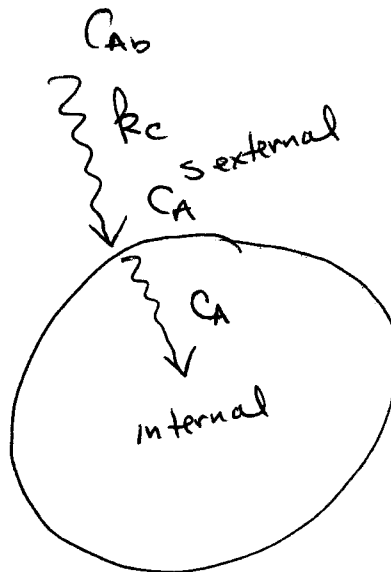


# Chapter 12 (Internal)



Goal for Steady-state

$$\frac{dF_A}{dV} = \eta r_{As}$$

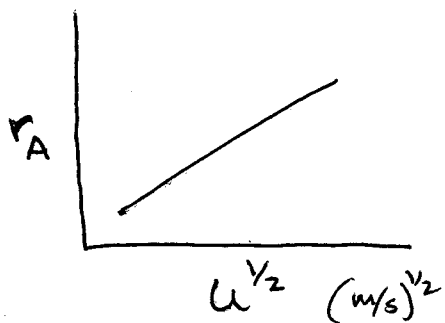
↑ a special rate that you know

or even combine Internal & external

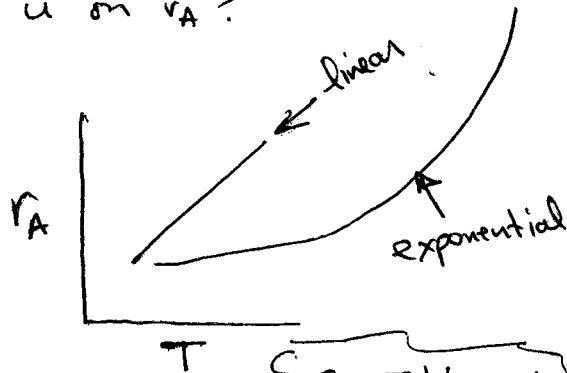
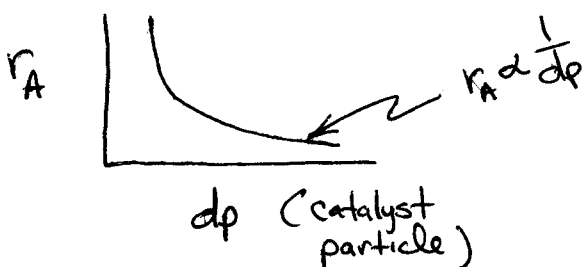
$$\frac{dF_A}{dV} = \Omega r_{Ab}$$

equation 12-67

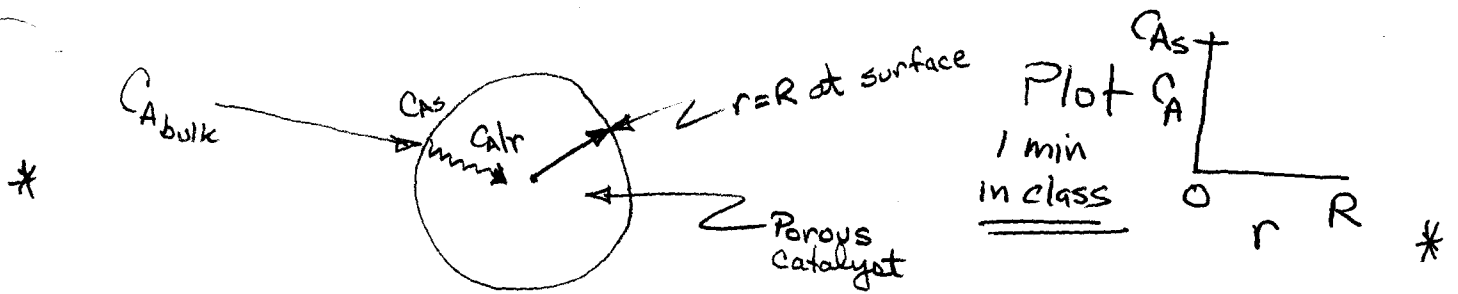
with no axial diffusion 12-69



← last lecture external diffusion  
what if no effect on increasing  $u$  on  $r_A$ ?

