Blackbody Radiation: Computer-Lab

We have discussed examples of modern physics which contradict our classical thoughts about the way nature behaves. It is important to recognize that our classical way of thinking is correct but only valid in specific cases. For example, classical mechanics is correct when the velocity of an object under consideration is slow with respect to the speed of light. Special relativity is a more general formalism. We have also discussed electromagnetic radiation. Over the years, our experiences have firmly convinced us that radiation behaves as a wave. However, when we magnify our perspective so that we are looking at individual events on a microscopic level such as the case where an electron is released from a polished metallic surface, we find that light must be considered as a particle, i.e. a photon. A photon represents the minimum energy that can be transferred by light at a given wavelength or frequency. Ultimately, it must be accepted that light behaves as both a wave and a particle dependent only on the way we probe it. Here the limits of use of one formalism over another is not as clear. So our classical idea of waves is valid if we perform an experiment where the wavelike nature is revealed.

Energy emitted from an object

Typically, when we look at an object we observe reflected light. At room temperature, the radiated light is infrared and not in the visible spectrum. As the temperature of the object increases, we finally see a red glow which is radiated light. If the temperature continues to increase, the visible radiated light tends to shorter wavelengths and we discern a change in color. To analyze this situation directly is difficult but by looking at the ideal case of a blackbody, analysis is possible since we only need consider the radiated light and not the reflected light.

As it turns out, blackbody radiation is an example where we must consider the quantum nature of radiation. A blackbody is an object which does not reflect light, i.e. a perfect absorber. A nearly ideal blackbody is the hole in a cavity containing electromagnetic radiation. Any light which is incident on this hole is absorbed in the sense that through multiple reflections inside the cavity, the energy is eventually absorbed and the light has a negligible chance of emerging from the hole as reflected light. The radiation which does emerge from the hole is a sampling of the light which is radiated from the walls of the cavity.

Any wave is caused by a vibrating object. The frequency of the wave is determined by the frequency of vibration. Electromagnetic waves are created by vibrating electric charges which are attached to atoms. The frequency at which an atom vibrates is related to the temperature of the wall since the temperature is a measure of the kinetic energy of the atom. The spectrum of wavelengths radiated from a blackbody can not be predicted from classical electromagnetic and thermodynamic considerations. This is demonstrated by the failure of the **Rayleigh-Jeans** formula:

$$R(\lambda) = \frac{2\pi ckT}{\lambda^4}$$

which is derived from these classical concepts to account for the radiancy (the electromagnetic energy intensity per wavelength) as measured using the experimental apparatus schematically represented on p78 of your text. This classical expression especially fails to predict the high frequency range of the observed radiation spectrum.

Max Planck improved the modeling of the measured data by assuming that the oscillating atoms in the walls of the cavity could only absorb and radiate energy in discrete bundles and forcing the upper limit on the energy that could be radiated to be the thermal energy of the atom, kT. This placed a limit on the highest frequency which could be emitted and removed one of the major downfalls of the Rayleigh-Jeans formalism, the infinity at high frequencies. **Planck's** result for the radiancy is:

$$R(\lambda) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/(\lambda kT)} - 1}$$

Each of these expressions demonstrate that the intensity at a given wavelength depends only on the temperature, T. In what follows you will numerically confirm that Planck's formula accurately predicts the two following laws:

Stephan's Law: $I(T) = \sigma T^4$ (σ = 5.67 x 10^{-8} W/m^2K^4)
Wien's Displacement Law: $\lambda_{max}T$ = 2.898x10-3mK (m stands for meter! not milli)

Part I: Comparing Radiancy of Rayleigh-Jeans with that of Planck for T=100 K:
Create Data in Excel

1. Open Excel program.
2. Click on A2, Enter 1
3. Go to the A3 Field, enter: = A1+1 in then hit Enter Button
4. Select the A2 field, move your cursor to the lower right edge of the field till it becomes a black cross, click and drag it down to A201 (field should have value 200).
5. Click on B2 and enter: =A2*1e-6. Again drag it down to B201.
(Creates wavelength in meter)
6. Click on C2 and enter: =2.6e-14*100/(B2^4) Click and drag down to C201.
(Creates y-axis Rayleigh-Jeans radiancy). Hereby, 2.6e-14 stands for the constant *2πck*, 100 for *T=100 K* and B2 for the wavelength. [**NOTE**: if you see a #NUM sign, don't be shocked, it says the program got too high a number. Keep on with the instructions].
7. Click on D2 and enter: =3.71e-16/(B2^5)/(EXP(0.01425/(B2*100))-1). Click and drag down to D201 (Creates y-axis for Planck's radiancy)
8. Label your columns in row 1.

