Heat Transfer Tutorial:

1. Following the steps given in the documentation of the Comsol Heat Conduction in a Slab tutorial (See next page). Submit a plot similar to Figure 5-2 in the documentation.

2. After completing the tutorial modify your solution to solve the Cutlip & Shacham Problem 6.12 on page 246 using Comsol Multiphysics. Part b can be done by increasing the number of mesh elements.

This simple example covers the heating of a finite slab and how the temperature varies with time. You can first set up the problem in FEMLAB and then compare it to the analytical solution as given in [1].

MODEL DEFINITION

The model domain is defined between $x = -b$ and $x = b$. The initial temperature is a constant temperature, T_0, over the whole domain, see the figure below. At time $t = 0$, the temperature is decreased to T_1 at both boundaries.

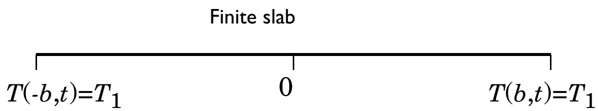


Figure 5-1: Depiction of the modeling domain.

In order to compare the modeling results to the literature [1], we can introduce new dimensionless variable according to the following definitions:

$$\Theta = \frac{T_1 - T}{T_1 - T_0} \qquad \eta = \frac{x}{b} \qquad \tau = \frac{\alpha t}{b^2} \tag{5-13}$$

The model equation then becomes:

$$\frac{\partial \Theta}{\partial \tau} = \frac{\partial^2 \Theta}{\partial \eta^2} \tag{5-14}$$

with the corresponding boundary and initial conditions:

$$\begin{array}{ll} \tau = 0 & \Theta = 1 \\ \eta = \pm 1 & \Theta = 0 \end{array} \tag{5-15}$$

RESULTS

The figure below shows the temperature as a function of position at the time steps corresponding to 0.01, 0.04, 0.1, 0.2, 0.4, and 0.6. In this plot, the center of the slab is situated at x=0, while the surfaces are situated at x=-1 and x=1. The temperature profiles obtained in this graph are identical to the analytical solution given in Carslaw and Jaeger [1].

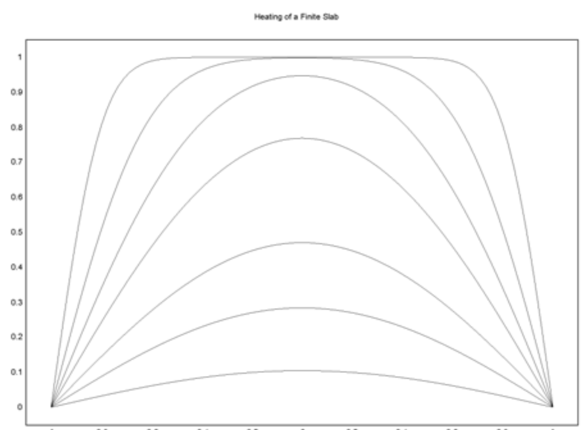


Figure 5-2: Results from the simulation at different time-steps. The top graph is at the smallest time-step, while the lowest graph is at the last time-step.

REFERENCES

[1] H. S. Carslaw and J. C. Jaeger, *Basic Principles of Membrane Technology*, Oxford University Press, p. 101, 1959.

Model Library path: `Chemical_Engineering_Module/Energy_Transport/slab`

Modeling using the Graphical User Interface

1 Start FEMLAB.
2 Select `1D` in the **Space dimension** list.
3 Select the **Energy balance, Conduction, Transient analysis** mode from the **Chemical Engineering Module**.
4 Click **OK**.

Geometry Modeling

1 Select **Line** from the **Specify objects** menu item in the **Draw** menu.
2 Type `-1 1` in the **x** edit field for **Coordinates**.
3 Type `slab` in the **Name** edit field.
4 Click **OK**.
5 Click the **Zoom Extents** button.

Boundary Conditions

1 Select **Boundary Settings** from the **Physics** menu.
2 Select both boundaries and choose **Zero temperature** in the **Boundary condition** list.
3 Click **OK**.

Subdomain Settings

1 Select **Subdomain Settings** from the **Physics** menu.
2 Select subdomain 1 and enter the coefficients according to the following table.

