SpaceTime Lab

Find the SpaceTime button and select the program. A small window will appear with several icons on it. One of them will be labeled Spacetime. Double-click on the Spacetime icon. A description window will appear. Point the mouse arrow at the OK button and click. Now you see a HIGHWAY where objects move at speeds large enough that relativistic theories must be applied.

The large blank area in the middle of the screen can be thought of as the snapshot or a movie still of a “cosmic superhighway” running from left to right across the screen. Different lanes on the highway are for objects moving at different speeds. Objects lying on the horizontal line through he middle of the screen are on the “median strip” of the highway and are considered our stationary reference frame. Objects above the center move to the right; the farther above the center, the faster they move. Objects in the very top lane move to the right with the speed of light, and only light flashes (and neutrinos) can occupy this top lane.

Objects below the center of the screen move to the left; the farther below the center, the faster they move in the negative direction. Objects in the very bottom lane move to the left with the speed of light, and only light flashes (and neutrinos) can occupy this bottom lane.

The vertical ruler at the left of the screen shows a measure of the speed of the lanes. The scale shows β, the velocity as a fraction of the speed of light: $\beta \equiv v/c$. β goes from +1 at the top (light moving to the right) to -1 at the bottom (light moving to the left). Notice that the β scale is not linear; equal vertical lane separations do NOT correspond to equal changes in β. This is done so that more of the interesting velocities (as β approaches one) can be easily observed on the screen.

Test Drive on HIGHWAY

Create a clock on the highway. Do this by hitting F1 and select (C)lock from the menu. Move the cross up and down (change of speed) or left and right (different position) until the position you like. Then hit enter. The number in the clock image is the time reading of the clock.

If the new clock is not directly over or under the clock on the center strip, you will notice that the new clock does not read zero, as the center-strip clock does. This is a relativistic effect called the relative synchronization of clocks.

Now place a rod on the highway. Hit F1 and select (R)od. Move the rod on the highway. Make several rods at different position on the highway to the both sides (positive and negative velocities).

You may have been surprised that the rod icon got smaller and larger as you moved it across the highway. The image of the rod is narrower when in one of the outer, faster lanes and longest when the rod is at rest on the center strip. This reduction in length of moving objects along their direction of motion is called length contraction (some texts call it Lorentz contraction).

You can "JUMP TO" the reference frame of the moving rod. Click F6 (select), hit the number of the rod you want to JUMP TO, then hit F4 (Transform) and hit (J)ump to. This will give you the perspective of an observer in the “moving” reference frame.

Now put a light flash onto the highway (select F1, then (F)lash). This time the flash can be placed only in the top or bottom lanes of the highway - the fastest lanes. Light can move only with the speed of light in free space! But it can move either to the right (top lane) or to the left (bottom lane). Place the light flash in either of these lanes, as you wish.

The present display is a snapshot of the highway at time zero. Now step time forward. Do this by clicking on the ↑ button at the lower right of the screen. Click on the ↑ button several times. Watch the objects move, each according to the speed of its lane. Read the time in the upper right corner of the screen. Try stepping time backward by clicking on the ↓ button. Changing the position just click on → and ← buttons.

You have now (hopefully) mastered the basic operation of the relativistic HIGHWAY.

Length Contraction (Lorentz Contraction)

Length contraction is the apparent reduction of length of a moving object along its direction of motion. This effect is obvious when create a rod from on the highway in the familiarization procedure above. The rod is very short near the edges of the highway and of maximum length at the zero-relative-velocity center strip. Create a rod on the center strip and then create a second rod in a moving lane. The moving rod is shown shorter in length from right to left than the rod at rest (but same width from top to bottom). The factor by which the length is shorter is γ [$\equiv(1-(v/c)^2)^{0.5}$]. Another way to illustrate length contraction is to create two objects (such as clocks) in the same lane. The distance between these two clocks will change as you jump between them and objects in other lanes.

→ *Create two clocks in the same reference frame at a distance of 10 units. Create another clock in a moving reference frame. Jump to the other reference frame and estimate the distance between the clocks in units.*

→*Calculate the contracted length of an object 10 meters long if it is traveling at a velocity of 0.9c. Use the program to verify your calculation. Explain how the number from the program verifies your calculation.*

Time Dilation

"Moving clocks run slow" is the result of time dilation. This implies that if a moving clock and a clock at rest are set to read the same time at some initial time, then as the moving clock passes an observer standing on a stationary clock farther along the string, the reading on the moving clock will be less than the reading on the stationary clock.

Start with a new screen. Clean the screen by selecting F6 objects and then Deleting them (F1 → Delete) or by F5 → (N)ew Screen (don't save). Create a clock string on the center strip (*select* the middle clock and go to the menu "Objects" and *select* "Clock String"). Now create a series of clocks in different lanes but all initially lines up vertically above and below the reference clock in the middle of the screen. Now click on the +t or -t buttons and watch the different clocks move and change time. (If you create a clock anywhere but directly above or below the reference clock, the initial reading of the moving clock will be different from zero. This is due to the relative synchronization of clocks).

→ *Calculate the reading of a clock moving at v = 0.87c and v = -0.9798c if the clocks are initially synchronized, and the stationary clock indicates that 1 hour has elapsed? Verify this result using the HIGHWAY. Notice that the fastest moving clocks have the smallest elapsed time reading.*

→ *Why do moving clocks at positiosn other than the resting clock show different times even for t=0?*

Trip to Alpha Centauri

The star nearest to our sun is Alpha Centauri, about 4 ly distant from us. Assume that the distance units shown along the center strip are ly and the time units on the clocks are years. Create a New Screen (under File menu, don't save).