→ Check your two columns: the Rayleigh-Jeans' radiancy (column C) should have a high value at low wavelength and drop continuously. The Planck's radiancy should start low and have a maximum at a certain wavelength.

Plot the Radiancy curves vs wavelength

Click on "Chart Wizard"-symbol from the tools,

1. choose a XY scatter (type without markers), enter next.
2. change from data value to series; choose y-axis in following format: =Sheet1!$D$2:$D$201 [means data point from sheet1 from column D, point 1 to 201; choose x-axis similar: =Sheet1!$B$2:$B$201; name curve “Planck”; add another curve with same x-axis but C-colums as y-axis, name “Rayleigh”. [your graph might look strange now, don’t worry, the y-scale is still wrong].
3. Go to “titles” and give the graph and the axis the correct names.
4. Place chart as new Sheet! [You can change between sheet by the clicking on the names at the lower left corner].
5. Now, your graph looks not very good, since it is in auto-scale mode of the axis and the y-values of Raleigh-Jeans are very high for small wavelength (remember: UV-catastrophy).
6. Click on the numbers on the y-axis, go to “scale” and reduce “maximum” to 300000 and “minimum” to zero. Now both graphs should be well visible.

→ Print your Chart and add on the print-out the temperature.
→ Describe the shapes of the curves and compare their dependence on wavelength.

Part II: Wien’s Displacement law

1. Determine from the data in column D the wavelength at which the radiancy is at maximum (look for the highest value in column D and note wavelength at same point from column).
2. Note the wavelength and the temperature (T=100K).
3. Create additional columns using Planck’s radiation law for temperatures T=30K, 50K, 300K and 500K.
4. For each wavelength determine the wavelength at which the radiancy is maximal.
5. Create on a new sheet a table with your five temperatures and wavelength.
6. Make a new column for the product λ_{max} * T. All of the numbers in that column should be about equal and have a value of 2.9×10^{-3}.

→ Compare your results with the accepted value. Discuss possible errors.

→ Plot and Print out a chart with three graphs of radiancy versus temperature for different temperatures.

Part III: Stefan’s law:

We want to verify now, that the total intensity, which is the integral over the radiancy $R(\lambda)$, follows the Stefan’s law: $I = \sigma T^4$.

1. Go back to sheet 1, add all the data for the columns, in which you have Planck’s radiation curves for different temperatures. This will be your total intensity. [for example: go to cell D205, enter: =SUM(D2:D201)]. Only adding the data, however, will

not give you the correct value for the intensity, since numerically the integral and the sum are somewhat different:

$$I(T)=\int_0^\infty R(\lambda,T)d\lambda \cong \sum_{n=1}^{200} R(\lambda,T)\Delta\lambda$$

2. To get the correct *I(T)* value, you have to multiply your sum by $\Delta\lambda$, which is the step width of the wave length (1e-6).
3. Note the five values for the five temperatures and make a new table on a new sheet.
4. Compute on a third column the quotient: I/T^4 . This should give you a constant number for all five temperature ranges.
5. Compare that value with the value of the Stefan-Boltzmann constant, which is $\sigma = 5.7\text{x}10^{-8}\ W/m^2K^4$.

→ Compare your results and discuss eventual sources of error, if you get any deviations.

Questions.

1. What wavelength (not a numerical value) do you observe with your eyes at high temperatures?

2. Estimate the temperature at which you would expect the dominant wavelength to be about 700nm.

3. At what wavelength does the Sun emit its peak radiancy? The temperature of the Sun is about 6000K. How does this compare with the peak sensitivity of the human eye?