SUBDOMAIN	1
k	1
ρ	1
C_p	1
Q	0

SUBDOMAIN	1
h_{trans}	0
T_{ext}	0
C_{trans}	0
$T_{ambrans}$	0

3 Click the **Init** tab and specify the initial condition according to following table:

SUBDOMAIN	1
$T(t_0)$	1

4 Click **OK**.

Mesh Generation

1 Click the **Mesh** button to initialize the mesh.

2 Click the **Refine Mesh** button once.

Computing the Solution

1 Select **Solver Parameters** in the **Solve** menu.

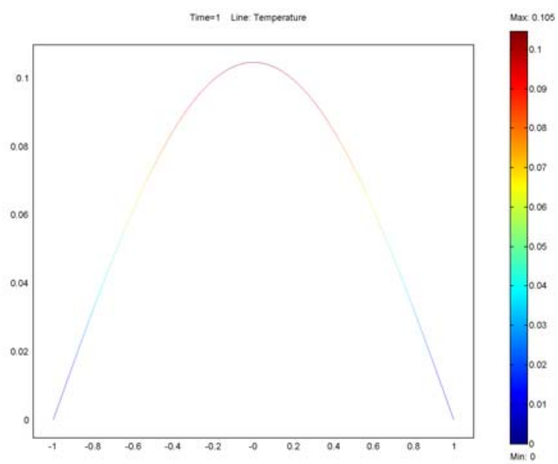
2 Type `0:0.01:1` in the **Times** edit field in the **General** page.

3 Click **OK**.

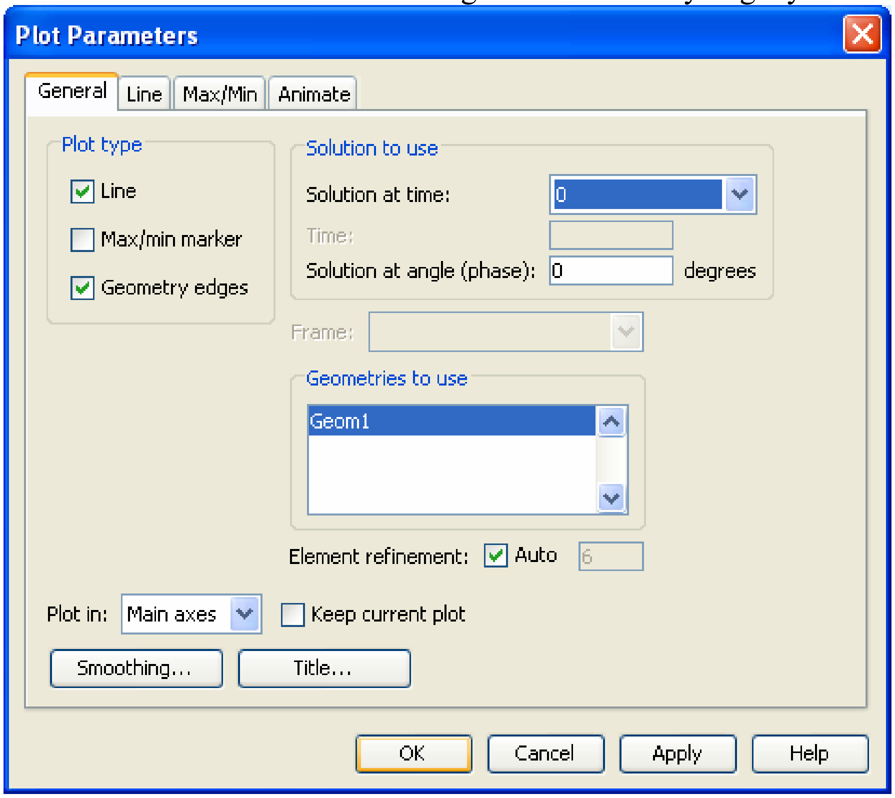
4 Click the **Solve** button in the Main toolbar.

Postprocessing and Visualization

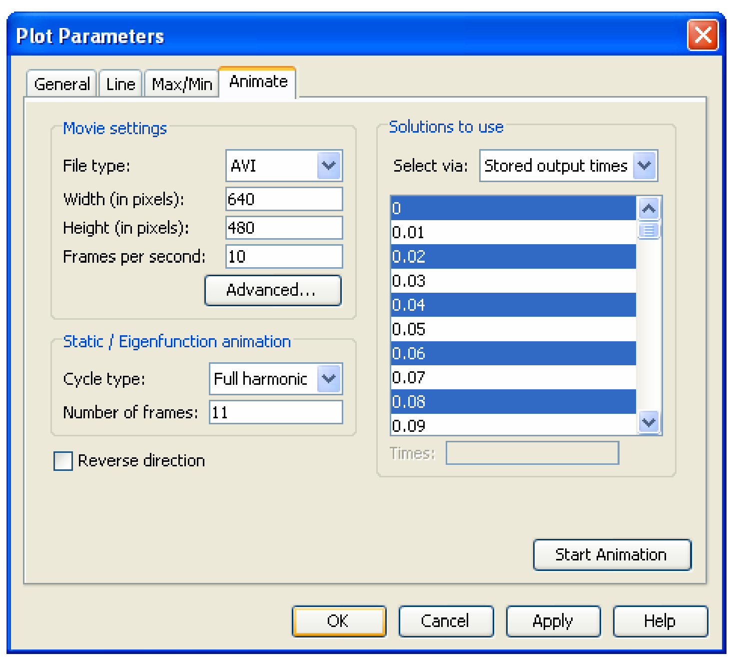
The default plot settings gives the temperature distribution at t = 1, see below.



