CEE 160L – Introduction to Environmental Engineering and Science

Lecture 5 and 6

Mass Balances

Mass Balance (MB)

- Very important tool
  - Track pollutants in the environment
  - Reactor/treatment design

- Basis: Law of Conservation of Mass
  - **Mass can neither be created nor destroyed.**

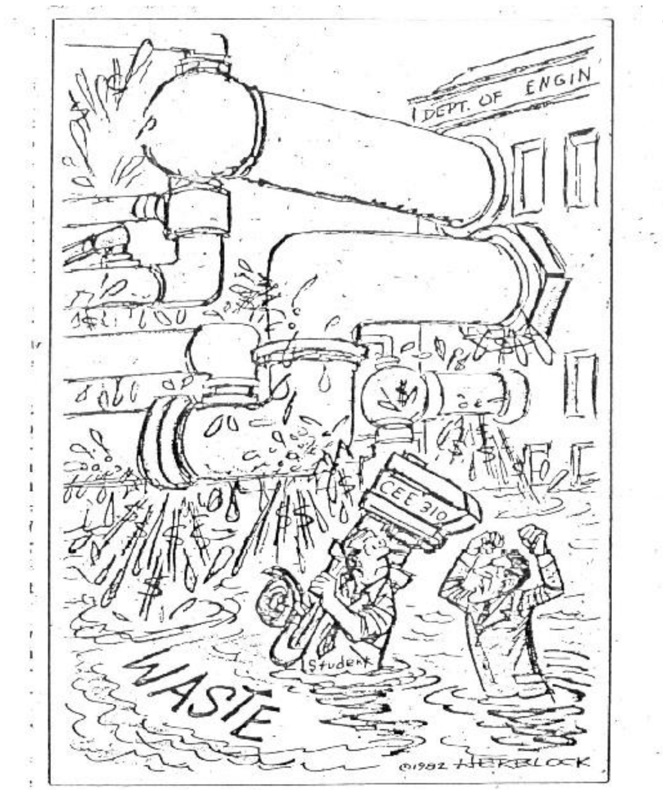
Control Volume

- MB performed over specific control volumes
  - Well-defined system boundaries

- Control volume (CV)
  - Specific region in space for which MB is written
  - **Define mass flowrates into and out of system.**

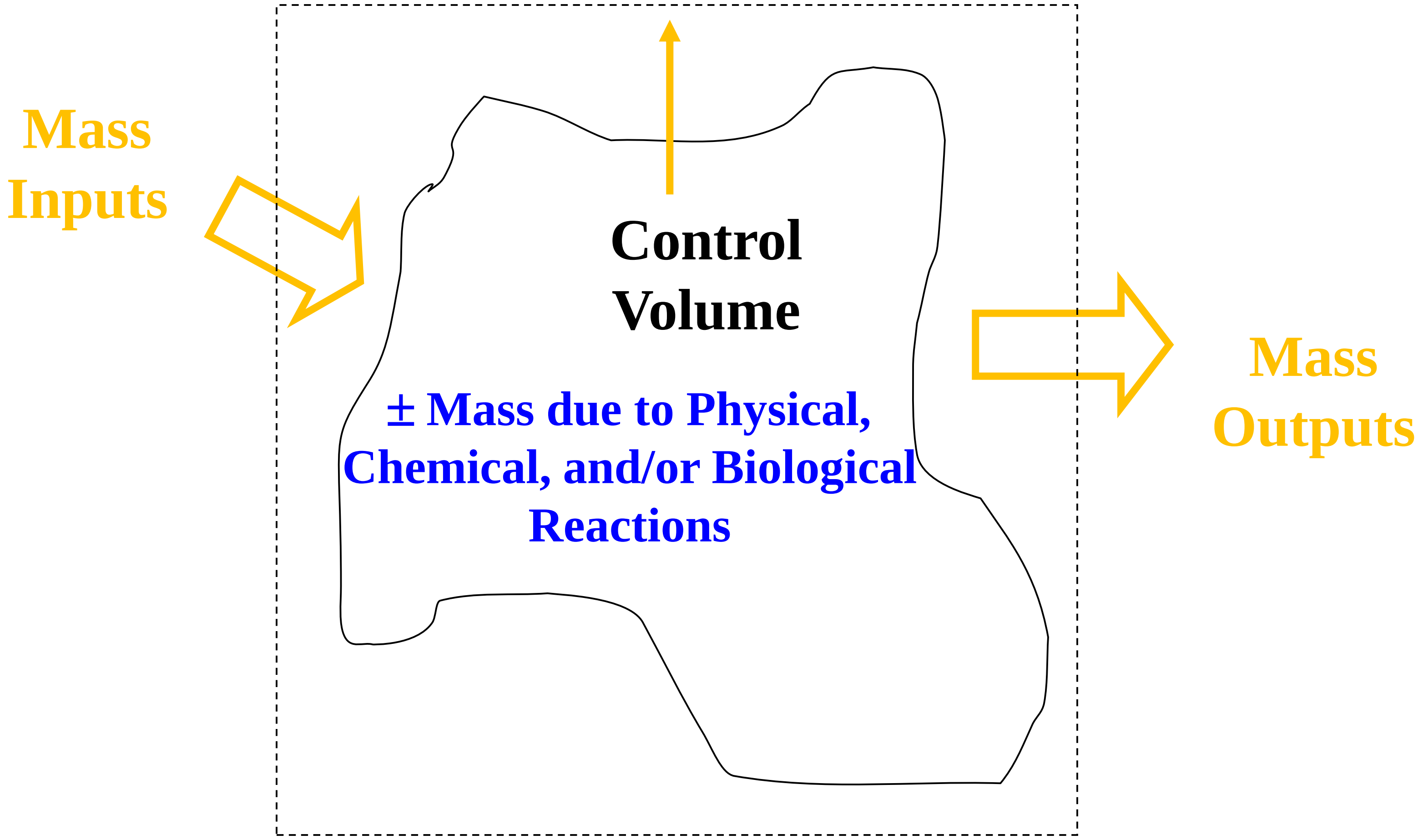
Control Volume Examples

- Entire Earth
- Watershed
- Airshed
- Lake
- Sand filter
- Sediment Particle



I don't care about the leaks, where the heck is your control volume!

