# Non-Newtonian Flows – Modified from the Comsol ChE Library module.

Modified by Robert P. Hesketh, Chemical Engineering, Rowan University

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http://ciks.cbt.nist.gov/~garbocz/SP946/node8.htm

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# 6. Expressions for Describing Steady Shear Non-Newtonian Flow

The expressions shown here are used to characterize the <u>non-Newtonian</u> behavior of fluids under equilibrium, *steady shear flow* conditions. Many phenomenological and empirical models have been reported in the literature. Only those having a direct and significant implication for suspensions, gels and pastes have been included here. A brief description of each relationship is given with examples of the types of materials to which they typically are applied. In defining the number of parameters associated with a particular model, the term "parameter" in this case refers to adjustable (arbitrary) constants, and therefore excludes measured quantities. Some of these equations have alternative representations other than the one shown. More detailed descriptions and alternative expressions can be found in the sources listed in the bibliography.

# Bingham

$$\begin{aligned} \sigma &= \sigma_{\rm B} + \eta_{\rm pl} \dot{\gamma} \\ \dot{\gamma} &= 0 \ \text{for} \ \sigma < \sigma_{\rm B} \end{aligned}$$

Where  $\dot{\gamma}$  is the shear rate (e.g. dv/dx). The Bingham relation is a two parameter model used for describing <u>viscoplastic</u> fluids exhibiting <u>a yield response</u>. The ideal Bingham material is an <u>elastic</u> solid at low <u>shear stress</u> values and a <u>Newtonian</u> fluid above a critical value called the Bingham <u>yield stress</u>,  $\mathbf{c}_{\text{B}}$ . The <u>plastic viscosity</u> region exhibits a linear relationship between shear stress and <u>shear rate</u>, with a constant <u>differential viscosity</u> equal to the plastic viscosity,  $\eta_{\text{pl}}$ .

## Carreau-Yasuda (This is used in Comsol)

$$\frac{\eta - \eta_{\infty}}{\eta_0 - \eta_{\infty}} = \left[1 + \left(\lambda \dot{\gamma}\right)^a\right]^{(n-1)/a}$$

A model that describes <u>pseudoplastic</u> flow with asymptotic viscosities at zero  $(\P_0)$  and infinite  $(\P_0)$  <u>shear rates</u>, and with no <u>yield stress</u>. The parameter  $\lambda$  is a constant with units of time, where  $1/\lambda$  is the critical shear rate at which viscosity begins to decrease. The <u>power-law</u> slope is (*n*-1) and the parameter *a* represents the width of the transition region between  $\P_0$  and the power-law region. If  $\P_0$  and  $\P_{\text{becare not known independently from experiment, these quantities may be treated as additional adjustable parameters.$ 

**CARREAU MODEL:** A mathematical expression describing the shear thinning behavior of polymers. It is more realistic than the power-law model because it fits the data very well at both high and low shear rates.



where:  $\eta_0$ ,  $\lambda$ , n are curve fitting parameters and  $\frac{1}{2}$  is the shear rate. Due to the mathematical complexities it is not possible to obtain analytical solutions with this model, but it is excellent for numerical simulations of flow processes.

### Casson

$$\sigma^{1/2} = \sigma_y^{1/2} + \eta_1^{1/2} \dot{\gamma}^{1/2}$$
$$\dot{\gamma} = 0 \text{ for } \sigma < \sigma_y$$

A two parameter model for describing flow behavior in <u>viscoplastic</u> fluids exhibiting a <u>yield</u> <u>response</u>. The parameter  $\sigma_y$  is the <u>yield stress</u> and  $\eta_{pl}$  is the differential high shear (<u>plastic</u>) viscosity. This equation is of the same form as the <u>Bingham</u> relation, such that the exponent is  $\frac{1}{2}$  for a Casson plastic and 1 for a Bingham plastic.

#### Cross

$$\frac{\eta - \eta_{\infty}}{\eta_0 - \eta_{\infty}} = \frac{1}{\left(1 + \lambda \dot{\gamma}^m\right)}$$

Similar in form to the <u>*Carreau-Yasuda*</u> relation, this model describes <u>*pseudoplastic*</u> flow with asymptotic viscosities at zero ( $\eta_0$ ) and infinite ( $\eta_{mc}$ ) <u>shear rates</u>, and no <u>*yield stress*</u>. The parameter  $\lambda$  is a constant with units of time, and *m* is a dimensionless constant with a typical range from 2/3 to 1.

$$\eta = \frac{\eta_0}{1 + \left(\frac{\sigma}{\sigma_2}\right)^{\alpha - 1}}$$

A two parameter model, written in terms of <u>shear stress</u>, used to represent a <u>pseudoplastic</u> material exhibiting a <u>power-law</u> relationship between shear stress and <u>shear rate</u>, with a low shear rate asymptotic viscosity. The parameter  $\sigma_2$  can be roughly identified as the shear stress value at which Thas fallen to half its final asymptotic value.