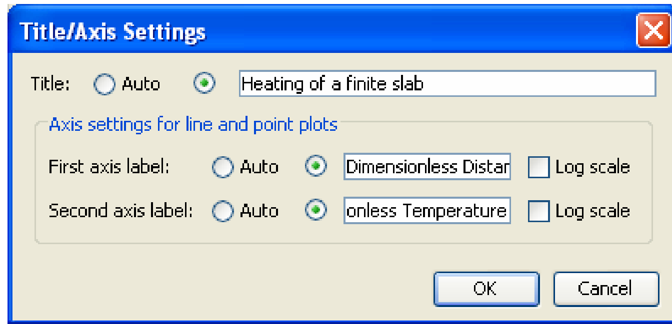
Check your solution initial conditions. Select Post processing, Plot parameters and chose the solution at time 0 as shown in the figure below. Did you get your initial conditions?



1. Now make a move. Choose the **Animate** tab
2. Select even times from 0 through 0.2 and then chose 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0. These can be selected by using the shift key for groups and pressing the Ctrl key and holding to select individual times.
3. Select **Start Animation** to play the movie.

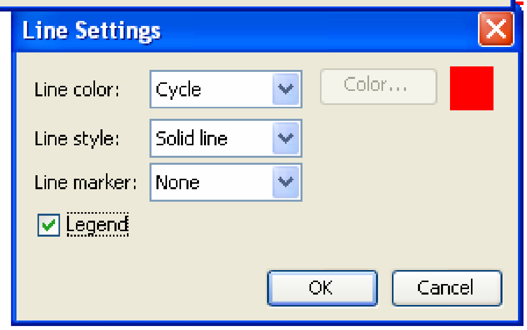


Next make a figure showing the temperature profile at the following times.

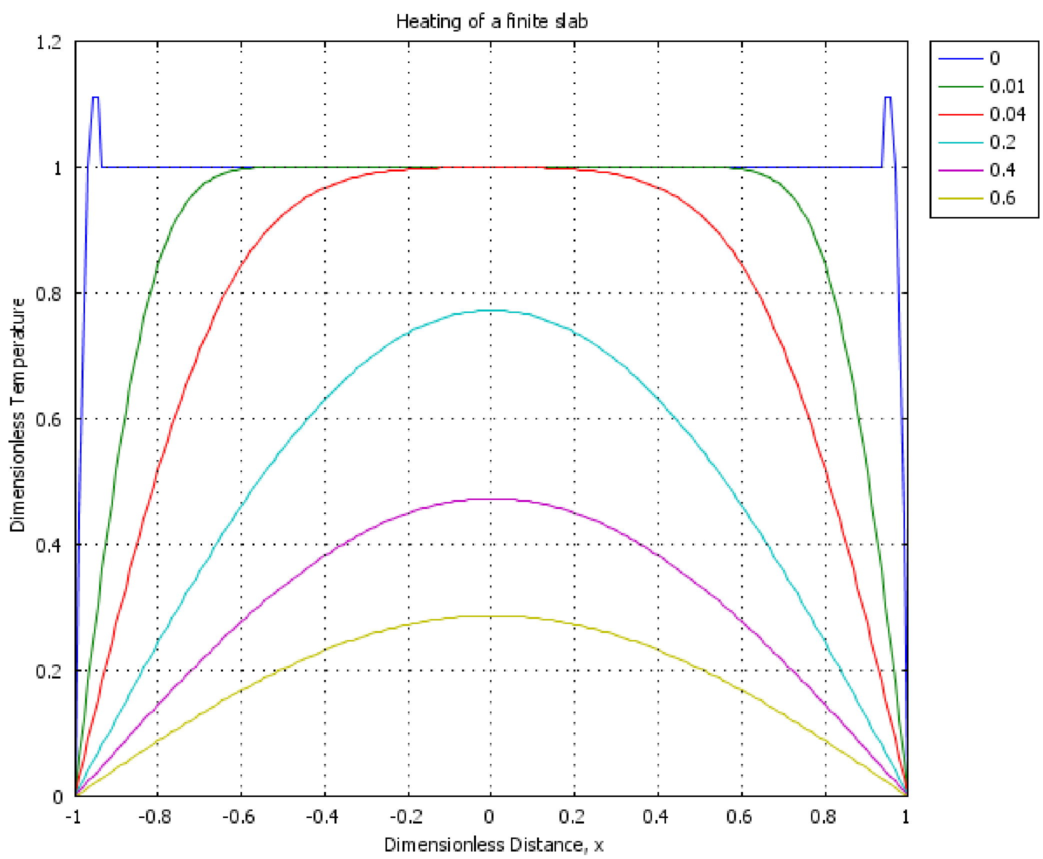


The FEMLAB solution can be compared to an analytical solution by plotting the temperature at a given set of output times. This is done using the following procedure:

1. Select **Domain Plot Parameters** from the **Postprocessing** menu.
2. Select the **General** tab and select the **Line/extrusion plot** check box under **Plot type**.
3. Select the following time-steps, 0.01, 0.04, 0.2, 0.4, 0.6 from the **Solutions to use** list by holding down the Ctrl key.
4. Add a title using the **Title/Axis** button. "Heating of a finite slab". Then add x and y-axis labels. Then click OK.
5. Show a legend in the Line/Extrusion menu, by choosing the line Settings button and select Show legend. Then click OK
6. Click OK.