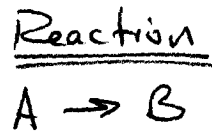
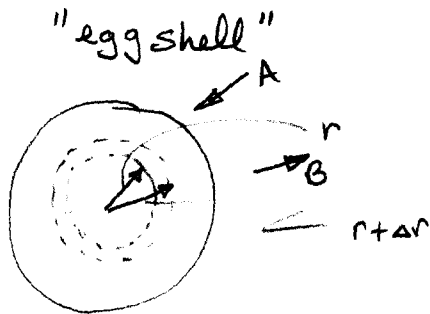


See Table 12-1  
page 849



\*  $D_e$  - effective diffusivity for a porous pellet

\* Shell Balance (similar to plug flow model)



$$S_{a,p} = A_i$$

$$A_i = \frac{m^2}{m^3} \left( \frac{\text{Internal + external surface area}}{\text{pellet volume}} \right)$$

$$r_A [E] \frac{\text{mol}}{\text{s m}^2}$$

(internal + external area)

Accumulation = in - out + gen

$$* (4\pi r^2 \Delta r) \frac{d[C_A]}{dt} = \underbrace{W_A}_{\substack{\uparrow \\ \text{mol A} \\ \text{s m}^2}} \left. 4\pi r^2 \right|_{r=r} - W_A \left. 4\pi r^2 \right|_{r=r+\Delta r} + r_A 4\pi r^2 \Delta r S_{a,p}$$

Area thickness

"in" is in the opposite direction to  $W_A \rightarrow W_A$  will be negative

Assumptions

Reaction is constant within shell as well as T and P

steady-state:  $\frac{dC_A}{dt} = 0$

$$4\pi \frac{(W_A r^2 |_{r+\Delta r} - W_A r^2 |_r)}{4\pi r^2 \Delta r} = r_A S_{a,p}$$

limit as  $\Delta r \rightarrow 0$

$$\frac{1}{r^2} \frac{d(W_A r^2)}{dr} = r_A A_i [E] \frac{\text{mol}}{\text{s m}^3}$$

$$* W_A = \overbrace{y_A (W_A + W_B)}^{\text{Convective}} - \overbrace{D_e C_T \frac{dy_A}{dr}}^{\text{Diffusion}}$$

Remember for the external case we neglected diffusion.

Assumptions:

- \* if EMD  $-W_A = W_B$
- or dilute concentration of A in inert  $y_A \rightarrow 0$

$$\frac{\text{mol}}{\text{m}^2 \text{ s}} W_A = -D_e C_T \frac{dy_A}{dr} \quad D_e [L^2] = \frac{\text{mol}}{\text{m}^2 \text{ s}} \frac{\text{m}^2}{\text{mol}} = \frac{\text{L}^2}{\text{s}} \quad \text{or } \left(\frac{\text{m}^2}{\text{s}}\right)$$

mixing up the gas and solid phase units!

$$\frac{1}{r^2} \frac{d}{dr} \left[ -D_e C_T \frac{d(y_A)}{dr} r^2 \right] = r_A S_A \rho_p$$

this is not diffusion of A in a gas!!



$$C_A = y_A C_T$$

Assume:  $D_e$  - constant (uniform through out pellet)

$$-D_e \frac{1}{r^2} \frac{d}{dr} \left( \frac{dC_A}{dr} r^2 \right) = r_A S_A \rho_p$$

\* Book example  $r_A = -k C_A^n$    
 variable units  $k [L^{n-1} / (s \cdot \text{mol})] \frac{\text{L}^{\text{gas}}}{\text{m}^2 \text{ surface}}$  for  $n \geq 0$

expanding and plugging in for  $r_A$

$$* \frac{d^2 C_A}{dr^2} + \frac{2}{r} \frac{dC_A}{dr} - \frac{k_n S_A \rho_p}{D_e} C_A^n = 0$$

\* Boundary Conditions

$$C_A = C_{A_s} \text{ at } r=R$$

$$\left\{ \begin{array}{l} C_A \text{ is finite at } r=0 \\ \text{or} \\ \frac{dC_A}{dr} = 0 \text{ at } r=0 \end{array} \right\}$$

\* Look At dimensionless Groups

11-3

$$* \psi = \frac{C_A}{C_{A_s}} \quad \lambda = \frac{r}{R}$$

$$d\psi = \frac{dC_A}{C_{A_s}} \quad d\lambda = \frac{dr}{R}$$

$$d^2\psi = \frac{d^2C_A}{C_{A_s}}$$

$$\frac{C_{A_s}}{R^2} \frac{d^2\psi}{d\lambda^2} + \frac{2}{R\lambda} C_{A_s} \frac{d\psi}{d\lambda} - \frac{k_n S_a \rho_p}{De} C_{A_s}^n \psi^n = 0$$

$$* \frac{d^2\psi}{d\lambda^2} + \frac{2}{\lambda} \frac{d\psi}{d\lambda} - \left( \frac{k_n S_a \rho_p R^2 C_{A_s}^{n-1}}{De} \right) \psi^n = 0$$

B.C.  $C_A = C_{A_s}$  at  $r=R \Rightarrow \lambda=1 \quad \psi=1$   
 $C_A$  is finite at  $r=0 \quad \lambda=0 \quad \psi$  is finite

let us call this  $\phi_n^2$

if  $n=1 \quad \phi_1 = \sqrt{\frac{k_1 S_a \rho_p R^2}{De}}$

$$\frac{\left( \frac{1}{s} \frac{L}{m^2} \right) \left( \frac{m^2}{kg \text{ cat}} \right) \left( \frac{kg \text{ cat}}{m^3 \text{ pellet}} \right)}{m^2 \text{ pellet}} m^2 \text{ pellet}$$