General Control Volume



General Guidelines for Solving Mass Balance Problems

1. Draw system as a diagram.
   - **Include flows (inputs and outputs) as arrows.**
2. Add numerical information to diagram.
   - Flow rates, concentration, reaction rates, etc.
3. Draw dotted line around process component(s) to be balanced.
   - **This is the CV!**

General Guidelines for Solving Mass Balance Problems (cont'd)

4. Decide what material is to be balanced.
   - **Air, water, pollutant**
5. Write MB equation.
   - Then, substitute numbers into equation.
6. If only one unknown, solve for that variable.
7. If > one unknown, repeat the procedure.
   - **Use different CV**
   - **Or use different material for same CV**

Overall mass balance between times t and t+Δt

(mass at time t+Δt) =

(mass at time t)
+
(mass entered system between t and t+Δt)
-
(mass exited between t and t+Δt)
+
(mass generated/consumed by reaction processes between t and t+Δt)

Overall mass accumulation ($\Delta M/\Delta t$) between times t and t+Δt

$$\frac{\text{(mass at time } t+\Delta t\text{) - (mass at time } t\text{)}}{\Delta t} =$$

$$+$$

$$\frac{\text{(mass entered system between } t \text{ and } t+\Delta t\text{)}}{\Delta t}$$

$$-$$

$$\frac{\text{(mass exited between } t \text{ and } t+\Delta t\text{)}}{\Delta t}$$

$$+$$

$$\left[\frac{\text{mass generated/consumed by reaction processes between } t \text{ and } t+\Delta t}{\Delta t}\right]$$

In the limit ($\Delta t \rightarrow 0$) gives the overall rate of mass accumulation rate

(rate mass accumulation) =

+

(rate mass influx)

-

(rate mass outflux)

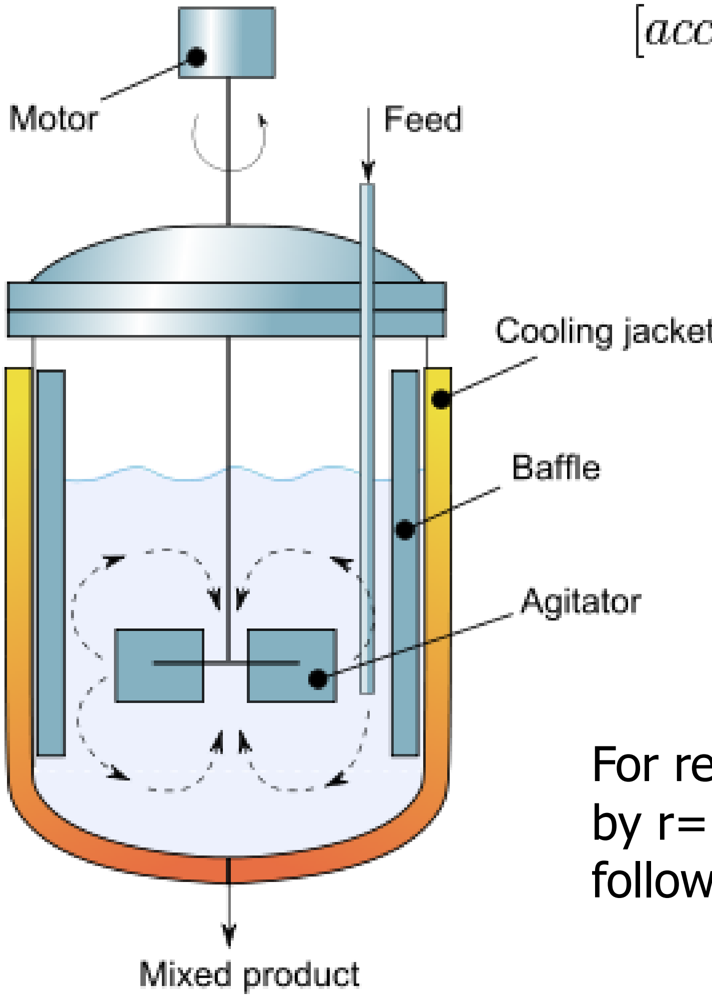
+

(rate net mass production/consumption)

Reaction term

$$\boxed{dM/dt = dM_{in}/dt - dM_{out}/dt + dM_{rxn}/dt}$$

Continuous stirred tank reactors (CSTR)



$$[accumulation] = [in] - [out] + [generation]$$

$$\frac{dN_i}{dt} = F_{io} - F_i + \boxed{V\nu_i r_i}$$ Rxn term

F_{io} is the molar flow rate inlet of species *i*,
F_i the molar flow rate outlet
V is the tank volume
ν_i is the stoichiometric coefficient.
τ_i is the residence time (the average amount of time a discrete quantity of reagent spends inside the tank)