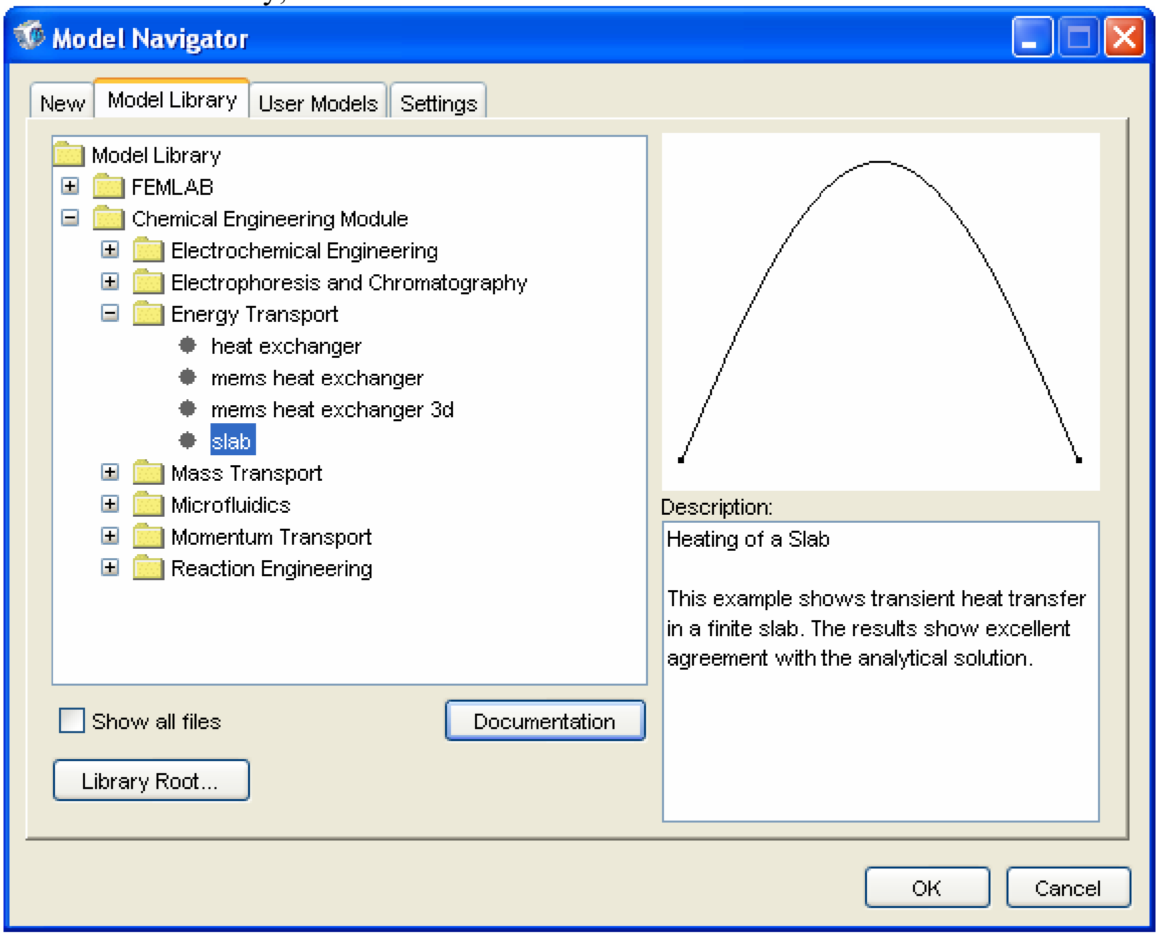


The generated plot looks as follows:



Paste this plot into a word document to be emailed to the professor at the end of the laboratory.

The documentation on the slab tutorial can also be found by selecting the documentation button under the user library, Slab model.



