CSTR in Series (4.3.2)

Assume 1) Well-mixed
2) liquid $v = v_0$
3) Rxn rate is pseudo first order
4) Steady-state

$$0 = F_{A0} - F_A + r_A V$$

$$F_A = C_A v$$

$$F_{A_0} = C_{A_0} v_0$$

$$0 = C_{A_0} v_0 - C_A v + r_A V$$

Since $v = v_0$

$$0 = C_{A0} v_0 - C_A v_0 + r_A V$$

$$\frac{C_A - C_{A0}}{r_A} = \frac{V}{v_0} = \tau$$ space time eqn 2-24 p 66 (Similar to a mean residence time)

plug in $r_A = -kC_A$

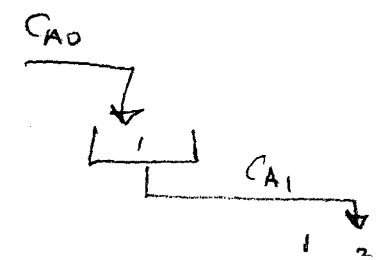
$$\frac{C_A - C_{A0}}{-kC_A} = \tau \quad \rightarrow \quad C_A - C_{A0} = \tau(-kC_A)$$

$$C_A = \frac{C_{A0}}{1 + k\tau}$$

or

$$C_A = \frac{C_{A0}}{1 + \tau k}$$

2 CSTR's in Series



$$\left\{ v_0 = v_1 = v_2 \quad \text{and} \quad \underline{V_1 = V_2} \right\}$$

from 1. $C_{A_1} = \dfrac{C_{A0}}{1+\tau k}$

from 2: $F_{A_1} - F_{A_2} + r_{A_2} V = 0$

$$v_0 (C_{A_1} - C_{A_2}) - k C_{A_2} V = 0$$

Solving for τ

$$\tau = \frac{C_{A_2} - C_{A_1}}{-k C_{A_2}}$$

Solving for C_{A_2}

$$C_{A_2} = \frac{C_{A_1}}{1 + k\tau}$$

pluging in for C_{A_1} from mole balance in Reactor 1

$$C_{A_2} = \frac{C_{A0}}{(1+k\tau)} \frac{1}{(1+k\tau)} = \frac{C_{A0}}{(1+k\tau)}$$

for n CSTR's with v = constant $V_1 = V_2 = V_3 \dots = V_n$ equal sized

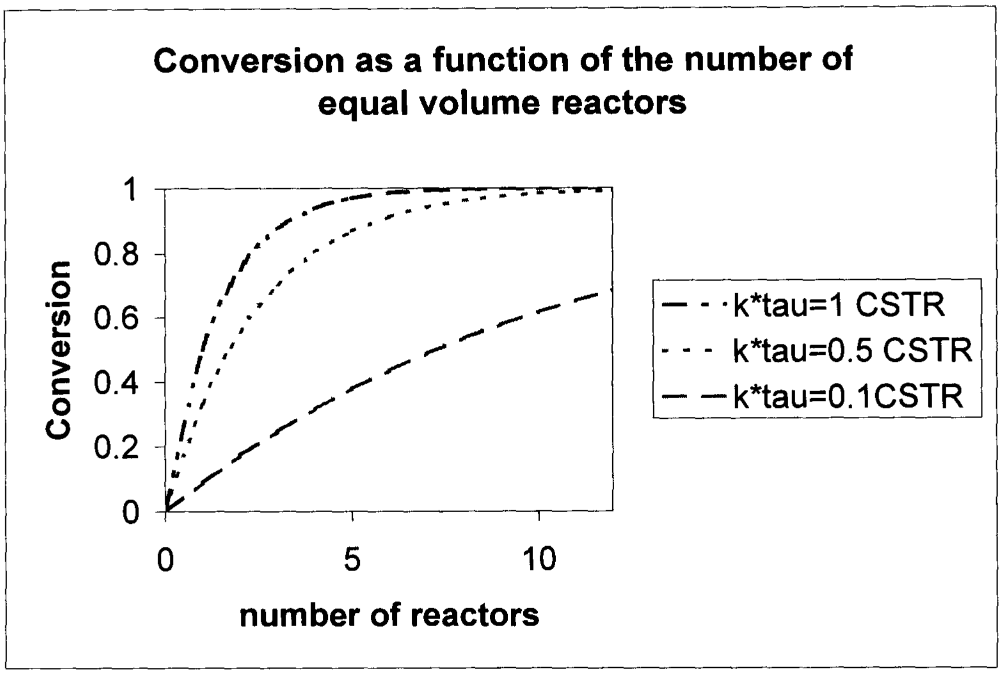
$$C_{A_n} = \frac{C_{A0}}{(1+k\tau_i)^n}$$