\* Thiele modulus

$$* \text{ or for } n \quad \phi_n = \sqrt{\frac{k_n S_a \rho_p C_{A_s}^n R}{De (C_{A_s} - 0)/R}} = \sqrt{\frac{\text{" } r_A \text{ | } (S_a \rho_p) R \text{ "}}{C_A = C_{A_s}}} \left[ \Rightarrow \frac{\text{mol/m}^2 \cdot \text{m}}{\text{s m}^2 (\text{mol/m}^3)^n} m_p \right]$$

$$\text{" } W_A \text{ " } \left[ \Rightarrow \frac{\text{kg mol}}{m_p \text{ s kg}} \frac{1}{m_p} \right]$$

if  $C_A=0$  at  $r=0$

$$* \phi_n = \sqrt{\frac{\text{Reaction if } C_A = C_{As} \text{ everywhere}}{\text{diffusion rate}}} = \sqrt{\frac{\text{a surface rxn rate}}{\text{a diffusion rate}}}$$

\*  $\phi_n \rightarrow \text{large} \Rightarrow \text{diffusion rate limited}$   
 $\rightarrow \text{small} \Rightarrow \text{reaction rate limited}$

\* Book gives example:

$$r_A = - \frac{k(C_A - C_B/k)}{(k_A C_A + C_B/k_B + 1)}$$

- ①  $k \rightarrow \text{large}$
- ② Surface reaction is controlling (compared to adsorption or desorption steps)
- ③  $k_A \rightarrow \text{small}$  } weakly adsorbed }  $(k_A + C_B/k_B) \ll 1$   
 $k_B \rightarrow \text{large}$  }  
 $C_A \text{ and } C_B \text{ are small}$

$r_A \approx -k C_A$  Book 12-21

following section 12.1.4 and ~~Appendix A.4~~

Solution  $\frac{C_A}{C_{As}} = \psi = \frac{\sinh(\phi_1 \lambda)}{\lambda \sinh \phi_1}$

Small  $\phi_1$   
Reaction control

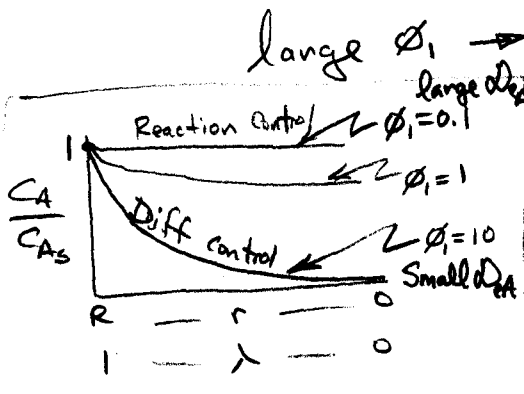
$\sinh(\phi_1 \lambda) \approx \phi_1 \lambda$   
 $\frac{\phi_1 \lambda}{\lambda \phi_1} \approx 1$

$\sinh x = \frac{e^x - e^{-x}}{2}$

$\approx \frac{e^x}{2}$

Note: Diffusional resistance causes a gradient ( $dy/dr$ )

$r = k C_A$   $C_{As}$  is lower, but there is more A inside the catalyst.