Cutlip and Shacham Problem 6.12 Unsteady-State Conduction within a Semi-Infinite Slab

The problem that you are solving in Cutlip is the conduction of heat through up to 80 ft of soil. The equation that you will use is the unsteady-state conduction equation

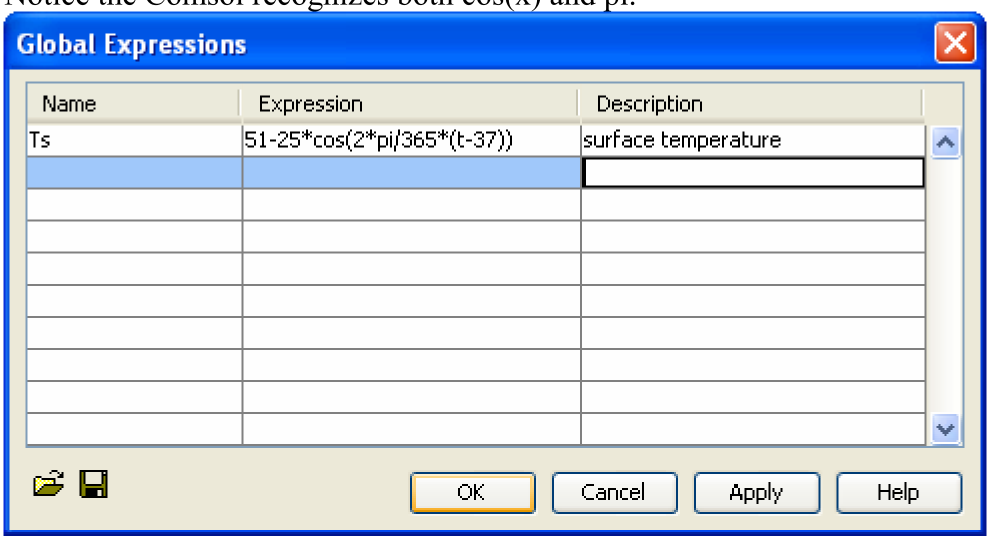
$$\frac{\partial T}{\partial t} = \left(0.9\, ft^2 / day\right)\frac{\partial^2 T}{\partial x^2} \qquad (1)$$

. The soil surface has a boundary condition for Chicago of

$$T\big|_{surface} = 51^{\circ} F - 25^{\circ} F \cos\left[\frac{2\pi}{365 days}\left(t - 37 days\right)\right] \qquad (2)$$

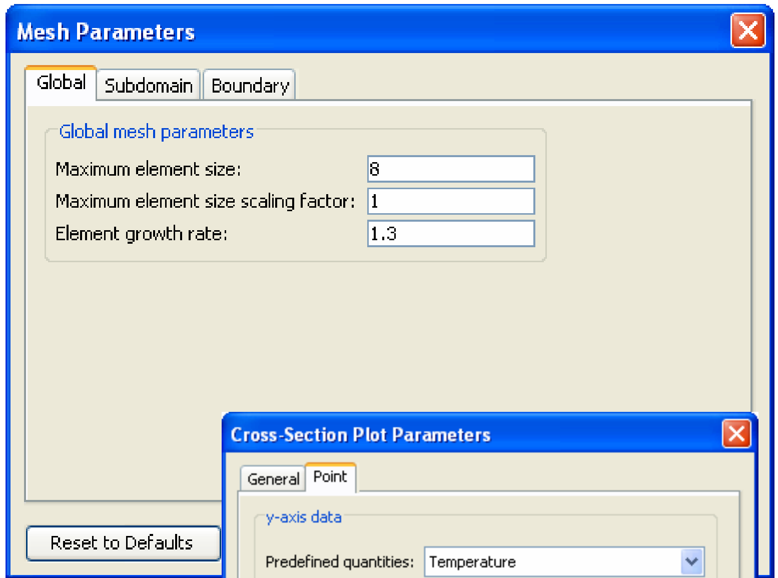
. In other words this is the surface temperature of the soil as a function of each day throughout the year.

1. Select the Draw mode (Triangle and pencil button)
2. Change the geometry coordinate system by moving the slab so that one end is at zero and the other is at 2.
3. Chose **Draw**, **Scale** and scale by a factor of 40 to match the Cutlip and Shacham problem. In this problem the soil is modeled for a distance of 80 ft from the surface.
4. Change the boundary conditions using the **Physics**, **Boundary Settings** menu.
   a. The surface at x=0 has a sinusoidal temperature condition for Chicago Illinois. Notice the Comsol recognizes both cos(x) and pi.