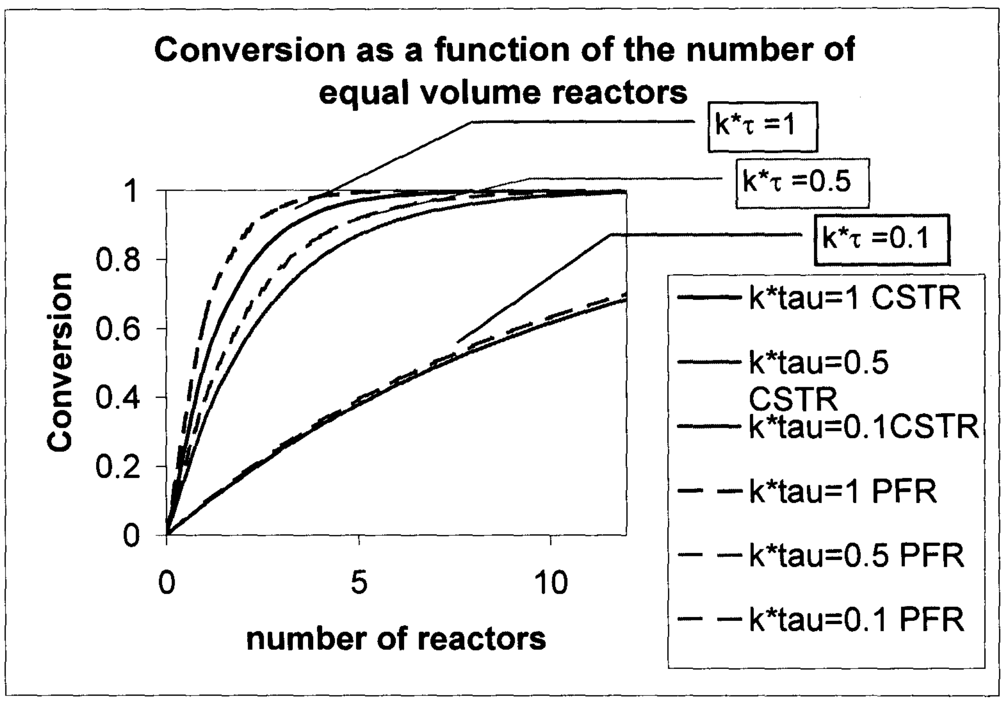
$C_A = C_{A0} - C_{A0} X_A = C_{A0}(1 - X_A)$

(Don't use X to get this) or $X_A = 1 - \dfrac{1}{(1+k\tau_i)^n}$

$\frac{C_A}{C_{A0}} = 1 - X_A$

$X_A = 1 - \frac{C_A}{C_{A0}}$


Conversion as a function of the number of equal volume reactors
Conversion
1
0.8
0.6
0.4
0.2
0
0
5
10
number of reactors
k*tau=1 CSTR
k*tau=0.5 CSTR
k*tau=0.1CSTR


Conversion as a function of the number of equal volume reactors
k*τ =1
k*τ =0.5
k*τ =0.1
Conversion
1
0.8
0.6
0.4
0.2
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5
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number of reactors
k*tau=1 CSTR
k*tau=0.5 CSTR
k*tau=0.1CSTR
k*tau=1 PFR
k*tau=0.5 PFR
k*tau=0.1 PFR