For reaction A→products reaction rate is given by $r=kC_A$ Consumption of reactant A generally follows

$$C_A = \frac{C_{Ao}}{1 + k\tau}$$

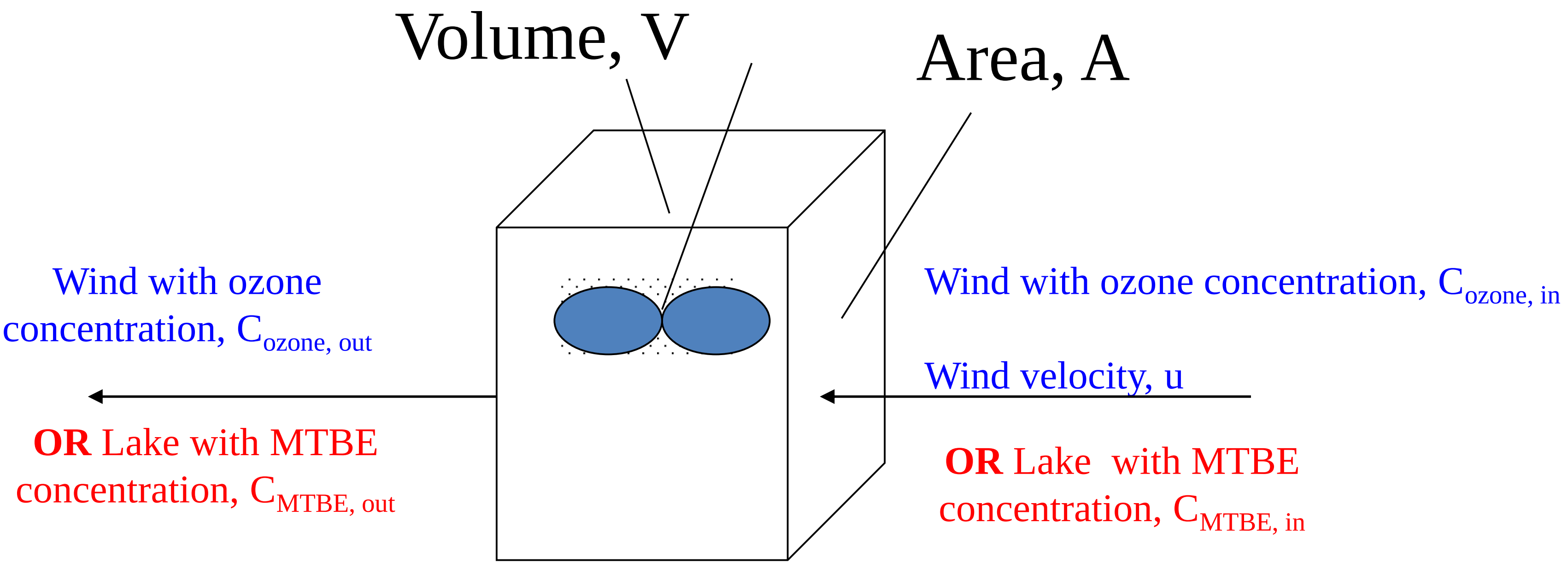
http://en.wikipedia.org/wiki/Continuous_stirred-tank_reactor

Commercial uses of CSTR

- Fermentors for biological processes in many industries
- Brewing
- Antibiotics production
- Cell culture
- Waste treatment
- http://encyclopedia.che.engin.umich.edu/Pages/Reactors/CSTR/CSTR.html

A Simple Control Volume

A Well-Mixed Airshed or Lake



Continuously Stirred Tank Reactor (CSTR) Model

CSTR Model

- Fluid particles entering the reactor are instantaneously mixed throughout the reactor.
  - **No concentration or thermal gradients exist.**
  - $C_{reactor} = C_{effluent}$
- Rapid dilution of influent concentration
- Smoothes time-varying input flow and concentrations

Mass Accumulation Term

Concentration of
water, air, pollutant

$$\text{Accumulation} = \frac{dm}{dt} = \frac{d(VC)}{dt} \qquad \left[\frac{M}{T}\right]$$

$$\text{For Constant Volume, } \frac{dm}{dt} = V\frac{dC}{dt} \qquad \left[\frac{M}{T}\right]$$

Steady State

- **No change in mass in reactor with time**
- All flow rates, T, P, concentrations, and liquid levels are constant with time.
- At steady state
  - Mass can be entering and leaving reactor
  - Reactions not necessarily at equilibrium
  - **However, inputs, outputs and rates of consumption and generation balance each other.**

Mass Balance Terms

Accumulation Term

$$\text{Steady state} = \text{system does not change with time} \Rightarrow \frac{dm}{dt} = 0$$

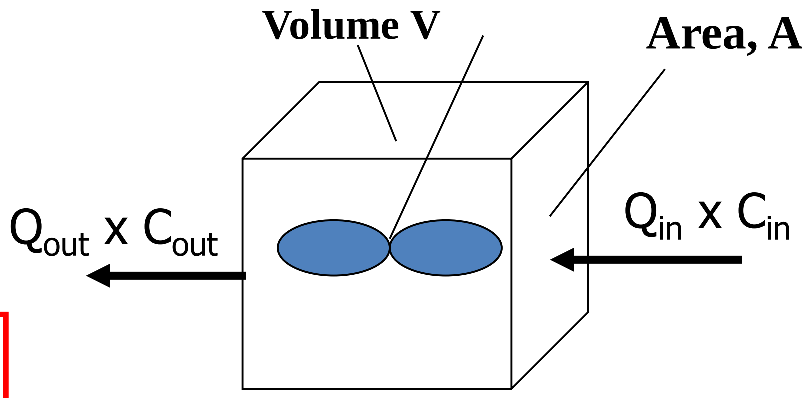
$$\text{Unsteady state} = \text{system changes with time} \Rightarrow \frac{dm}{dt} \neq 0$$

