

# Packed Bed Reactors

1) majority of commercial gas-phase catalytic processes

2) Exceptions - fluidized bed

- continuously regenerate catalyst
  - improve heat removal
- cracking  
 syn { acrylonitrile  
       ethylene dichloride  
 naphthalene oxidation  
       to phthalic  
       anhydride

3) Fixed bed reactors

- simple technology (adiabatic reactors)

- exothermic rxns } need multibed reactors  
 - endothermic rxns }

a) multitubular FBR



2.5cm, ID  
20,000 tubes



b) multibed reactor



## Scale-up

- 1) empirical (semi-) lab - pilot ..... Commercial
- 2) fundamental

need modeling

problems: complex hydrodynamics  
 lack of detailed & accurate kinetics

procedure - find kinetics without diffusional limitations  
 catalyst deactivation



# PFR

① Show transparency

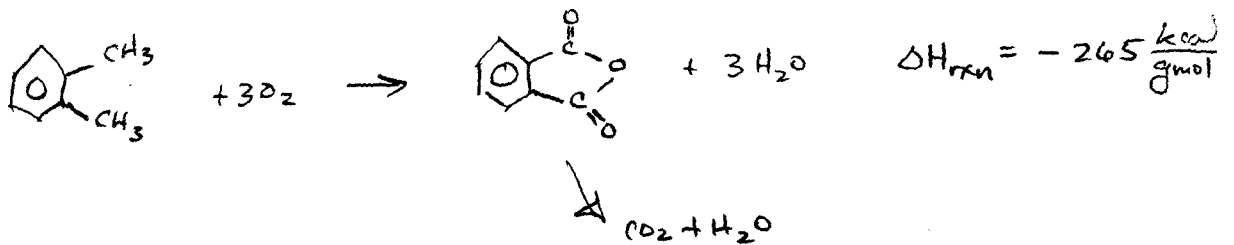
Chemical Engineering  
Science (1985)

Phthalic Anhydride manufacture → exothermic

Vertical shell & tube heat exchanger

Catalyst in tubes (solid sphere, rings etc.)

molten salt in shell side { Na-K & nitrates



45,000 metric ton/yr → ~20,000 tubes with ID ~ 2.5cm  
tube length 3.65m

Previous work — keep mixture below flammable limits

current reactor — working above the flammability limits }

V<sub>2</sub>O<sub>5</sub> catalyst with TiO<sub>2</sub> + promoters } 10 wt%

non-porous carborundum }

$$d_p = 6 \text{ mm}$$

$$G/ZG = \frac{d_p}{ID_{\text{tube}}} = 0.23$$

$$\rho_c \approx 1,780 \frac{\text{kg}_c}{\text{m}^3_{\text{cat}}}$$

assume  $\phi = 0.45$

$$\rho_{\text{bulk}} = \rho_c (1 - \phi)$$

$$\frac{\text{kg}_{\text{cat}}}{\text{m}^3_{\text{cat}}} \quad \frac{\text{m}^3_{\text{cat}}}{\text{m}^3_{\text{bed}}}$$

$$\rho_{\text{bulk}} = 980 \frac{\text{kg}}{\text{m}^3_{\text{bed}}} \quad \left( \frac{\text{m}^3_{\text{cat}}}{\text{m}^3_{\text{bed}}} \right)$$

Perry's Figure 5-70

Cylindrical Rings —  $\phi = 0.47$

Spheres —  $\phi \approx 0.45$

Raschig Ring —  $\phi = 0.6$

# PFR & Pressure Drop

ΔP-1

"Fixed" Bed  
"Packed" Beds } need to consider pressure drop in calculations of X.

(Read Section 4.3 to see the effect of gas expansion on conversion).

Rase H.F., Fixed-Bed Reactor Design and Diagnostics, Gas Phase Reactions, Butterworths 1990

High Range: ~~once through~~ Reactors, I.  $0.01 P_T \leq \Delta P \leq 0.10 P_T$   
nonadiabatic reactors with high velocities provide heat transfer efficiency for controlling hot spots.  
low Range: Recycle or large flows of gas or multiple adiabatic beds

Flow through an empty tube

laminar flow (Hagen-Poiseuille):

$$v = \frac{R^2}{8\mu} \left( \frac{-\Delta P}{L} \right)$$

$$\Delta P = -\frac{8v\mu L}{R^2}$$

turbulent flow:

$$\frac{\Delta P}{L} = \underbrace{\left( \frac{2Pf}{D} \right)}_{C_v} v_{\infty}^2$$

where  $v_{\infty} = \frac{Q}{A}$   
superficial velocity

Flow through a packed Bed & Ergun Equation



3 eqns

$$\frac{dP}{dL} = -\frac{f_p \rho}{D_p} \left[ \frac{(1-\phi)}{\phi^3} \right] v_{\infty}^2 \quad \text{and} \quad f_p = \frac{150}{Re_p} + 1.75$$