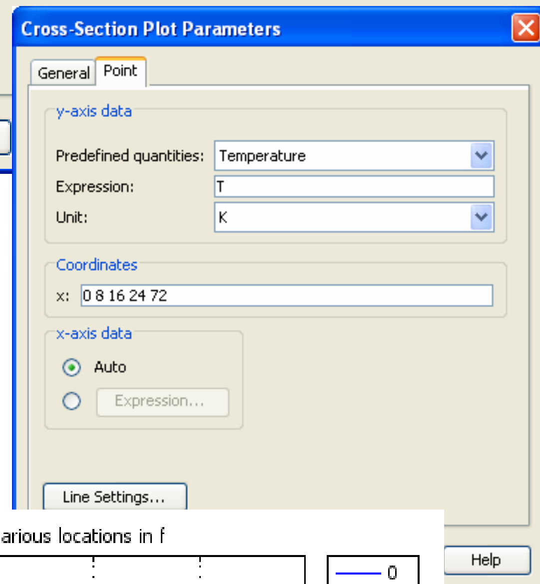


   b. What should the other surface boundary condition be set at? Try a few and see what happens. Should it be constant temperature that you specify or insulated?
5. Change your constants in the **Physics**, **Subdomain Settings** options.
6. Set your initial condition to a reasonable value. Should it be the mean temperature of the earth for that region? Should it be freezing? Does it matter?
7. Now set your time steps in the Solver Parameters. Change from 0:0.01:1 (start at zero, increment by time steps of 0.01, and end at 1) to values of your choosing. In Cutlip and Shacham an end value of at least 4 years is suggested.

8. Note that the initialized mesh will consist of 15 nodes. To change this to that given in the Cutlip & Shacham problem set the maximum size to 8. (The default is set to 1/15 of the maximum size). Go to **Mesh**, **Mesh Parameters** put a value of 8 in the maximum element size.



9. To produce the plot required in part b you will need to make a cross section plot for the following times as shown below:



10. Remember to label you x and y-axis, title and show a legend of the distances being plotted.
11. To produce the second plot you should keep the current plot and plot in a new plot.