## Herschel-Bulkley

$$\sigma = \sigma_y + k \dot{\gamma}^n$$

A three parameter model used to describe <u>viscoplastic</u> materials exhibiting a <u>yield response</u> with a <u>power-law</u> relationship between <u>shear stress</u> and <u>shear rate</u> above the <u>yield stress</u>,  $\mathbf{a}_y$ . A plot of  $log (\mathbf{a} - \mathbf{a}_y)$  versus  $log \overset{i}{\mathbf{y}}$  gives a slope *n* that differs from unity. The Herschel-Bulkley relation reduces to the equation for a *Bingham* plastic when n=1.

## **Krieger-Dougherty**

$$\eta_{\rm r} = \left(1 - \frac{\Phi}{\Phi_{\rm m}}\right)^{-[\eta]\Phi_{\rm m}}$$

A model for describing the effect of particle self-crowding on suspension viscosity, where  $\Phi$  is the particle volume fraction,  $\Phi_m$  is a parameter representing the maximum packing fraction and [ $\eta$ ] is the *intrinsic viscosity*. For ideal spherical particles [ $\eta$ ]=2.5 (i.e. the Einstein coefficient). Non-spherical or highly charged particles will exhibit values for [ $\eta$ ] exceeding 2.5. The value of [ $\eta$ ] is also affected by the particle size distribution. The parameter  $\Phi_m$  is a function of particle shape, particle size distribution and *shear rate*. Both [ $\eta$ ] and  $\Phi_m$  may be treated as adjustable model parameters.

The aggregate volume fraction (representing the effective volume occupied by particle aggregates, including entrapped fluid) can be determined using this equation if  $\Phi_m$  is fixed at a reasonable value (e.g. 0.64 for random close packing or 0.74 for hexagonal close packing) and [ $\mathbf{T}$ ] is set to 2.5. In this case,  $\Phi$  is the adjustable parameter and is equivalent to the aggregate volume fraction.

Meter

$$\eta = \eta_{\infty} + \frac{\eta_0 - \eta_{\infty}}{1 + \left(\sigma/\sigma_2\right)^{\alpha-1}}$$

Expressed in terms of <u>shear stress</u>, used to represent a <u>pseudoplastic</u> material exhibiting a <u>power-law</u> relationship between shear stress and <u>shear rate</u>, with both high  $(\neg \infty)$  and low  $(\neg_0)$  shear rate asymptotic viscosity limits. The parameter  $c_2$  can be roughly identified as the shear stress value at which  $\neg$  has fallen to half its final asymptotic value. The Meter and <u>Carreau-Yasuda</u> models give equivalent representations in terms of shear stress and shear rate, respectively. If  $\neg_0$  and  $\neg \infty$  are not known independently from experiment, these quantities may be treated as additional adjustable parameters.

#### **Powell-Eyring**

$$\eta = \eta_{\infty} + \left(\eta_0 - \eta_{\infty}\right) \frac{\sinh^{-1}(\tau \dot{\gamma})}{\tau \dot{\gamma}} -$$

Derived from the theory of rate processes, this relation is relevant primarily to molecular fluids, but can be used in some cases to describe the <u>viscous</u> behavior of polymer solutions and <u>viscoelastic</u> suspensions over a wide range of <u>shear rates</u>. Here, Teois the <u>infinite shear viscosity</u>  $T_0$  is the <u>zero shear viscosity</u> and the fitting parameter represents a characteristic time of the measured system. If  $T_0$  and Teoare not known independently from experiment, these quantities may be treated as additional adjustable parameters.

## Power-law [Ostwald-de Waele]

$$\sigma = K \dot{\gamma}^n$$

A two parameter model for describing <u>pseudoplastic</u> or <u>shear-thickening</u> behavior in materials that show a negligible <u>yield response</u> and a varying <u>differential viscosity</u>. A log-log plot of  $\mathbf{a}$ versus  $\dot{\gamma}$  gives a slope n (the *power-law* exponent), where n<1 indicates pseudoplastic behavior and n>1 indicates shear-thickening behavior.