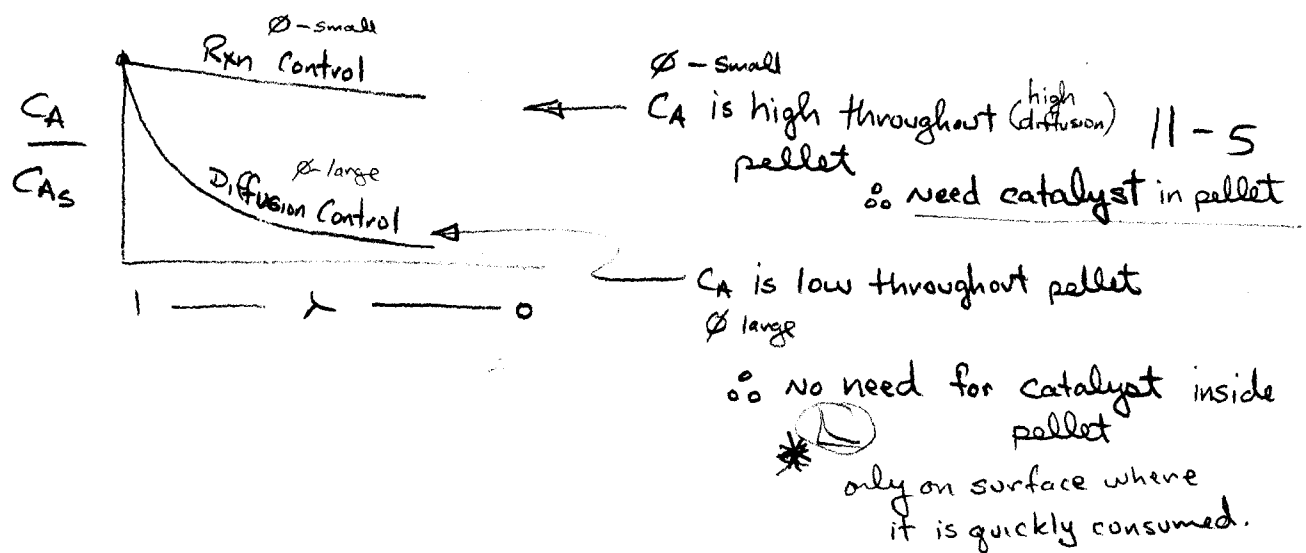


large  $\phi_1 \rightarrow$  large  $dC_A$

$\approx \frac{e^{(\phi_1 \lambda)} - e^{-\phi_1 \lambda}}{2 e^{\phi_1}} = \frac{e^{\phi_1(\lambda-1)}}{\lambda}$

$\lambda < 1$

so  $e^{\phi_1(\lambda-1)} = \frac{1}{e^{\phi_1(1-\lambda)}}$



Define  $\eta$

$$\eta = \frac{\text{Overall Reaction Rate (including diffusional resistance in pores)}}{\text{Reaction rate with inside and outside surfaces at } C_A = C_{A_s} \quad T = T_s}$$

$$\eta = \frac{r_{obs}}{r_A|_{C_A = C_{A_s}}} = \frac{\frac{1}{W} \int (r_A dW)}{r_A|_{C_A = C_{A_s}}} \quad \left. \vphantom{\frac{1}{W} \int (r_A dW)} \right\} \text{Rate both inside and outside surfaces.}$$

What is  $r_A|_{C_A = C_{A_s}}$

At steady-state the mass flux to the surface must equal the consumption rate of  $r_A$  in the pellet.

$$-W_A|_{r=R} (4\pi R^2) = -r_{obs} S_{tp} \frac{4}{3}\pi R^3$$

$$-W_A(4\pi R^2) \Big|_{r=R} = (4\pi R^2) \left( \rho_e \frac{dC_A}{dr} \Big|_{r=R} \right) = +4\pi R^2 \rho_e \frac{C_{As}}{R} \frac{d\psi}{d\lambda} \Big|_{r=R}$$

$$\frac{d\psi}{d\lambda} = \frac{d}{d\lambda} \left( \frac{1}{\lambda} \frac{\sinh(\phi, \lambda)}{\sinh \phi_1} \right) \quad \text{from soln.}$$

$$= -\frac{1}{\lambda^2} \frac{\sinh(\phi, \lambda)}{\sinh \phi_1} + \frac{\phi_1}{\lambda} \frac{\cosh(\phi, \lambda)}{\sinh \phi_1}$$

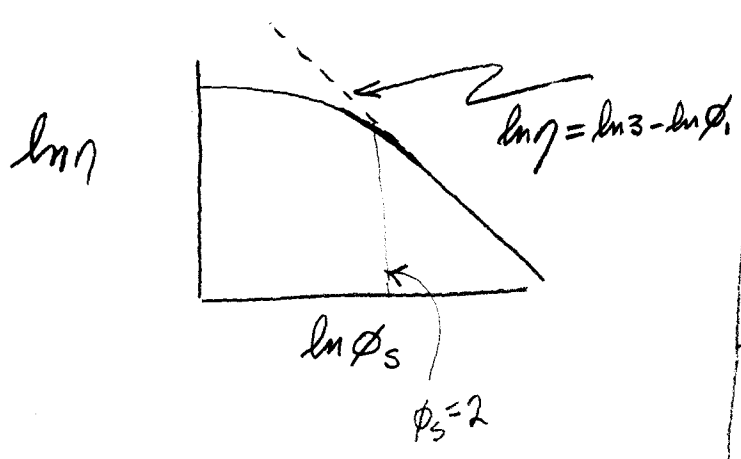
$$\frac{d\psi}{d\lambda} \Big|_{\lambda=1} = -1 + \phi_1 \coth(\phi_1)$$

$$-W_A(4\pi R^2) = +4\pi R^2 \left( \phi_1 \coth(\phi_1) - 1 \right) \rho_e C_{As} = -r_{obs} S_a \rho_p \frac{4}{3} \pi R^3$$