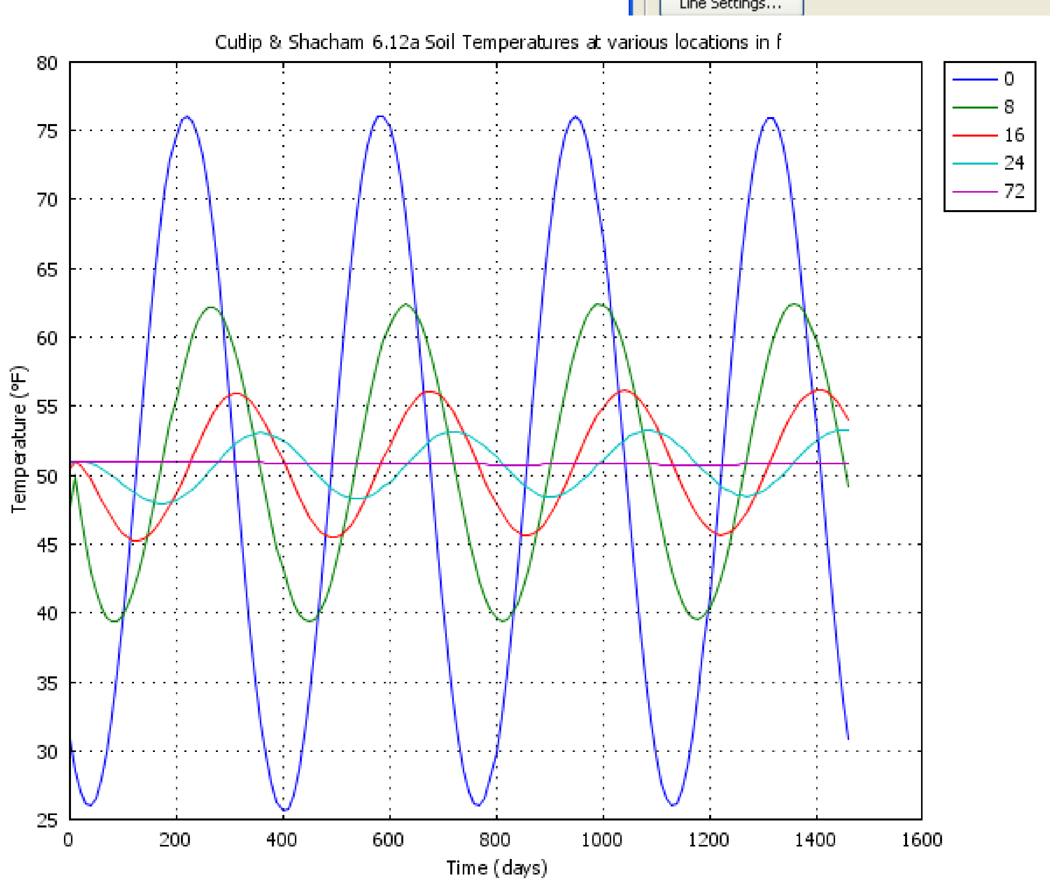


Figure 1: C&S 6.12 a Soil Temperatures for 4 years

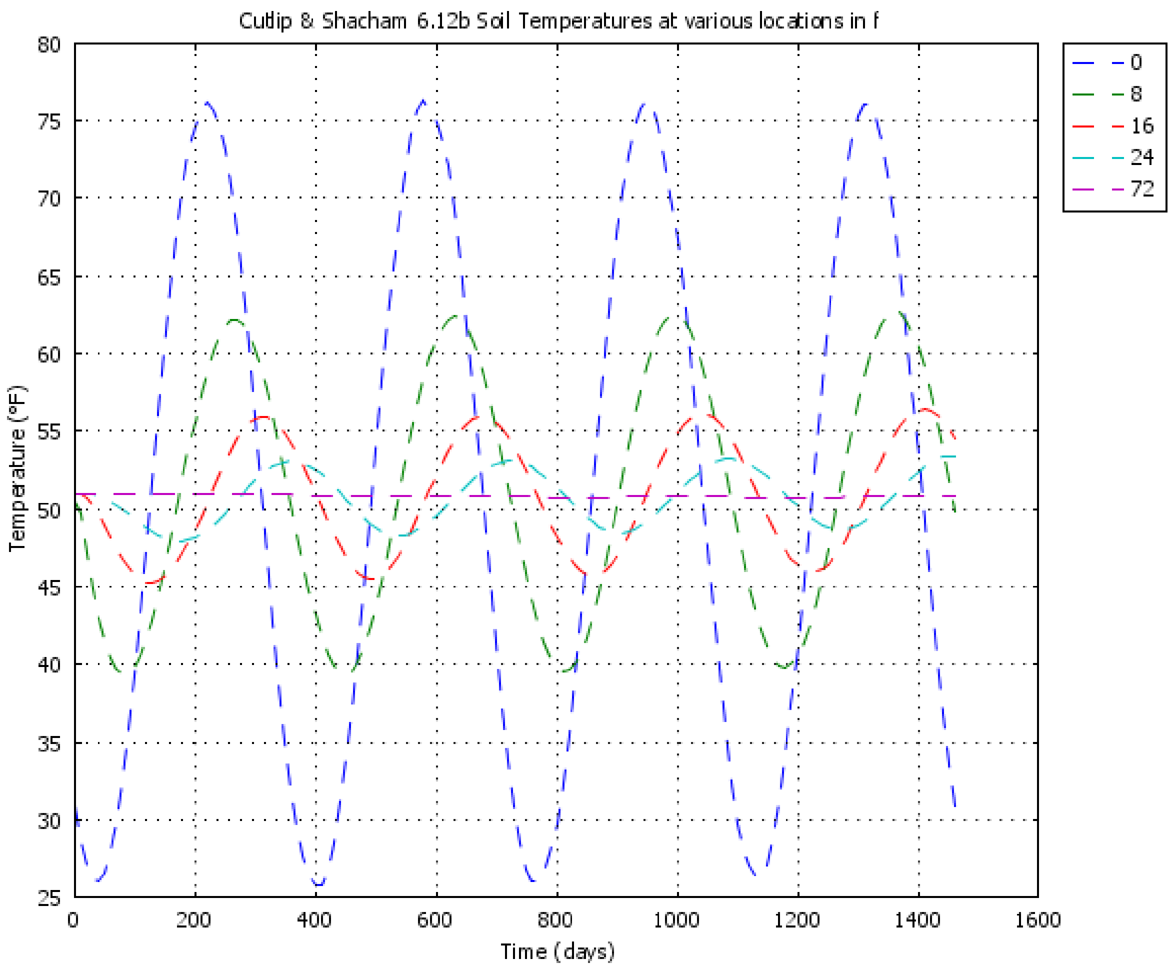


Figure 2: C&S 6.12b Soil Temperatures - 20 elements

Comparison:

To compare the two solutions using a graphical solution, then plot the second solution on top of the first figure. Then using the zoom window feature you can look at the differences between solutions. Are they identical?

After completing the C&S 6.12 explore the effect of increasing the distance into the earth. Double the value from 80 to 160 ft.

