepsilon)}{\rho g_c D_p \varepsilon^3}\left[\frac{150(1-\varepsilon)\mu}{D_P G}+1.75\right]. \tag{6.111}$$

For example, when $D_p=0.005$ m, $G=50$ kg/m^2 sec, $g_c=1$ kgm/N sec^2, $\rho=800$ kg/m^3, $\mu=0.010$ N sec/m^2, and $\varepsilon=0.4$, the gradient is $\Delta P/L=0.31(10^5)$ Pa/m.

An improved correlation is that of Sato (1973) and Tallmadge [*AIChE J.* **16,** 1092 (1970)] shown on Figure 6.8(a). The friction factor is

$$f_p=150/\mathrm{Re}_p+4.2/\mathrm{Re}_p^{1/6} \tag{6.112}$$

with the definitions of Eqs. (6.108) and (6.110). A comparison of Eqs. (6.109) and (6.112) is

Re_p	5	50	500	5000
f_p (Ergun)	31.8	4.80	2.05	1.78
f_p (Sato)	33.2	5.19	1.79	1.05

In the highly turbulent range the disagreement is substantial.

TWO-PHASE FLOW

Operation of packed trickle-bed catalytic reactors is with liquid and gas flow downward together, and of packed mass transfer equipment with gas flow upward and liquid flow down.

Concurrent flow of liquid and gas can be simulated by the homogeneous model of Section 6.8.1 and Eqs. 6.109 or 6.112, but several adequate correlations of separated flows in terms of Lockhart–Martinelli parameters of pipeline flow type are available. A number of them is cited by Shah (*Gas-Liquid-Solid Reactor Design,* McGraw-Hill, New York, 1979, p. 184). The correlation of Sato (1973) is shown on Figure 6.9 and is represented by either

$$\phi=(\Delta P_{LG}/\Delta P_L)^{0.5}=1.30+1.85(X)^{-0.85},\quad 0.1<X<20, \tag{6.113}$$

or

$$\log_{10}\left(\frac{\Delta P_{LG}}{\Delta P_L+\Delta P_G}\right)=\frac{0.70}{[\log_{10}(X/1.2)]^2+1.00}, \tag{6.114}$$

where

$$X=\sqrt{(\Delta P/L)_L/(\Delta P/L)_G}. \tag{6.115}$$

The pressure gradients for the liquid and vapor phases are calculated on the assumption of their individual flows through the bed, with the correlations of Eqs. (6.108)–(6.112).

The fraction h_L of the void space occupied by liquid also is of interest. In Sato's work this is given by

$$h_L=0.40(a_s)^{1/3}X^{0.22}, \tag{6.116}$$

where the specific surface is

$$a_s=6(1-\varepsilon)/D_p. \tag{6.117}$$

Additional data are included in the friction correlation of Specchia and Baldi [*Chem. Eng. Sci.* **32,** 515–523 (1977)], which is represented by

$$f_{LG}=\frac{g_c D_p \varepsilon}{3\rho_G u_G^2(1-\varepsilon)}\left(\frac{\Delta P}{L}\right), \tag{6.118}$$