$$r_{obs} = \frac{-3(\phi_1 \coth(\phi_1) - 1) \rho_e C_{As}}{S_a \rho_p R^2}$$

$$\phi_1 \equiv \sqrt{\frac{k_1 S_a \rho_p R^2}{\rho_e}}$$

$$\eta = \frac{r_{obs}}{r_A} \Big|_{C_A=C_{As}} = \frac{-3 \rho_e C_{As} [\phi_1 \coth \phi_1 - 1]}{S_a \rho_p R^2} \cdot \frac{1}{-k_1 C_{As}} = \frac{1}{\phi_1^2} 3(\phi_1 \coth \phi_1 - 1)$$



large  $\phi_1$

$$\coth \phi_1 = \frac{\cosh \phi_1}{\sinh \phi_1} \rightarrow \frac{e^{\phi_1}}{e^{\phi_1}} \approx 1$$

$$\eta \approx \frac{3\phi_1}{\phi_1^2} = \frac{3}{\phi_1}$$

$$r_{obs} = \eta \left. r_A \right|_{C_A=C_{A_s}}$$

$$\phi_1 = R \sqrt{\frac{k_1 p_p S_a}{D_e}}$$

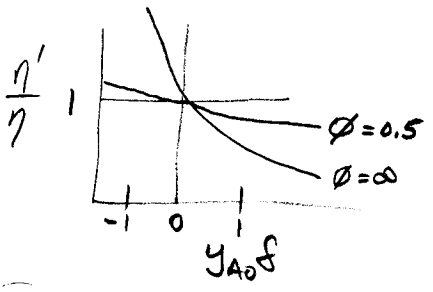
CASE I again for  $\phi \rightarrow$  large Diffusion control

$$r_{obs} = \frac{3}{\phi_1} = \left( \frac{3}{R} \sqrt{\frac{D_e}{k_1 p_p S_a}} \right) (k_1 C_{A_s}) = \frac{3}{R} \sqrt{\frac{k_1 D_e}{p_p S_a}} C_{A_s}$$

CASE II rxn control  $\eta = 1$   $r_{obs} = r|_{C_{A_s}}$

CORRECTIONS

①  $y_{A_0} \delta \neq 0$  Figure 11-6 Volume Change



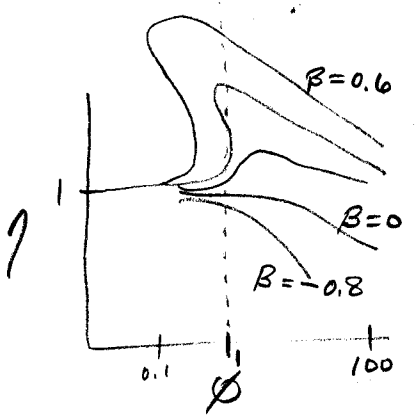
$y_{A_0} \delta > 0$  increase in # of moles (pressure), but dilutes reactants  
Reduce  $r_{obs}$

$y_{A_0} \delta < 0$  decrease in moles (pressure)  
 increases  $r_{obs}$

(multiple steady-states)  
 ↓

②

NONISOTHERMAL  
 $\eta$  only valid for isothermal conditions



exothermic  $\rightarrow$  external surface  $T < T_{inside}$  pellet  
 exothermic reaction takes place

gives  $\eta > 1$

$$B = \frac{C_{A_s} (-\Delta H_r) D_e}{k_i T_s} = \frac{\Delta T_{max}}{T_s}$$

$\downarrow$   
 thermal conductivity

for  $r = k C_A^n$   
 See expressions 12-20  $\phi_n =$   
 12-35  $\eta =$

$$\gamma = \frac{E}{RT_s} = 30$$

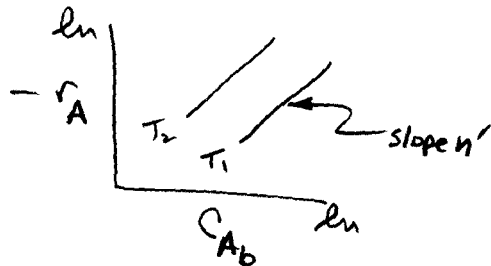


Q.3 Falsified kinetics :

If internal diffusion is limiting the reaction rate :

$\phi \rightarrow$  large

CSTR Data - No external mass transfer limitations



No external limitations

$$C_{A_b} = C_{A_s}$$

CSTR:  $0 = F_{A_0} - F_A + r_A W$

$$r_A = \left( \frac{F_A - F_{A_0}}{W} \right)$$

$$r_A = -k C_{A_b}^{n'}$$

observed rate  
(=  $\eta' r_{actual}$ )

$$C_{A_b} = F_A \phi \Rightarrow C_{A_s}$$

$$\ln(-r_A) = \ln k' + n' \ln C_{A_s}$$

slope from lines above

For Power Law Expressions

from equation 11-36

$$\eta = \left( \frac{2}{n+1} \right)^{1/2} \frac{3}{\phi_n}$$

$$= \left( \frac{2}{n+1} \right)^{1/2} \frac{3}{R} \sqrt{\frac{D_e}{k_n S_a \rho}} C_{A_s}^{(1-n)/2}$$

$$-k' C_{A_s}^{n'} = r_{obs} = \eta r_{A_s} = \frac{3}{\phi_n} \left( \frac{2}{n+1} \right)^{1/2} (-k_n C_{A_s}^n)$$

