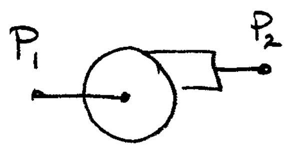
Pump Selection

- highest efficiency at operating conditions



Pump Efficiency $\equiv \dfrac{\text{useful work}}{\text{total work}}$

de Nevers:

$$\underbrace{\frac{dW_{nf}}{dm}}_{\substack{\text{work flow}\\ \text{mass fluid}\\ \text{to pump}}} = \underset{\substack{\uparrow \\ \text{final}\\ -\text{initial}}}{\Delta}\underbrace{\left(\frac{P}{\rho} + gz + \frac{v^2}{2}\right)}_{\substack{\text{useful}\\ \text{work}}} + \underbrace{\mathcal{F}}_{\substack{\text{Useless}\\ \text{(lost work)}}}$$

In most books $W_{nf} = W_s$ Shaft work

assume $z_1 = z_2$

$v_1 = v_2$ (velocity in pipe of same diameter)

ρ = constant

$$\frac{dW_{nf}}{dm} = \Delta\left(\frac{P}{\rho}\right) + \mathcal{F}$$

this is hard to characterize

$$\left(\frac{dW_{nf}}{dm} - \mathcal{F}\right) = \Delta\left(\frac{P}{\rho}\right)$$

$$\left[dW_{nf} - \mathcal{F}\,dm = \frac{\Delta P}{\rho}\,dm\right]\frac{1}{dt}$$

$$\left(\frac{d W_{nf}}{dt} - \mathcal{F}\frac{dm}{dt}\right) = \frac{\Delta P}{\rho}\frac{dm}{dt} \qquad \frac{1}{\rho}\frac{dm}{dt} = \frac{d}{dt}(V\rho)\frac{1}{\rho}$$

$$= \frac{dV}{dt} = \varphi$$

$$\Big(\underbrace{\qquad}_{\text{What do we use?}}\Big) = \Delta P\,\varphi$$

$$\eta_m \dot{W}_m = \Delta P \varphi$$

or

$$\eta_m = \frac{\Delta P \varphi}{\dot{W}_m} = \frac{\Delta P \varphi}{\mathcal{P}_{m\,\text{supplied}}}$$

Power

motor

Book has $P_{o\,\text{supplied}}$

$$\eta_e = \frac{\Delta P \varphi}{\dot{W}_e} = \frac{\Delta P \varphi}{\mathcal{P}_{e\,\text{supplied}}}$$

$$\dot{W}_e > \dot{W}_m > \dot{W}_{nf}$$

Rate of work or Power

this is the most common

Named: Brake HorsePower

Based on the measurement device

Check $\Delta P Q \; [=] \; Pa \frac{m^3}{s} \stackrel{?}{=} W?$

$$\frac{Force}{m^2} \qquad \frac{N}{m^2}\frac{m^3}{s}$$

$$N = kg\frac{m}{s^2} \qquad \text{eg. } F = ma$$

$$Pa\frac{m^3}{s} = \frac{N}{m^2}\frac{m^3}{s} = kg\frac{m}{s^2}\frac{1}{m^2}\frac{m^3}{s} = \frac{kg\,m^2}{s^3}$$

Now W? $\quad W \; [=] \; \frac{J}{s} = \frac{Nm}{s}$

work = (Force)(distance)

$$W = \frac{kg\,m}{s^2}\frac{m}{s} = \frac{kg\,m^2}{s^3} \qquad N \cdot m$$

Units work out!

Gases $\rho \neq$ constant integration required

$$\eta_{Gases} = \frac{\dot{m}\int (dP/\rho)}{P_{o\,supplied}} = \frac{\int \Delta\left(\frac{P}{\rho}\right)\frac{dm}{dt}}{P_o}$$

Derivations:

Size requirements of motor

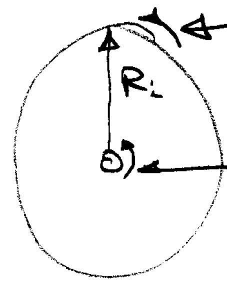
How big should the motor be for a given ΔP?

$P_{o\ \text{supplied}} = \quad \varphi? \quad \eta_m?$ impeller speed? impeller size?

Power delivered through shaft: $(\text{Torque})\,\omega$

angular velocity

angular momentum balance on fluid in pump



assume fluid leaves in a tangential direction

small angular momentum $\cong 0$

$$\text{Angular momentum} = \dot{m}\,\omega R_i^2 = \rho\,\varphi\,\omega R_i^2 = \text{Torque}$$

$$\underbrace{(\text{Torque})\,\omega}_{\text{Power from motor}} = \underset{\text{Power supplied}}{P_o} = \frac{\Delta P\,\varphi}{\eta_m}$$

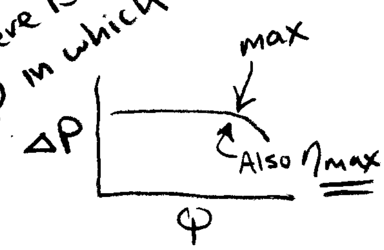
$$(\rho\,\cancel{\varphi}\,\omega R_i^2)\,\omega = \frac{\Delta P\,\cancel{\varphi}}{\eta_m}$$