Rate of mass coming in (Input)

$$dm_{in}/dt = Q_{in} \times C_{in}$$

Rate of mass going out (Output)

$$dm_{out}/dt = Q_{out} \times C_{out}$$



Overall Balance for a Simple Constant Volume CSTR

- Neglecting reactions, additional sources/sinks for now

$$[\text{Accumulation}] = [\text{Inputs}] - [\text{Outputs}] \pm \cancel{[\text{Generation/Consumption}]}$$

$$\frac{VdC}{dt} = Q_{in} C_{in} - Q_{out} C_{out}$$

where :

V = volume

C = concentration (mass/volume)

Q = volumetric flowrate

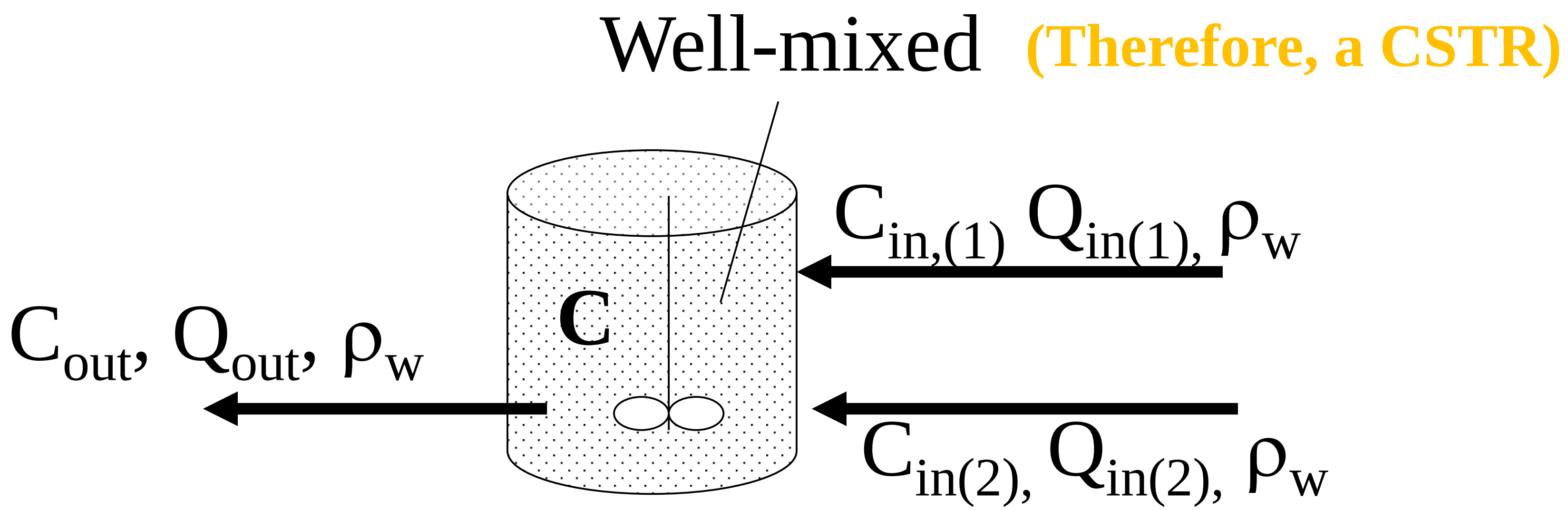
Concentration in the reactor

Consider an overflowing rain barrel

What is the volume?
What is Q_{in}?
What is C_{in}?

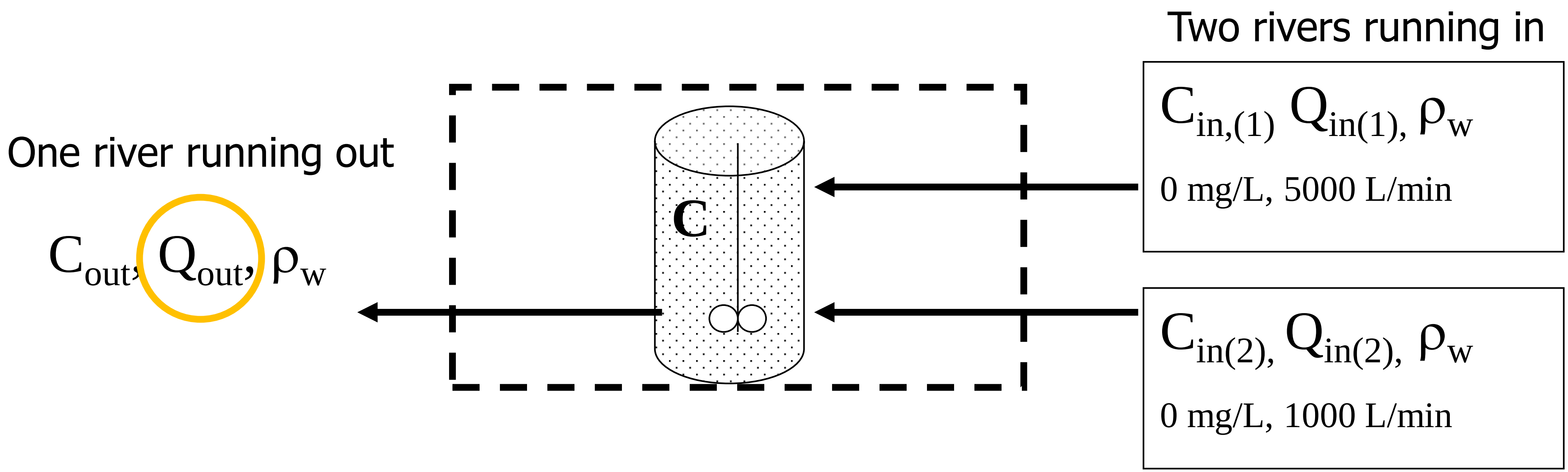
Is it well stirred?
What is Q_{out}?
What is C_{out}?