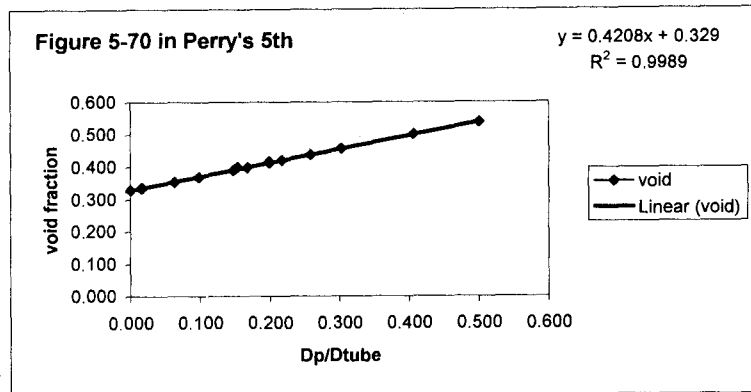
$$Re_p = \frac{\rho D_p}{\mu} \left[ \frac{v_{\infty}}{1-\phi} \right]$$

where  $\phi$  - void fraction =  $\frac{\text{void volume (gas)}}{\text{Reactor Volume}}$

Void Fraction Curve for spheres  
 Taken from Perry's 5th Edition  
 Figure 5-70

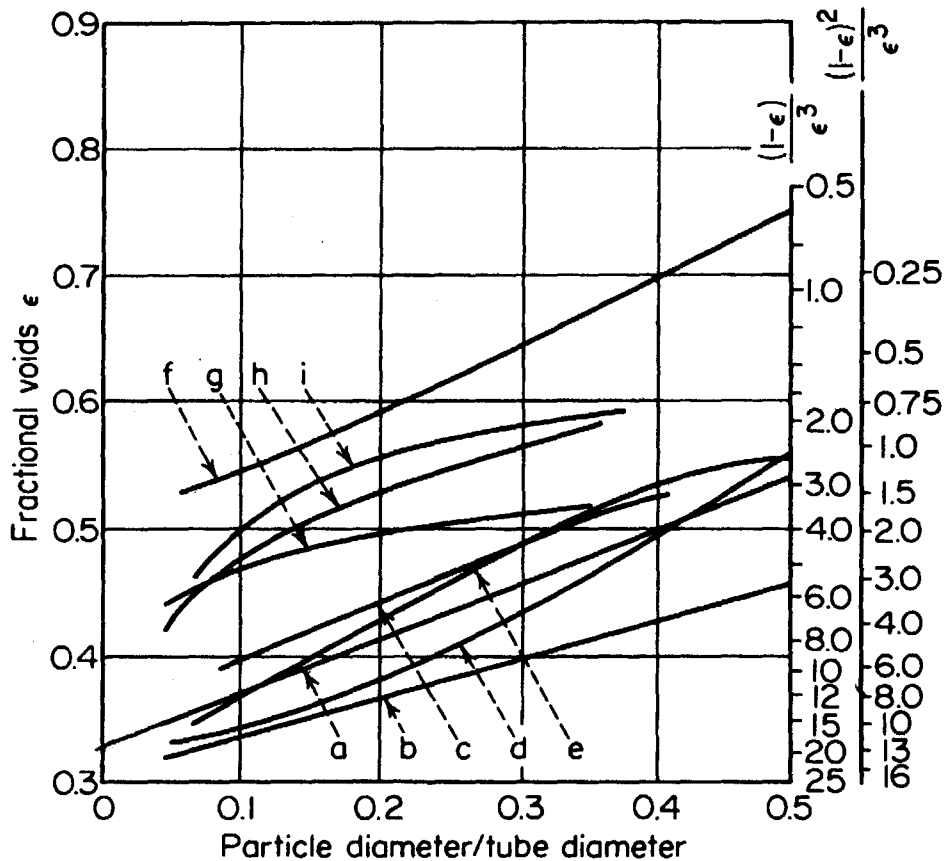
Scale x 0.003671  
 Scale y 0.003636

Dp/Dtube	void	Dp/Dtube	void
x	y	x	y
0	7.86	0.000	0.329
4.21	9.56	0.015	0.335
4.71	9.86	0.017	0.336
17.21	14.86	0.063	0.354
26.71	18.96	0.098	0.369
40.21	24.86	0.148	0.390
41.71	27.36	0.153	0.399
45.71	27.16	0.168	0.399
54.21	31.86	0.199	0.416
54.21	30.86	0.199	0.412
<b>59.11</b>	<b>32.96</b>	<b>0.217</b>	<b>0.420</b>
70.31	37.86	0.258	0.438
82.51	42.86	0.303	0.456
110.71	55.21	0.406	0.501
136.2	65.36	0.500	0.538