Now look at the actual rate.  $\phi_n = \sqrt{\frac{k_n S_a \rho C_{A_s}^{1-n/2}}{D_e}}$  this has been absorbed in the  $n'$  value

$$\frac{2n/2 C_{A_s}^{n'} C_{A_s}^{(1-n)/2}}{C_{A_s}^{(2n+1-n)/2}} = C_{A_s}^{n'}$$

$$\frac{2n+1-n}{2} = n'$$

$$\frac{n+1}{2} = n'$$

$$\underline{\underline{n' = \frac{1+n}{2}}}$$

see also  $E_{True} = 2 E_{apparent}$   
 $\uparrow$  true  $\quad \quad \quad \uparrow$  apparent

$$k' = k_n \left( \frac{2}{n+1} \right)^{1/2} \frac{3}{R} \sqrt{\frac{D_e}{k_n S_a \rho}}$$

$$= k_n^{1/2} [ ]$$

$$k' = A^{1/2} [ ] \left[ \exp\left(-\frac{E_a}{RT}\right) \right]^{1/2}$$

$$A' \exp\left(-\frac{E_a'}{RT}\right) = A^{1/2} [ ] \exp\left(-\frac{E_a}{2RT}\right)$$

$$\underline{\underline{E_a' = E_a}}$$

Measured  $\quad \quad \quad$  Intrinsic

11/13/99

## Review Thiele modulus

$$\phi_1 = \sqrt{\frac{k_1 S_a \rho_p R^2}{D_e}}$$

$\phi \rightarrow$  large     $\mathcal{D}$  control  
 $\phi \rightarrow$  small    rxn control

$$\eta \equiv \frac{r_{A,obs}}{r_A|_{C_A=C_{A_s} \text{ everywhere}}} = \frac{\frac{1}{W} \int r_A dW}{r_A|_{C_A=C_{A_s}}}$$

How could you use this?

$$\frac{dF_A}{dW} = r_{A,obs}$$

what is this?  $\therefore$  observed rate  
actual rate

$$r_{A,obs} = \eta r_A|_{C_A=C_{A_s}} = \underbrace{-\eta k C_{A_s}}_{\text{for first order rate}}$$

Do we know  $C_{A_s}$ ?

Section 11.4 Overall Effectiveness Factor  $\Omega$  (first order reactions) 11-9

At steady-state

$$W_A A_p = \cancel{-} r_{A,obs} (A_p + A_i)$$

↑ external area      ↑ Internal area

mol / m<sup>2</sup> s

Fogler incorrectly places a negative sign ( $W_A < 0$ ) for a reactant

Looking at the left hand side

$$W_A A_p = -k_c A_p (C_{Ab} - C_{As})$$

↑ external mass transfer coefficient

If the intrinsic reaction rate is  $r_A' = -k C_A$   
 By definition of  $\eta$   
 $r_{A,obs} = \eta r_A' = -\eta k C_{As}$

↑ concentration at any surface.

$$-(A_p + A_i) \eta k C_{As} = -k_c A_p (C_{Ab} - C_{As})$$

$$C_{As} = \frac{k_c A_p C_{Ab}}{k_c A_p + \eta k (A_p + A_i)}$$

$$r_A = \underbrace{-\eta k C_{As}}_{r_A} = -\eta k \frac{k_c A_p C_{Ab}}{k_c A_p + \eta k (A_p + A_i)}$$

$$r_A = -k C_{Ab} \left[ \frac{\eta}{1 + \eta \frac{k(A_p + A_i)}{k_c A_p}} \right]$$

Define  $\Omega \equiv \frac{r_{obs}}{\eta|_{C_A=C_{Ab}}} = \frac{-k C_{Ab} \left[ \frac{\eta}{1 + \eta \frac{k(A_p + A_i)}{k_c A_p}} \right]}{-k C_{Ab}} = \frac{\eta}{1 + \eta \frac{k(A_p + A_i)}{k_c A_p}}$

for PFR  $\frac{dF_A}{dV} = \eta_{A,obs} (A_p + A_i) = \Omega k C_{Ab} (A_p + A_i)$   
 (Go through Example 11-2) !!

$$\frac{r_{obs,1} \phi_{11}^2}{r_{obs,2} \phi_{12}^2} = \frac{\phi_{11} \coth \phi_{11} - 1}{\phi_{12} \coth \phi_{12} - 1}$$

$$r|_{C_A=C_{A,s}} = r|_{C_A=C_{A,s}}$$

we know  $r_{obs,1}/r_{obs,2}$

Look only at  $\phi$  ratio  
What about

$$\frac{\phi_{11}^2}{\phi_{12}^2} = \frac{R_1^2 \frac{k_p s_a}{\Delta_e}}{R_2^2 \frac{k_p s_a}{\Delta_e}}$$