Example 1: Lake Contaminated with MTBE *(Steady State)*



Where: C = concentration of MTBE [mg MTBE/L]
Q = volumetric flowrate of water [L/min]
ρ_w = density of water [mg/L]

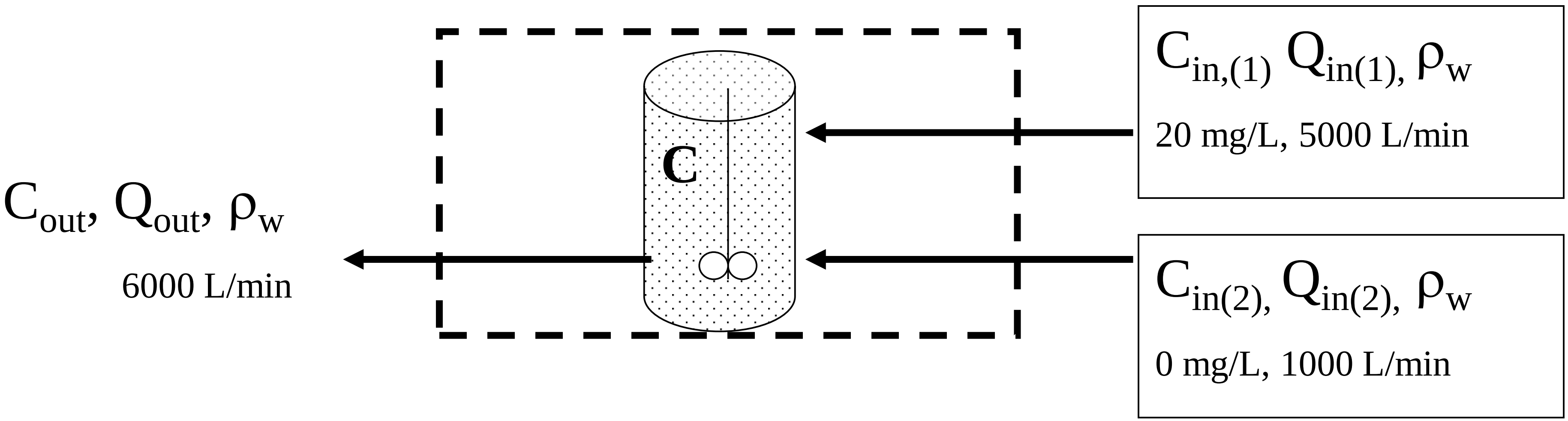
1. Steady State Mass Balance on Lake Water Only



At steady state there if no net accumulation or depletion of water in the lake

$Q_{out}=Q_{in(1)}+Q_{in(2)}=6000 L/min$

2. Mass Balance on MTBE Lake



At steady state then amount of water and MTBE that flows in has to equal the amount of water MTBE and water that flows out

To balance the mass then

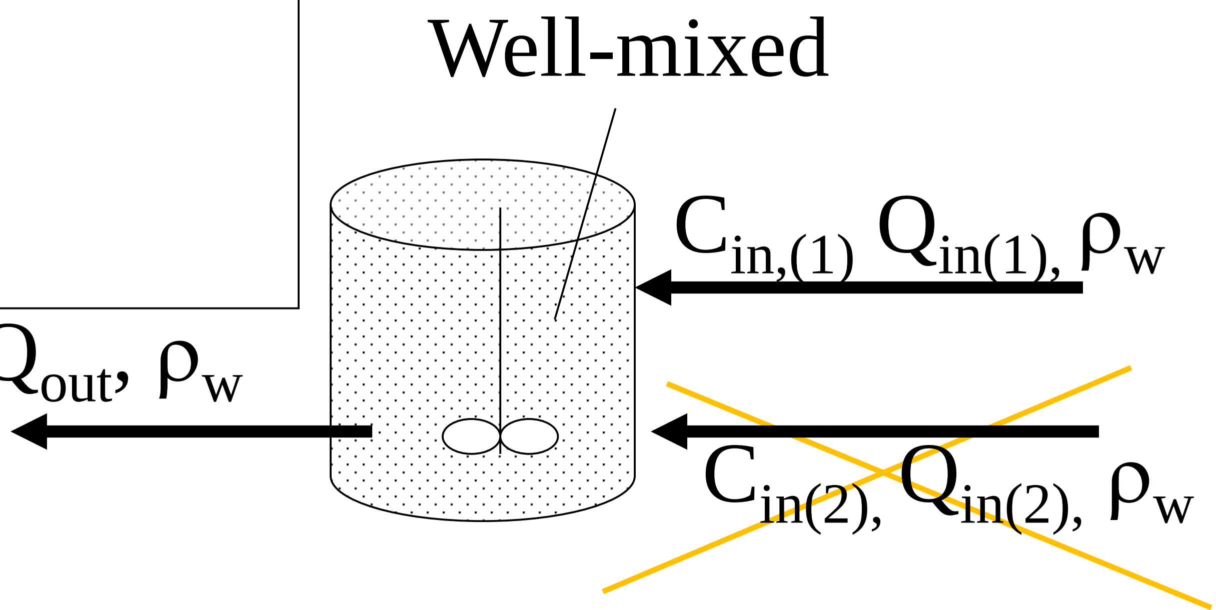
$C_{out}Q_{out}=[C_{in(1)}+C_{in(2)}]x[Q_{in(1)}+Q_{in(2)}]=20mg/L*6000L/min$