linear fit to spherical data  
 $y = 0.4208x + 0.329$

**BEDS OF SOLIDS 5-53**



**Fig. 5-70.** Voidage in packed beds. Spherical: *a*, smooth, uniform; *b*, smooth, mixed; *c*, clay. Cylindrical: *d*, smooth, uniform; *e*, alundum, uniform; *f*, clay Raschig rings. Granules: *g*, fused magnetite (synthetic ammonia catalyst); *h*, fused alundum; *i*, Aloxite. (Leva, "Fluidization," p. 54, McGraw-Hill, New York, 1959.)

Fogler Ergun

$$\frac{dP}{dz} = - \frac{G}{\rho D_p} \left( \frac{1-\phi}{\phi^3} \right) \left[ \frac{150(1-\phi)\mu}{D_p} + 1.75G \right] \quad 4-22$$

$\frac{\text{kg}}{\text{m}^2 \text{s}}$

BSL 6.4-13

$$\left( -\frac{dP}{dL} \right) \frac{\rho}{G_0^2} D_p \frac{\varepsilon^3}{1-\varepsilon} = 150 \left( \frac{1-\varepsilon}{D_p G_0 / \mu} \right) + 1.75 \quad 6.4-13$$

$$\frac{dP}{dL} = \frac{G_0^2}{\rho D_p} \left( \frac{1-\varepsilon}{\varepsilon^3} \right) \left[ \frac{150(1-\varepsilon)}{D_p G_0 / \mu} + 1.75 \right]$$

$$G_0 = \frac{\text{kg}}{\text{m}^2 \text{s}}$$

$$= \frac{G_0}{\rho D_p} \left( \frac{1-\varepsilon}{\varepsilon^3} \right) \left[ \frac{150(1-\varepsilon)}{D_p / \mu} + 1.75 G_0 \right]$$

Geankoplis eqn 31-20

$$\frac{\Delta P}{\Delta L} = \frac{150 \mu v' (1-\varepsilon)^2}{D_p^2 \varepsilon^3} + \frac{1.75 (v')^2 \rho}{D_p} \frac{1-\varepsilon}{\varepsilon^3}$$

$$= \frac{v'}{D_p} \frac{1-\varepsilon}{\varepsilon^3} \left[ \frac{150 \mu (1-\varepsilon)}{D_p} + 1.75 v' \rho \right]$$

$$v' = \varepsilon v$$

↳ superficial

$$v' A_c \rho = G A_c$$

$$v' = G / \rho$$

$$\frac{\Delta P}{\Delta L} = \frac{G}{\rho D_p} \frac{1-\varepsilon}{\varepsilon^3} \left[ \frac{150 \mu (1-\varepsilon)}{D_p} + 1.75 \frac{G \rho}{\rho} \right]$$

### Froment & Bischoff

$$-\frac{dP}{dz} = f \frac{\rho u_s^2}{d_p} \quad 11.5.1-3$$

$$f = \frac{1-\epsilon}{\epsilon^3} \left[ a + \frac{b(1-\epsilon)}{Re_p} \right] \quad 11.5.1-13$$

$$-\frac{dP}{dz} = \frac{1-\epsilon}{\epsilon^3} \frac{\rho u_s^2}{d_p} \left[ a + \frac{b(1-\epsilon)}{Re_p} \right]$$

$$\rho u_s = G$$

$$= \frac{1-\epsilon}{\epsilon^3} \frac{G^2}{\rho d_p} \left[ a + \frac{b(1-\epsilon)}{Re_p} \right]$$

$$= \frac{1-\epsilon}{\epsilon^3} \frac{G}{\rho d_p} \left[ Ga + \frac{Gb(1-\epsilon)}{Re_p} \right]$$

$$Re_p = \frac{d_p \rho u_s}{\mu}$$

$$= \frac{1-\epsilon}{\epsilon^3} \frac{G}{\rho d_p} \left[ Ga + \frac{Gb(1-\epsilon)\mu}{d_p \rho u_s} \right]$$

where  $a=1.75$      $b=150$

Tallmadge (1970)     $b = 4.2 Re^{5/6}$

$$\frac{1-\epsilon}{\epsilon^3} \frac{G}{\rho d_p} \left[ Ga + \frac{G(1-\epsilon) 4.2 Re^{5/6}}{Re_p} \right]$$

$$\left[ Ga + \frac{G(1-\epsilon) 4.2}{Re_p^{1/6}} \right]$$

Fronmunt ~~3~~<sup>3</sup>/  
Bischoff

Tallmadge (1970) cont.