$S_a = \frac{m^2}{kg}$   
 $\rho_p = \frac{kg}{m^3}$   
As long as decreasing the size does not effect  $S_a$

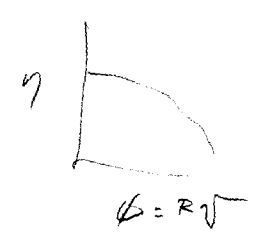
$$\frac{\phi_{11}^2}{\phi_{12}^2} = \frac{R_1^2}{R_2^2} = \frac{(0.01)^2}{(0.001)^2} \frac{1 \times 10^{-4}}{1 \times 10^{-6}} = 1 \times 10^2 = 100$$

i.e. the particles have a uniform pore structure

$\phi_{11} = 10 \phi_{12}$

Now plug in this relationship

$$\frac{(3 \times 10^{-5})}{(15 \times 10^{-5})} (100) = \frac{10 \phi_{12} \coth (10 \phi_{12}) - 1}{\phi_{12} \coth \phi_{12} - 1}$$



Solve for  $\phi_{12}$  by trial & error

$\phi_{12} = 1.646$        $\phi_{11} = 16.46$   
 $R_2 = 0.001$        $R_1 = 0.01$

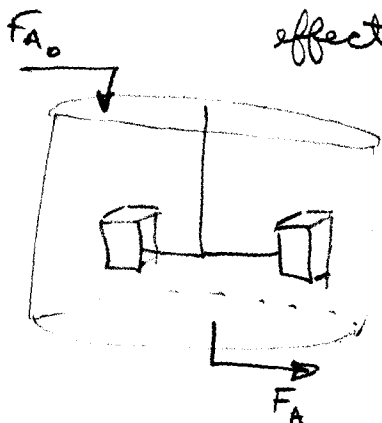
Now

$$\frac{\phi_{13}}{\phi_{11}} = \frac{R_3}{R_1} \quad R_3 = \frac{0.899}{16.46} (0.01 \text{ m}) = \frac{5.46 \times 10^{-4}}{0.00546 \text{ m}}$$

$\phi \quad \underline{\underline{0.55 \text{ mm}}}$

Example 12-2 pages 759-760

What size particle do we need to eliminate diffusion effects?



$$0 = F_{A0} - F_A + r_A W$$

$$r_A = \frac{F_A - F_{A0}}{W} \left. \begin{array}{l} \text{measure } F_A \\ \text{calculate } r_A \end{array} \right\}$$

2 RUNS	$r_A \times 10^5 \text{ (mol/grat s)}$	R (m)
1	3.0	0.01
2	15.0	0.001

Run under identical conditions T, P

(I)

first see what  $\phi$  would be at  $\eta = 0.95$ ;

$$\eta \approx 1 \text{ or } 0.95 = \frac{3(\phi_{13} \coth \phi_{13} - 1)}{\phi_{13}^2}$$

solve for  $\phi_{13}$  →

$$\phi_{13} = R_3 \sqrt{\frac{k_p a_s}{D_e}} \quad \phi_{13} = 0.899 = R_3 \sqrt{\frac{k_p a_s}{D_e}}$$

if uniform particle

we don't know  $a_s$  &  $D_e$  so we can not find this term

$\phi_{13} = 0.899 \quad \eta = 0.95$   
 $\phi_{13} = ? \quad \eta = 1.0$   
 you can not use 1!!

$$\eta \equiv \frac{r_{obs}}{r_l} = \frac{3(\phi_1 \coth \phi_1 - 1)}{\phi_1^2} \quad (C_A = C_{As})$$

(assume first order rxn)

(II)

Now TAKE RATIO OF 2 Rates

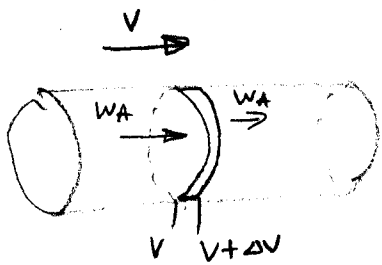
$$\frac{\eta_1}{\eta_2} = \frac{\frac{3(\phi_{11} \coth \phi_{11} - 1)}{\phi_{11}^2}}{\frac{3(\phi_{12} \coth \phi_{12} - 1)}{\phi_{12}^2}} = \frac{r_{obs1}/r_l |_{C_A=C_{As}}}{r_{obs2}/r_l |_{C_A=C_{As}}}$$

Combining 11-58, 11-59, 11-60 and 11-61

Approach

1.  $R = ? \quad \eta > 0.95$
2.  $\eta = \frac{3(\phi_1 \coth \phi_1 - 1)}{\phi_1^2}$
3.  $\phi_1 = R \sqrt{\frac{k_p a_s}{D_e}}$