$C_{out}6000L/m=(20mg/L)(5000L/min+1000L/min)$

$C_{out}=20*5000/6000=16.7mg/L$ (inflow of clean 2nd river dilutes MTBE outflow)

Example 2: Lake Contaminated with MTBE *(Steady State)*

What happens if $Q_{in(2)} = 0$?
At steady state, will C in lake will
Increase?
Decrease?
Remain the same?



Where: C = concentration of MTBE [mg MTBE/L]
Q = volumetric flowrate of water [L/min]
ρ_w = density of water [mg/L]

Example 2: Lake Contaminated with MTBE *(Steady State)*

What happens if $Q_{in(2)} = 0$?
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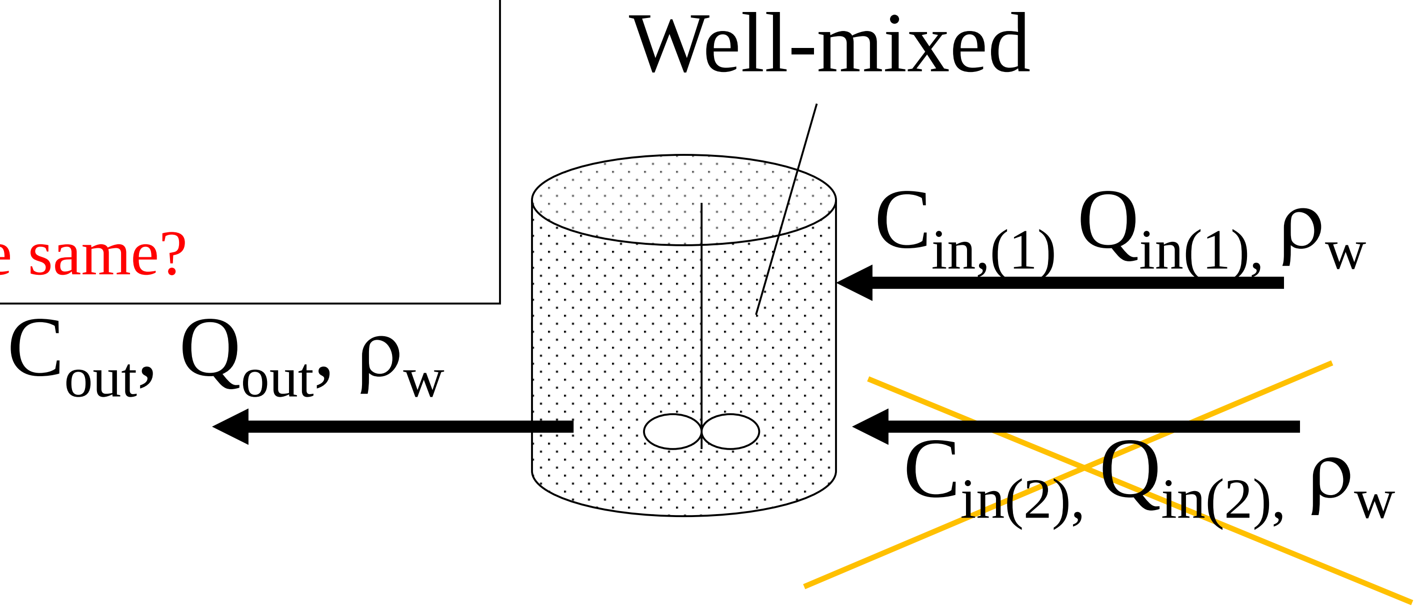


If 20mg/L of MTBE and 5000L/min water flows in, then at steady state 20mg/L of MTBE and 5000L/min water must flow out.

Residence Time

- Residence time=amount present/rate of removal

1. **Hydraulic residence time (HRT)**

$$\mathrm{HRT} = \theta_h = \frac{V}{Q_{out}} = \frac{\text{Volume in System}}{\text{Flow Rate Out of System}}$$

$$= \text{Average Time Fluid Stays in Reactor}$$

Example: HRT in drinking water distribution system = 1.3 d

Residence Time

- Residence time=amount present/rate of removal

2. **Pollutant residence time (PRT)**

$$\mathrm{PRT} = \frac{C_{\mathrm{in}}\mathrm{V}_{\mathrm{in}}}{C_{\mathrm{out}}\mathrm{V}_{\mathrm{out}}} = \frac{\text{Pollutant Mass in System}}{\text{Pollutant Mass Out of System}}$$

$$= \text{Average Time Pollutant Mass Stays in Reactor}$$

In the limit ($\Delta t \rightarrow 0$) gives the overall rate of mass accumulation rate

(rate mass accumulation) =

+

(rate mass influx)