$$= \frac{1-\varepsilon}{\varepsilon^3} \frac{G}{eD_p} \left[ Ga + \frac{G(1-\varepsilon)4.2 \mu^{1/4}}{(D_p \rho u_s)^{1/4}} \right]$$

Good correlation of  $\phi$  is given in Perry's

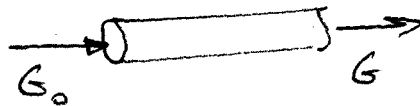
AP-2

Plug in  $f_p \neq Re_p$

$$\frac{dP}{dz} = - \frac{\rho v_{\infty}^2}{D_p} \frac{(1-\phi)}{\phi^3} \left[ \frac{150(1-\phi)M}{D_p \rho v_{\infty}} + 1.75 \right]$$

this form is not very easy to use

So convert to a form using a mass balance



$$G [=] \frac{\text{kg}}{\text{m}^2 \text{s}}$$

↑ total cross sectional area

$$G = \rho v_{\infty} A = \rho \left[ \frac{\phi}{A} \right] A$$

↑  $v_{\infty}$

3rd ed  
eqn 4-22

eqn  
4-23

$$\frac{dP}{dz} = - \frac{1}{\rho} \underbrace{\left[ \frac{G}{D_p} \frac{(1-\phi)}{\phi^3} \left[ \frac{150(1-\phi)M}{D_p} + 1.75G \right] \right]}_{\text{Constant with } z} \left( \frac{\rho_0}{\rho} \right)$$

↑ Variable density - T or # moles

↑  $\rho_0$

let  $\frac{dP}{dz} = - \frac{1}{\rho} [\rho_0 B_0]$

and  $\rho_0 =$  equation ~~4-21~~  
4-25 3rd ed.



Variable Density: (continued)

ΔP-4

$$\frac{P_0}{P} = \frac{P_0}{P} \frac{T}{T_0} \frac{F_T}{F_{T_0}} = \frac{P_0}{P} \frac{T}{T_0} \left[ 1 + \delta \frac{F_{A_0}}{F_{T_0}} X_A \right]$$

$\underbrace{y_{A_0}}_{\text{mole fraction}}$

Variable density (i) if  $y_{A_0} \rightarrow 0$  dilute gas }  $\neq T = T_0$   
or  $\delta = 0$  or small

$$\frac{P_0}{P} = \frac{P_0}{P} \frac{T}{T_0}$$

$$\frac{dP}{dz} = -\beta_0 \left[ \frac{P_0}{P} \frac{T}{T_0} (1 + \delta) \right]$$

$$\int P dP = -\beta_0 P_0 \frac{T}{T_0} \int dz$$

$$\frac{P^2}{2} \Big|_{P_0}^P = -\left[ \beta_0 P_0 \frac{T}{T_0} \right] (z) \quad \frac{P^2}{2} - \frac{P_0^2}{2} = -\beta_0 P_0 \frac{T}{T_0} z$$

$$P = \left[ P_0^2 - 2 \beta_0 P_0 \frac{T}{T_0} z \right]^{1/2}$$

Variable density (ii)  $T \neq T_0$  and  $y_{A_0} \neq 0$   $\delta_A \neq 0$

Numerical integration

Now we need a relation for  $\frac{P_0}{P}$  ΔP-3

$$\frac{dP}{dz} = -\beta_0 \left[ \frac{P_0}{P} \right] \leftarrow \text{I will give this equation on the exam.}$$

I assume  $\rho \approx \bar{\rho}$

$$\frac{dP}{dz} = -\underbrace{\frac{P_0}{P} \beta_0}_{\text{constant}} \quad \text{integrate} \quad P = P_0 - \left[ \frac{P_0 \beta_0}{P} \right] z$$

## II Variable Density

find a relation for  $\frac{P_0}{P}$

$$\text{mass balance: } \rho_0 v_{\infty 0} = \rho v_{\infty}$$

$$G_0 = G$$

$$\frac{\rho_0}{\rho} = \frac{v_{\infty}}{v_{\infty 0}} = \frac{\dot{Q}/A}{\dot{Q}_0/A}$$

$$\frac{P \dot{Q}}{P_0 \dot{Q}_0} = \frac{F_T R T}{F_{T_0} R T_0}$$

$$\frac{\rho_0}{\rho} = \frac{P_0}{P} \frac{F_T}{F_{T_0}} \frac{T}{T_0}$$

$$\frac{\dot{Q}}{\dot{Q}_0} = \frac{P_0}{P} \frac{F_T}{F_{T_0}} \frac{T}{T_0}$$