PFR - filled with catalyst



12-2  
example

$$0 = \underbrace{W_A|_V}_{\frac{\text{mol}}{\text{m}^2 \text{ s}}} - \underbrace{W_A|_{V+\Delta V}}_{\frac{\text{mol}}{\text{m}^2 \text{ s}}} + \underbrace{r_A}_{\frac{\text{mol}}{\text{m}^2 \text{ cat}}} \underbrace{S_a}_{\frac{\text{m}^2 \text{ cat}}{\text{m}^3 \text{ reactor}}} \underbrace{\rho}_{\frac{\text{m}^3 \text{ cat}}{\text{m}^3 \text{ reactor}}} (1-\phi) \frac{\Delta z}{m}$$

$$W_A = y_A \left( \underbrace{\sum W_i}_{\text{total } W_T} \right) - c_T D_{\text{gas}} \frac{dy_A}{dz}$$

neglect axial diffusion  
plug flow assumption!

$$W_A = \frac{F_A}{A}$$

Don't let the book confuse you p. 629

→ Same old PFR equation ←

$$\frac{dF_A}{dV} = r_A \rho_c (1-\phi) S_a$$

$r_A$  - the observed rate

$$r_A = r_{\text{obs}} = \sum_{C_A=C_{A_b}} r_A|_{C_A=C_{A_b}} = \sum k C_{A_b}$$

$\left( \frac{\text{m}^3}{\text{m}^2 \text{ s}} \right)$

eg 10-66 →  $k_c \propto \left( \frac{U}{d_p} \right)^{1/2} D_{AB}^{2/3} \left( \frac{\rho}{\mu} \right)^{1/6}$

→  $\mu_g \propto \frac{T^{3/2}}{T+\alpha}$

Table 10-2 p 558 →  $D_{\text{gas}} \propto \frac{T^{1.75}}{P}$      $D_{\text{eff}} \propto T^{1/2}$

$k = A \exp(-E/RT)$

$$\frac{dF_A}{dV} = \sum k C_{A_b} \rho_c (1-\phi) S_a$$

Now  $F_A = \phi C_{A_b}$

Assume  $\phi = \text{constant}$

$$\phi \frac{dC_{A_b}}{dV} = \sum k \rho_c (1-\phi) S_a C_{A_b}$$

$$\sum = \frac{1}{1 + \eta \frac{k(C_{A_i} + A_p)}{k_c A_p}}$$

$$\eta = \frac{3(\phi \coth \phi - 1)}{\phi^2}$$

If T and P are constant then → = constant!!

$$\phi = R \sqrt{\frac{k S_a \rho}{D_e}}$$

11-12

$$k_c \propto \frac{(u_p)^{1/2}}{d_p^{1/2}} \frac{D_{AB}^{2/3}}{\mu^{1/6}}$$

$$k_c \propto \left[ \frac{G^{1/2}}{d_p^{1/2}} \right] \left[ \frac{D_{AB}^{2/3}}{P} \right] \left[ \frac{T^{1.75}}{T^{3/2}} \right]^{2/3} \left( \frac{T+\alpha}{T} \right)^{1/6}$$

$$k_c \propto \frac{G^{1/2}}{d_p^{1/2}} \left( \frac{1}{P^{2/3}} \right) \left[ \frac{(T+\alpha)^{1/6}}{T^{1/4}} \right]$$

↑  
minor T dependence

$\frac{3}{2} \frac{1}{23}$

external mass transfer limited:

$$r_A = -k_c a_c C_A$$

↑  
 $\left( \frac{mol}{kg \cdot s} \right)$

$\frac{m^3}{m^2 \cdot s}$      $\frac{m^2}{kg}$      $\frac{mol}{m^3}$

$$a_c = \left( \frac{\pi d_p^2}{\frac{1}{6} \pi d_p^3} \right) [e_c]$$

$$= \left[ \frac{6}{d_p} \right] \frac{(1-\epsilon)}{\rho_B}$$

$$\frac{m^2}{m^3} \frac{m^3}{m^3_{Bed}} \frac{m^3_{Bed}}{kg}$$

↑  
weight of bed } bulk  
per m<sup>3</sup> of bed } density

$$r_A \propto \left( \frac{G}{d_p} \right)^{1/2} \frac{1}{d_p} C_A = \frac{G^{1/2}}{d_p^{3/2}} \frac{1}{P^{2/3}} \left( \frac{1}{T^{1/2}} \right) C_A$$

Fogler makes comment about T page 582 <sup>below</sup> equation 10-85

linear temperature dependence? - not if G is constant

From equation 11-35

internal diffusion:

$$r_A = \frac{3}{R} \frac{r_A}{C_A=C_{AS}} = \frac{3}{R} \sqrt{\frac{D_e k_1}{\rho_s}} C_{AS}$$

Reaction at internal:  
and external surface

independent of  $d_p$  or  $G$

$$r = k C_{AS} (A_i + A_p)$$

↙ external surface