-

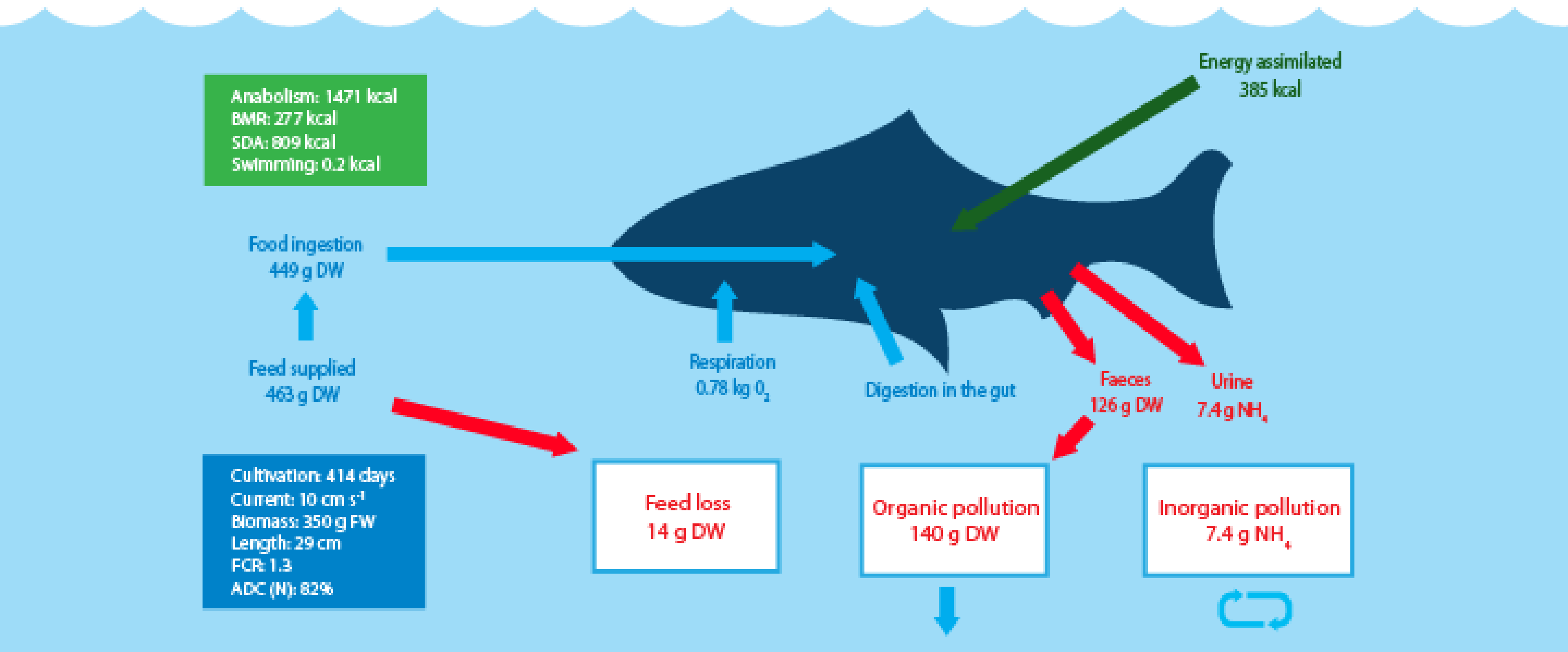
(rate mass outflux)