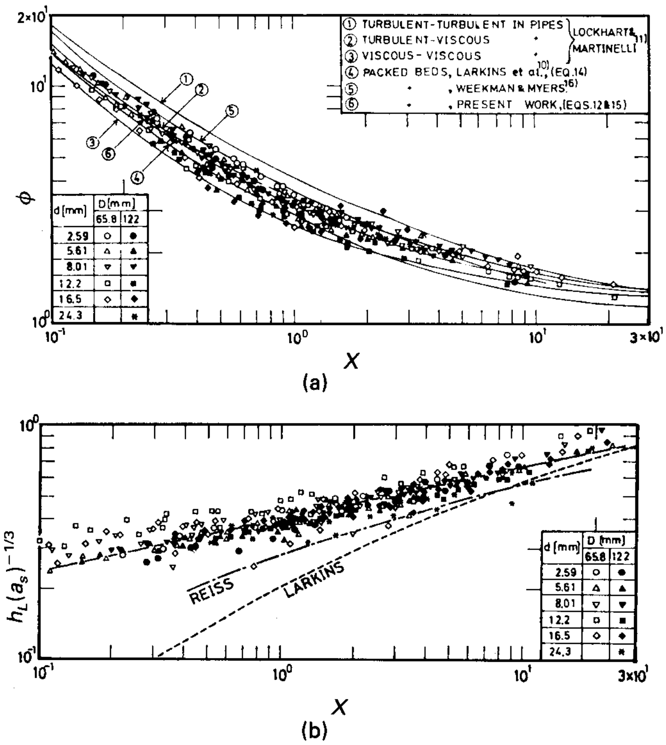


Figure 6.9. Pressure drop gradient and liquid holdup in liquid–gas concurrent flow in granular beds. [*Sato, Hirose, Takahashi, and Toda,* J. Chem. Eng. Jpn. **6,** *147–152 (1973)*]. (a) Correlation of the two phase pressure drop gradient $\Delta P/L$, $\phi=1.30+1.85X^{-0.85}$. (b) Correlation of frictional holdup h_L of liquid in the bed; a_s is the specific surface, 1/mm, d is particle diameter, and D is tube diameter. $h_L=0.4a_s^{1/3}X^{0.22}$.

$$\ln f_{LG}=7.82-1.30\ln(Z/\psi^{1.1})-0.0573[\ln(Z/\psi^{1.1})]^2. \tag{6.119}$$

The parameters in Eq. (6.119) are

$$Z=(\mathrm{Re}_G)^{1.167}/(\mathrm{Re}_L)^{0.767}, \tag{6.120}$$

$$\psi=\frac{\sigma_w}{\sigma_L}\left[\frac{\mu_L}{\mu_w}\left(\frac{\rho_w}{\rho_L}\right)^2\right]^{1/3}. \tag{6.121}$$

Liquid holdup was correlated in this work for both nonfoaming and foaming liquids.

$$\text{Nonfoaming,}\quad h_L=0.125(Z/\psi^{1.1})^{-0.312}(a_sD_p/\varepsilon)^{0.65}, \tag{6.122}$$

$$\text{Foaming,}\quad h_L=0.06(Z/\psi^{1.1})^{-0.172}(a_sD_p/\varepsilon)^{0.65}. \tag{6.123}$$

The subscript w in Eq. (6.121) refers to water.

Countercurrent flow data in towers with shaped packings are represented by Figure 13.37. The pressure drop depends on the viscosity of the liquid and on the flow rates and densities of the liquid and gas, as well as on characteristics of the packing which are represented here by the packing factor F. Nominally, the packing factor is a function of the specific surface a_s and the voidage ε, as

$$F=a_s/\varepsilon^3, \tag{6.124}$$

but calculated values are lower than the experimental values shown in the table by factors of 2–5 or so. Clearly the liquid holdup reduces the effective voidage to different extents with different packings. The voidages of the packings in the table range from 70 to

Good correlation of ϕ is given in Perry's

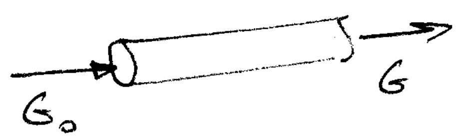
plug in f_p & Re_p

$$\frac{dP}{dz} = -\frac{\rho v_\infty^2}{D_p} \frac{(1-\phi)}{\phi^3}\left[\frac{150(1-\phi)\mu}{D_p \rho v_\infty} + 1.75\right]$$

Not Given in 4th Ed

this form is not very easy to use

So convert to a form using a mass balance



$G [=] \frac{kg}{m^2 s}$ ← total cross sectional area

$$G = \rho v_\infty A = \rho \left[\frac{Q}{A}\right] A$$

$\frac{Q}{A} = v_\infty$

3rd ed eqn 4-22

IN 4th Ed eqn 4-22

$$\frac{dP}{dz} = -\frac{1}{\rho}\left(\underbrace{\frac{G}{D_p}\frac{(1-\phi)}{\phi^3}}\left[\underbrace{\frac{150(1-\phi)\mu}{D_p} + 1.75G}\right]\frac{\rho_o}{\rho_o}\right) = \beta_o$$

constant with z

ρ: variable density – T or # moles

let $\frac{dP}{dz} = \frac{-1}{\rho}\left[\rho_o \beta_o\right]$ and β_o = equation ~~4-31~~ 4-25 3rd ed. 4th Ed