Create Alpha Centauri. The startup screen shows the "reference clock" in the center. This clock represents the Earth. Create a clock at the position of Alpha Centauri (x = 4.00, β = 0). Create a the shuttle (F1 and Shuttle) at the same position as the Earth (x=0) but in a rightward-moving lane whose velocity is 0.9c (β = 0.9). Step time forward by clicking on the ↑ or PgUp button a few times. Watch the shuttle move toward Alpha Centauri. Continue clicking on the ↑ button until the shuttle gets close to Alpha Centauri. The exact position x=4.00 may occur between one time step and the next step, so you cannot position the shuttle exactly. If this happens, try to get close with the ↑ button. Stop when the shuttle is lined up with Alpha Centauri (is doesn't have to be exact). Now we need to turn around. Select the shuttle (use F6)and pull down the F1 menu and select the Program Shuttle command. Now a cross enters at the position of the shuttle. Move the cross to the new velocity (negative) and hit enter This "programs" the shuttle to turn around when it reaches Alpha Centauri. Step forward in time to bring the shuttle back to earth (increase your time steps until you get close to Earth). Welcome back to Earth. Read the number on the Earth clock and the number on the shuttle clock. These are number of years.

Now go back to t=0 (Options menu) and take the trip on the shuttle (jump to shuttle). Ride the shuttle out and back.

What do you observe? → Discuss relevant screens.

Relative Additions of Velocities

In HIGHWAY, create a clock moving to right with half the speed of light relative to you and jump to it. Now create a second clock moving to the right with half the speed of light relative to you on the first clock and jump to the second clock. Now create a third clock moving to the right with half the speed of light relative to you on the second clock and jump to the third clock.

Will you now be moving 0.5 + 0.5 + 0.5 = 1.5 of the speed of light relative to the original frame? (NO!) Each velocity jump will increase the speed by 0.5 with respect to the current frame. However, with respect to the previous frame the increase is not as great. This outcome is required so that no object moves faster than the speed of light with respect to any frame. Jump to the original clock. How fast is the third clock moving with respect to the original clock? (select clock and read numbers displayed at bottom center of highway screen). _______________.

Velocities DO NOT ADD (in a Galilean sort of way) in special relativity.

→ Choose the speed of two clocks, which relative speed would classically add up to over c. Calculate the real relative speed. Verify your calculation using the HIGHWAY . Where did you put your clocks, what did you do?

Relative Synchronization of Clocks

Create a three clocks along the zero-speed medium strip of HIGHWAY (ground observer S). Each clock has the same reading. Now create two flashes, one above the left clock (positive speed) and one below the right clock (negative speed). These represent the lightnings at the train ends. Now create a third clock right above the middle clock in a different reference frame (train reference frame S'). Let the clock run positively until the light flashes hit the middle clock. Note the time passed for each light beam. Also note the x-values of the respective clocks.

Jump into the reference frame of the other clock, S'. Note down the time when each of the light beams hit the clock.

→ Do the light flashes hit the middle of the train at the same time?

→ Set up a calculation, which supports the time values of your data.

Clocks are synchronized in S. The flashes hit front and back at the same time. In S', the clocks and platform are moving past the train. In the S' frame the clocks in frame S are not synchronized. The clock at the front of the train indicates a later time than the clock at the back of the train.. The clock at the front "chases" the clock in the back as viewed by observer in S'. Click on the -t button (while observer is in S' frame) until the flash hits front of train (as observed by S'). The clock in S should read 0.00. Does the back clock (in S) read the same? (no, it reads a time earlier since the flash hasn't hit the back yet as observed by S'). Now click the +t button and note how the clock at the front "chases" the clock in the back since they are moving in the negative direction. The clock at the front reads a later time than the clock at the back (as observed by S' frame).

$\rightarrow$ The clocks are synchronized in S but not in S'. The chasing clocks leads (shows a later time) by an amount $\Delta t_s = L_p \frac{V}{c^2}$. in the frame which they are moving. Δt_s is the time the clocks are out of synchronization in another frame, which is moving with a velocity V. L_p is the proper distance between clocks (i.e. distance between clocks in the frame they are synchronized).

Doppler Shift

Create a beacon in a fast lane on the HIGHWAY display (create a clock, select it, go to objects and select beacon). A beacon is a clock that emits two light flashes, one in each direction, every unit of its own time, starting at zero time. Advance time and notice how the light flashes are crowded together in the direction in which the beacon is moving. In contrast, light flashes traveling in the direction opposite to the beacon's direction of motion are spread out. Someone standing at rest on the center strip of the highway will receive the crowded-together flashes in rapid sequence. Another median-strip observer looking in the opposite direction will receive the stretched-out flashes with a greater separation in time.

This piling-up and stretching-out of periodic light flashes in opposite directions is called the Doppler shift. Notice that the Doppler shift combines two effects. (1) The emitter is "chasing its own flashes" in its direction of motion and "running away from its own flashes" in the opposite direction. The beacon clock emits forward and backward light flashes every unit of its own time. But the beacon clock - like all other moving clocks - runs slow compared with clocks in the current rest frame. This means that the beacon emits fewer light flashes forward -

and also backward- than would be expected from the lapse of time in the current rest frame.