+

$\boxed{\text{(rate net mass production/consumption)}}$ Reaction term

$$\boxed{dM/dt = dM_{in}/dt - dM_{out}/dt + dM_{rxn}/dt}$$

Mass balance of fish farm

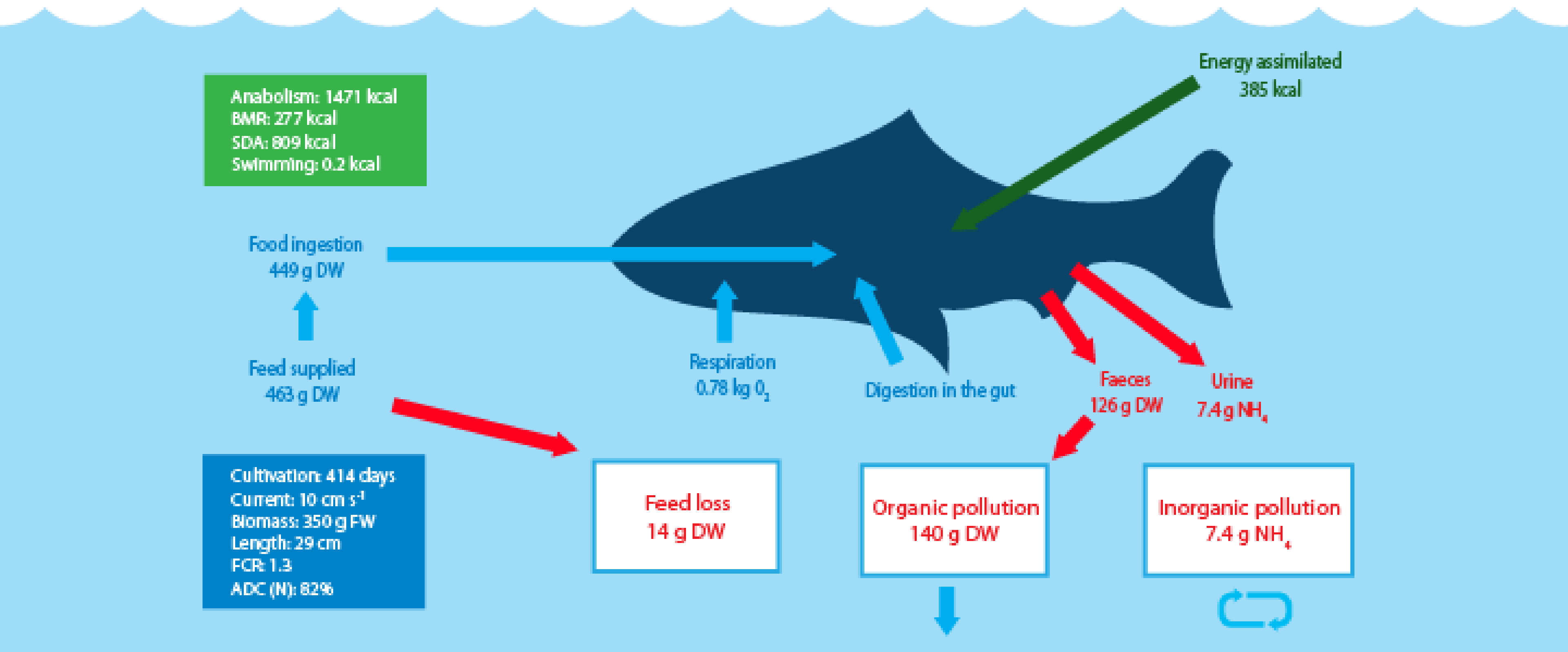


Mass influx?

Mass outflux?

Reaction terms?

Mass balance of fish farm



Mass influx : mass fish added initially, fish feed added, inspired O2

Mass outflux: uneaten feed, fish death, fish waste, respired CO2

Reaction terms: mass increase of fish

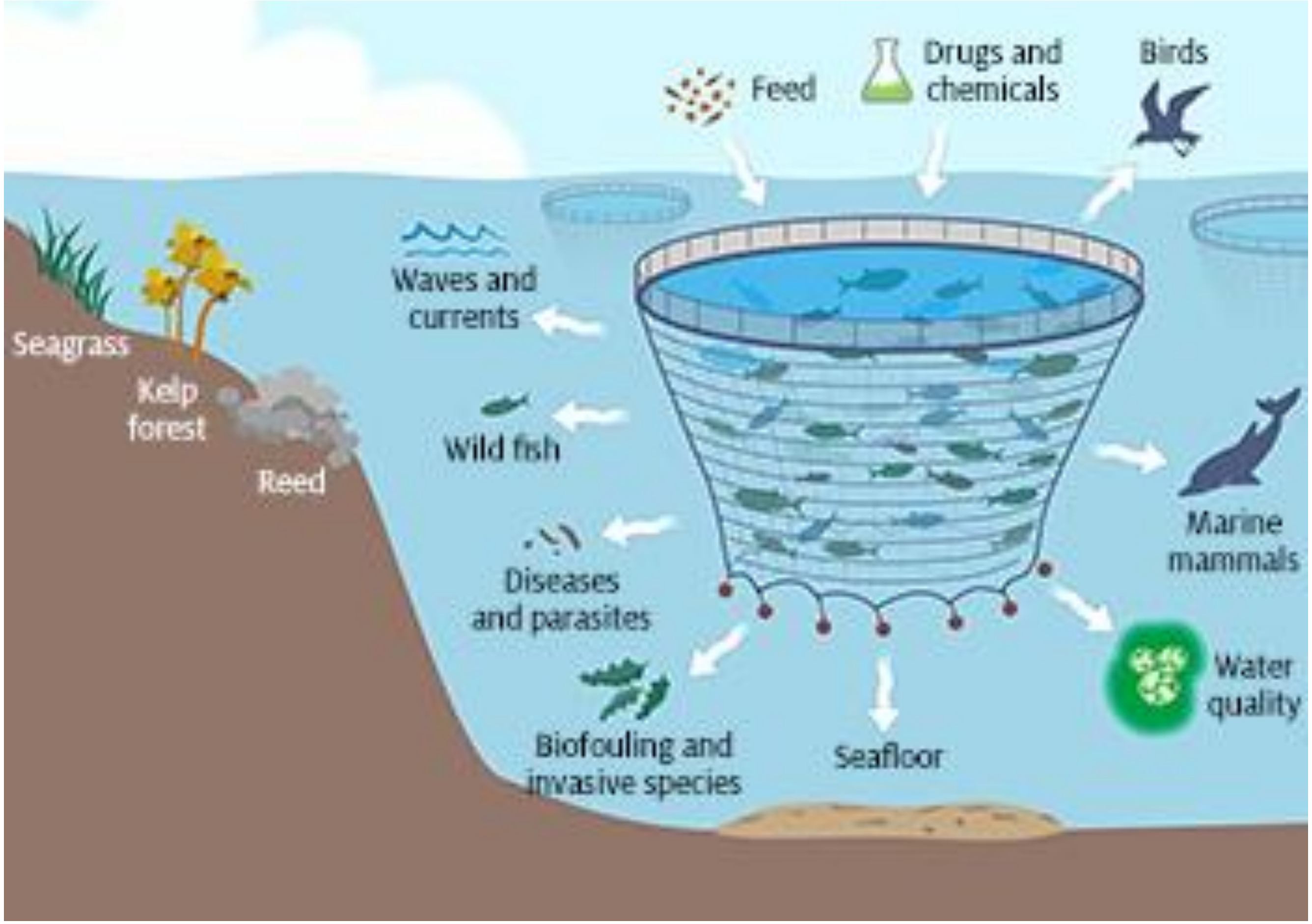
Consider other factors

Mass accumulation?

Mass consumption?

In flows?

Out flows?



http://www.waikatoregion.govt.nz/Environment/Natural-resources/coast/Coastal-pressures/Marine-farming/Marine-farming-science-projects/