Fogler Ergun

$$\frac{dP}{dz} = -\frac{G}{\rho D_p}\left(\frac{1-\phi}{\phi^3}\right)\left[\frac{150(1-\phi)\mu}{D_p} + 1.75\, \underbrace{G}_{\frac{kg}{m^2 s}}\right] \qquad 4\text{-}22$$

BSL 6.4-13

$$\left(-\frac{dP}{dL}\right)\frac{\rho}{G_0^2} D_p \frac{\varepsilon^3}{1-\varepsilon} = 150\left(\frac{1-\varepsilon}{D_p G_0/\mu}\right) + 1.75 \qquad 6.4\text{-}13$$

$$\frac{dP}{dL} = \frac{G_0^2}{\rho D_p}\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\left[150\frac{(1-\varepsilon)}{D_p G_0/\mu} + 1.75\right]$$

$$G_0 = \frac{kg}{m^2\, s}$$

$$= \frac{G_0}{\rho D_p}\left(\frac{1-\varepsilon}{\varepsilon^3}\right)\left[\frac{150\,(1-\varepsilon)}{D_p/\mu} + 1.75\, G_0\right]$$

Geankoplis eqn 3.1-20

$$\frac{\Delta P}{\Delta L} = \frac{150\,\mu v'}{D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.75\,(v')^2\rho}{D_p}\frac{1-\varepsilon}{\varepsilon^3}$$

$$= \frac{v'}{D_p}\frac{1-\varepsilon}{\varepsilon^3}\left[\frac{150\,\mu(1-\varepsilon)}{D_p} + 1.75\, v'\rho\right]$$

$v' = \varepsilon v$ $\qquad v' A_c \rho = G A_c$

superficial

$v' = G/\rho$

$$\frac{\Delta P}{\Delta L} = \frac{G}{\rho D_p}\frac{1-\varepsilon}{\varepsilon^3}\left[\frac{150\,\mu(1-\varepsilon)}{D_p} + 1.75\,\frac{G\cancel{\rho}}{\cancel{\rho}}\right]$$

Froment & Bischoff

$$-\frac{dP}{dz} = f \, \frac{\rho u_s^2}{d_p} \qquad 11.5.1\text{-}3$$

$$f = \frac{1-\varepsilon}{\varepsilon^3}\left[a + \frac{b(1-\varepsilon)}{Re_p}\right] \qquad 11.5.1\text{-}13$$

$$-\frac{dP}{dz} = \frac{1-\varepsilon}{\varepsilon^3}\,\frac{\rho u_s^2}{d_p}\left[a + \frac{b(1-\varepsilon)}{Re_p}\right]$$

$$\rho u_s = G$$

$$= \frac{1-\varepsilon}{\varepsilon^3}\,\frac{G^2}{\rho\, d_p}\left[a + \frac{b(1-\varepsilon)}{Re_p}\right]$$

$$= \frac{1-\varepsilon}{\varepsilon^3}\,\frac{G}{\rho D_p}\left[Ga + \frac{Gb(1-\varepsilon)}{Re_p}\right]$$

$$Re_p = \frac{D_p \rho u_s}{\mu}$$

$$= \frac{1-\varepsilon}{\varepsilon^3}\,\frac{G}{\rho D_p}\left[Ga + \frac{\cancel{G} b(1-\varepsilon)\mu}{D_p \cancel{\rho u_s}}\right]$$

where $a = 1.75 \quad b = 150$

Tallmadge (1970) $\quad b = 4.2\, Re^{5/6}$

$$\frac{1-\varepsilon}{\varepsilon^3}\,\frac{G}{\rho D_p}\left[Ga + \frac{G(1-\varepsilon)\, 4.2\, Re^{5/6}}{Re_p}\right]$$

$$\left[Ga + \frac{G(1-\varepsilon)}{Re_p^{1/6}}\, 4.2\right]$$

Froment & Bischoff 3/

Tallmadge (1970) cont.

$$= \frac{1-\varepsilon}{\varepsilon^3} \frac{G}{\rho D_p} \left[Ga + \frac{G(1-\varepsilon) 4.2 \mu^{1/6}}{(D_p \rho u_s)^{1/6}} \right]$$