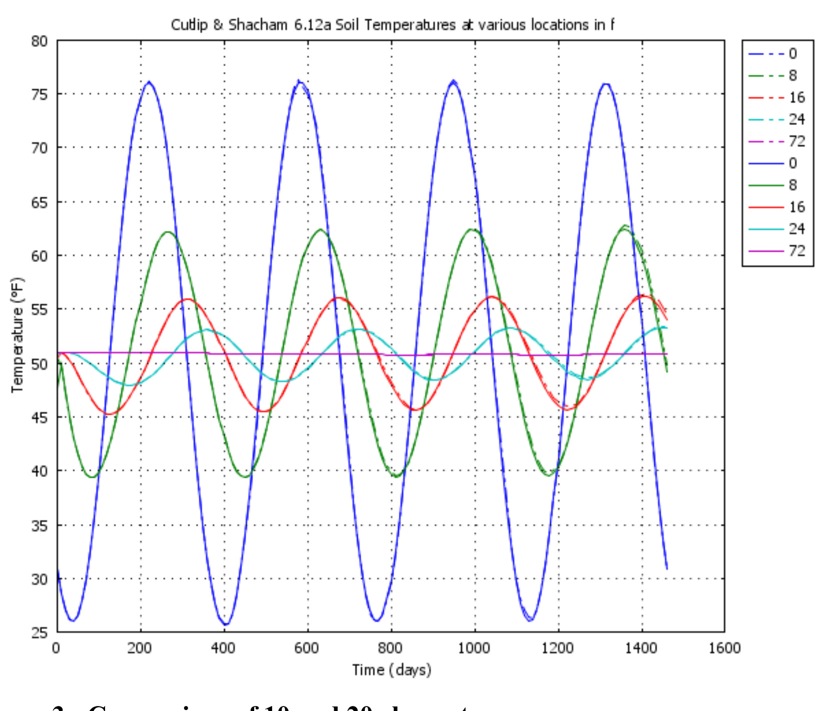


Figure 3: Comparison of 10 and 20 elements

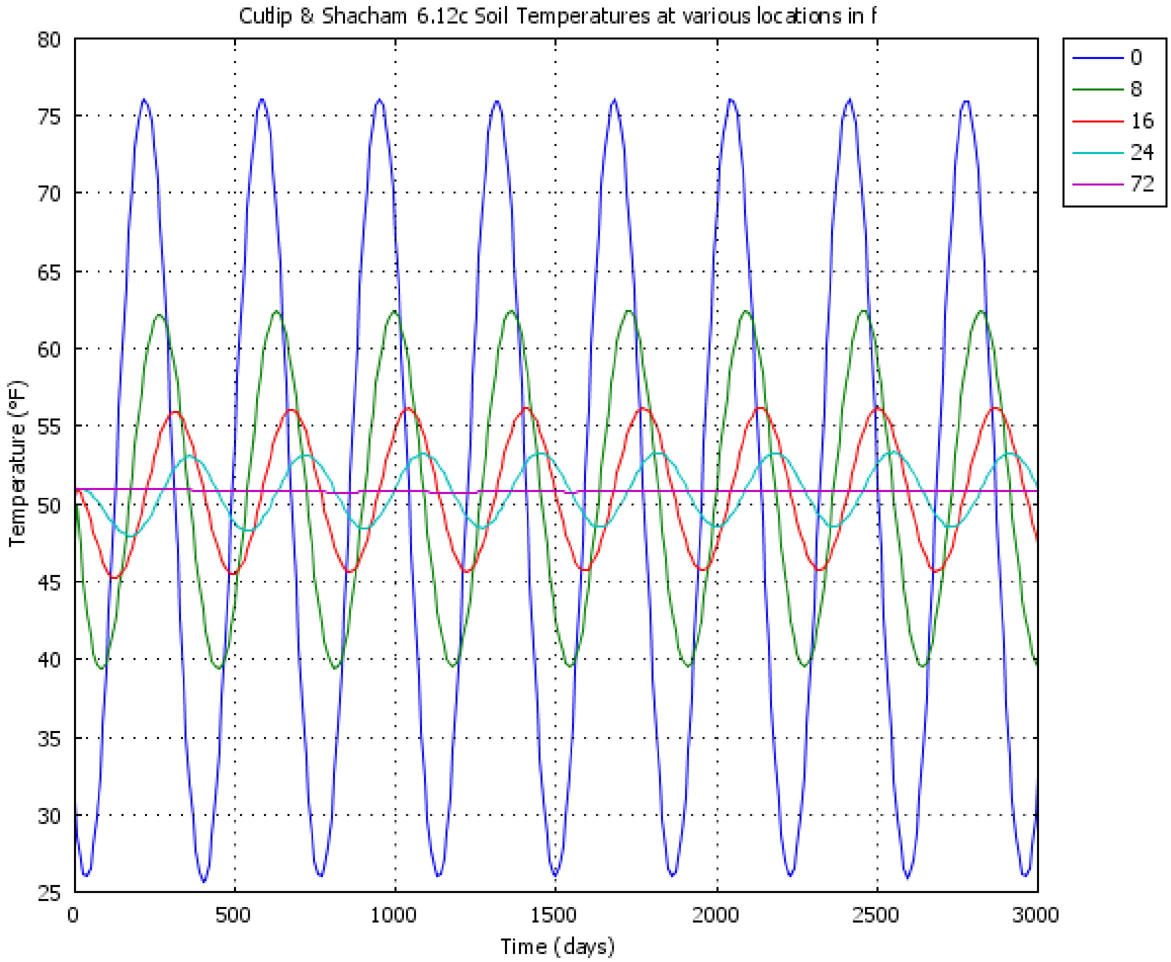


Figure 4: C&S 6.12c Examination of Phase Shift

To be emailed to the instructor:

1. Plot from finite slab
2. Plots requested in Cutlip and Shacham problem 6.12
    a. 10 elements
    b. 20 elements
    c. Comparison of 10 and 20 elements
    d. Figure that shows a phase change in the temperatures between the surface and interior values.
    e. Figure showing the effect of extending the simulation from 80 ft to 160 ft.
3. Comment on the depth of soil needed to model this cyclic suface temperature phenomena.