$$\Delta P = \eta_m\,\rho\,\omega^2 R_i^2$$

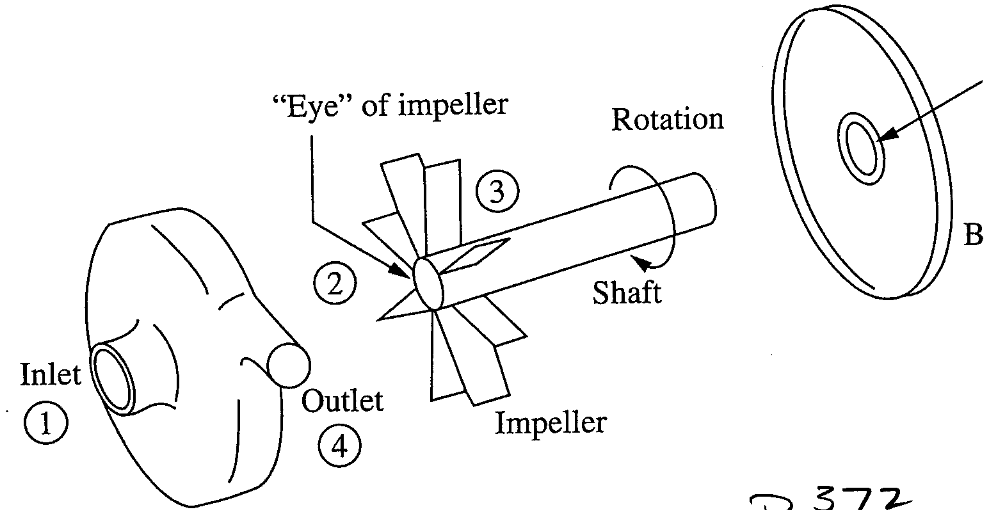
η_m: efficiency of design; ω: speed; R_i: size

Independent of φ!

Except - there is a maximum φ in which $\Delta P \downarrow$



10.3.1



Front of housing, called diffuser or volute

P 372

FIGURE 10.7

inlet (1) Eye of impeller

Book derivation

$1 \to 2$ $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 + \rho \mathcal{F}_{1-2}$ $\Delta h = 0$

(Centrifuge) $2 \to 3$ section 2.9 eqn 2.34

$$P_3 - P_2 \int_{r_2}^{r_3} \rho \omega^2 r\,dr = \frac{\rho \omega^2}{2}(r_3^2 - r_2^2)$$

Diffuser

$$P_4 - P_3 = \frac{\rho}{2}(v_3^2 - v_4^2) - \rho \mathcal{F}_{3\text{-}4}$$

Adding equations
dividing by ρg
$v_2 = \omega r_2$ $v_3 = \omega r_3$

this term stays since $ID_{outlet} > ID_{suction}$

$$H = \frac{P_4 - P_1}{\rho g} = \frac{\omega^2}{g}(r_3^2 - r_2^2) + \frac{(v_1^2 - v_4^2)}{2g} + \frac{(\mathcal{F}_{3\text{-}4} + \mathcal{F}_{1\text{-}2})}{g}$$

ΔP is sometimes referred to as the pump head

$$\Delta P = \varrho g \underbrace{H}_{\text{pump head}}$$

$H \equiv \dfrac{\Delta P}{\varrho g}$ this is the height of fluid that the pump could elevate fluid

Example in text Example 10.5 – (Neglects $\mathcal{F}$)

plug in numbers and get $H = 30.4\,m$

Actual value is about twice this value.

Cavitation & NPSH

- Vapor lock
- Cavitation will destroy a pump ← loud sound
  - boiling within pump

At higher fluid temperatures this is more likely to occur (See 10.3.2 Fig 10.10)

Vapor lock

$$\dot{W}_m = \frac{\Delta P Q}{\eta_m} = \frac{\rho g H Q}{\eta_m}$$

$\rho_{gas} \simeq 1\ kg/m^3$

$\rho_{liq} \simeq 10^3\ kg/m^3$

$$\Delta P Q = (1\ kg/m^3)(9.81\ m/s^2)(H)Q \text{ — flow stops}$$

$$\Delta P Q = (10^3\ kg/m^3)(9.81\ m/s^2) H Q \text{ — } \underline{\text{flow ok}}$$

this would be small

these are the same

Prevent vapor lock - always fill pump with fluid
this is known as priming

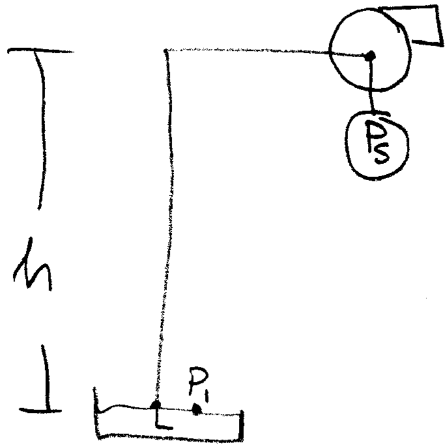
prevent cavitation have $P_{pump\ inside} > P^{vap}$

minimum required Net Positive Suction Head - $NPSH_R$

available NPSH or NPSHA - head available at suction head

$$NPSHA > NPSHR + (P^{vap}{}_{head})$$

$$(\text{Suction Pressure})\ P_s = (\rho g)\,NPSH + P^{vap}$$



$$P_1 + 0 + \rho g(0) = P_s + \tfrac{1}{2}\rho v_s^2 + \rho g h + \rho F$$

solve for h

$$h = \frac{P_1 - P_s - \tfrac{1}{2}\rho v_s^2 - \rho F}{\rho g} =$$

Substituting for P_s

$$h = \frac{P_1 - \rho g(NPSH) - P^{vap} - \tfrac{1}{2}\rho v_s^2 - \rho F}{\rho g}$$

Note NPSH for pumps is given at 60°F with discharge line valve fully open

1. Small values of NPSH mean that the value of h is larger – you can "suck" liquid from lower heights

2. suction ID's are larger than outlet this makes v_s smaller and gives larger h's (more suction power)