# **Computer Laboratory Exercises:**

- 1. Open the Documentation for the Non-Newtonian Fluids Module. You must allow blocked content to see the documentation.
- 2. Then select New from the Model Navigator window
- Select Axial Symmetry ((2D), Non-Newtonian Flow Steady-state analysis
- 4. Using the Graphical User Interface construct and complete this module. Conduct a



parametric study of pressure to reproduce Figure 4-17.

2. Complete Cutlip & Shacham problem 5.4 a, 5.4 d and then see hints for 5.4c and complete this simulation. Compare the COMSOL solutions with those generated by POLYMATH.

Wodel Navigator	Select New,	X
New Model Library User Models Settings         Space dimension:       Axial symmetry (2D) <ul> <li>Chemical Engineering Module</li> <li>Energy balance</li> <li>Mass balance</li> <li>Momentum balance</li> <li>Momentum balance</li> <li>Brinkman Equations</li> <li>Compressible Euler</li> <li>Darcy's Law</li> <li>K-s Turbulence Model</li> <li>Incompressible Navier-Stokes</li> <li>Swirl Flow</li> <li>Non-Isothermal Flow</li> <li>Non-Newtonian Flow</li> <li>Steady-state analysis</li> <li>Transient analysis</li> </ul> Dependent variables:         u v p           Application mode name:         nn           Element:         Lagrange - P <sub>2</sub> P <sub>1</sub> V         Stagrange - P <sub>2</sub> P <sub>1</sub> Image:         Lagrange - P <sub>2</sub> P <sub>1</sub>	$P = f(\nabla \mathbf{v}) + f \qquad \text{Sector}$	elect Axial Symmetry ((2D), Ion-Newtonian Flow Steady- tate analysis
	OK Cancel	

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#### Hints for 5.4c for power law fluid with n>1 (n=2)

From COMSOL: The problem with power law models with an exponent >1 is that the viscosity is zero at zero shear rate. In the center of the channel where vr=0, the shear rate is zero. This means that without artificial stabilization, there is no hope for convergence.

After applying anisotropic diffusion, pressure stabilization and also splitting the domain into two subdomain's (this provides access to the mesh parameters on the center line) and meshing a bit more agressive toward the center of the channel we obtained a maximum of 0.641 compared to the theoretical value of 0.644.

#### Create 2 domains and mesh as shown. Then select the refine mesh button once

Now create subdomain and boundary conditions appropriate to this problem. Finally add the following Artificial Diffusion conditions recommended by COMSOL for this problem:



You should have 2 subdomains that you specify the Artificial Diffusion in both:
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Subdomain Settings - Nor	n-Newtonian Flow (n	n) (	
Equations			
$\rho \partial \mathbf{u} / \partial \mathbf{t} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} = \nabla \langle [-p\mathbf{I} \cdot \mathbf{v}] \rangle$	$+ n(\nabla \mathbf{u} + (\nabla \mathbf{u})^{T})] + \mathbf{F}$		
$\nabla \mathbf{u} = 0$			
Subdomains Groups	General Power Law Carreau Init Element Color		
Subdomain selection	Fluid properties and sources/sinks		
	Library material:	Load	
2	Quantity	Value/Expression Description	
	ρ	1000 Density	
	Viscosity mode	Power law	
✓	η	1 User defined viscosity	
Group:	Fr	0 Volume force, r-dir.	
Select by group	Fz	0 Volume force, z-dir.	
Active in this demain	Artificial Diffusion		
Boundary Settings - Non-	Newtonian Flow (nn	) 🛛 🔀	
Equation			
u = 0			
Boundaries Groups	Coefficients Color/Sty	le	
Boundary selection	Boundary conditions		
1	Boundary condition:	No slip 🗸	
3	Quantity	Value/Expression Description	
4	u <sub>o</sub>	0 r-velocity	
5	v <sub>o</sub>	0 z-velocity	
7 🗸	P <sub>0</sub>	0.2 Pressure	
Group:			
Select by aroup			
Select by group			
<ul> <li>Select by group</li> <li>Interior boundaries</li> </ul>			
Interior boundaries			

The boundary 4 is the center line between the 2 subdomains.

Make sure you are using the stationary non-linear solver in the Solver Parameters dialog box.

# Submit at the end of the period (or unless otherwise instructed):

- 1. Figure 4-17: Parametric study of the process, sweeping the inlet pressure from 10 kPa to 210 kPa, while investigating a Cross-sectional viscosity plot. The greater the inlet pressure (and pressure differential) the less the viscosity and more varied its distribution through the cross-section.
- 2. Answer all questions for C&S 5.4 a, c and d and submit appropriate plots.
- 3. Compare your COMSOL simulation results with those obtained in POLYMATH for C&S 5.4 for each of the 3 simulations.