Power Loss Analysis at a Step Discontinuity of a Multimode Optical Waveguide

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Abstract—In this paper, we analyze the radiated, reflected, and transmitted power due to an abrupt step discontinuity in a multimode optical waveguide by a numerical technique without any physical approximations other than neglecting the evanescent radiation modes. The amount of power loss depends primarily on the number of modes the portion of the waveguide on each side can support. The range of thickness mismatch for which reasonably low loss occurs depends on the wavelength. The technique can be used to analyze step discontinuities of symmetric or asymmetric, single or multimode optical slab waveguides with arbitrary cross section accurately.

Index Terms—Multimode waveguides, optical waveguides, optical waveguide theory, power loss, radiation loss, slab waveguides, step discontinuity.

I. INTRODUCTION

With the increased use of integrated optics in computer and telecommunication systems, it has become important to understand the implications of nonideal optical devices on overall system performance. Real optical devices exhibit material inhomogeneities as well as size variations. This is especially true in integrated optics which are not as well developed as optical fiber. In addition, integrated optic systems have numerous optical discontinuities as the optical path traverses different devices, e.g., optical fiber to optical waveguide.

Furthermore, the anticipated use of optics in board-level electronic systems is likely to include multimode optics [1], [2]. This is because modal dispersion is insignificant over typical board-level distances for bit rates up to 10 Gb/s. A further advantage of multimode optics over single-mode optics is their lower manufacturing costs. Therefore, detailed analysis of power losses in multimode optical systems is essential for designing future board-level integrated-optic systems.

Many different techniques exist for the analysis of optical waveguide systems with axial inhomogeneities. A review of coupled mode theory [3], Marcuse’s step transition method [4], [5], the beam propagation method and a brute force numerical technique is given in Suchoski and Ramaswamy [6]. In coupled mode theory, coupling to reflected guided modes, to backward traveling radiation modes, and between the different radiation modes is ignored. In Marcuse’s step transition method, it is assumed that the radiation modes on both sides of the discontinuity are orthogonal, and the power in the reflected guided mode is neglected. Both techniques ignore the evanescent radiation modes as well. Suchoski and Ramaswamy [6] also introduce an exact numerical technique. The reason the technique is called exact is that none of the physical approximations used in the other techniques are made except ignoring the evanescent radiation modes. In this paper, we will use the terminology “exact numerical technique” to refer to the work of Suchoski and Ramaswamy with the understanding that the evanescent radiation modes are not included in the derivation. The technique is shown to give better results compared to the results obtained by using coupled mode theory and Marcuse’s step transition method for the analysis of single-mode waveguide step discontinuities and tapers.

The three advantages of the technique stated over existing techniques are applicability of the exact numerical technique to step discontinuity problems regardless of the waveguide cross section, the accuracy of the technique regardless of the geometry mismatch between input and output waveguide sections, and the ability to decrease the panel size to ensure the accuracy [6]. The method has not been applied to the multimode case.

In the analysis of multimode optical waveguides, previous work generally has ignored radiation loss [7]–[9]. The only loss mechanism considered was coupling to higher order guided modes.

In this paper, we employ the exact numerical technique to analyze the radiated, reflected, and transmitted power due to an abrupt step discontinuity of a multimode optical waveguide. For generalization, the derivation is done for asymmetric multimode case. We will carry out the analysis for transverse electric (TE) modes, however, the analysis for transverse magnetic (TM) modes is similar.

II. TE MODES OF THE SLAB WAVEGUIDE

Before the derivation, well known TE modes [3], [5] of symmetric slab waveguides will be stated for completeness.

A side view of an asymmetrical slab waveguide is shown in Fig. 1. The refractive indexes \( n_s, n_f \) and \( n_c \) correspond to the substrate, the film, and the cover regions, respectively. Throughout the derivation the following relation is assumed among the refractive indexes without loss of generality:

\[
\eta_f > \eta_s > \eta_c
\]

Letting \( \eta_s = \eta_c \), one can obtain the results for the symmetric case. It is also assumed that the field components are
independent of one spatial coordinate, which is chosen as the $y$ direction in the analysis

$$\frac{\partial}{\partial y} = 0. \tag{1}$$

In this case, $E_y$ components of the modes supported by the optical waveguide are the solutions of the scalar Helmholtz equation

$$\frac{\partial^2 E_y}{\partial z^2} + (k_c^2 n_f^2 - \beta^2) E_y = 0. \tag{2}$$

Once the $E_y$ component is known, $H_x$ and $H_z$ can be calculated by the following equations:

$$H_x = -\frac{\beta}{\omega \mu_0} E_y \tag{3}$$

$$H_z = \frac{j}{\omega \mu_0} \frac{\partial E_y}{\partial x}. \tag{4}$$

$E_x$, $E_z$, and $H_y$ components vanish for TE case.

A. Guided Modes

Guided modes show standing wave behavior in the film region and decay exponentially in the cover and the substrate regions. The propagation constant $\beta$ can take allowed discrete values in the range $k_c n_f < |\beta| < k_c n_s$. Continuity of the tangential component of the electric field $E_y$ at $x = d$ and $x = -d$ requires

$$E_y = \begin{cases} A \cos(\kappa_f d + \phi) \exp[-\alpha_c(x - d)], & \text{for } x \geq d \\ A \cos(\kappa_f x + \phi), & \text{for } |x| \leq d \\ A \cos(\kappa_f d - \phi) \exp[\alpha_s(x + d)], & \text{for } x \leq -d \end{cases} \tag{5}$$

where

$$\alpha_s = \sqrt{\beta^2 - k_c^2 n_s^2}$$

$$\kappa_f = \sqrt{k_c^2 n_f^2 - \beta^2}$$

$$\alpha_c = \sqrt{\beta^2 - k_c^2 n_c^2}.$$

Matching the tangential component of the magnetic field $H_z$ at $x = d$ and $x = -d$ requires

$$\phi = \arctan(\alpha_c/\kappa_f) - \kappa_f d$$

and yields the following eigenvalue equation

$$\tan(2\kappa_f d) = \frac{\kappa_f (\alpha_c + \alpha_s)}{\kappa_f^2 - \alpha_c \alpha_s}. \tag{6}$$

Allowed $\kappa$ values satisfy the eigenvalue equation, and allowed $\beta$ values are related to $\kappa$ values with the equation above.

The orthogonality relation between two guided modes is given as

$$\int_{-\infty}^{\infty} E_{ym} E_{yn}^* \, dx = \frac{2 \omega \mu_0}{\beta n} P \delta_{mn} \tag{7}$$

where $P$ is the power carried, per unit length of $y$, by the mode in the $z$ direction and $\delta_{mn}$ is the Kronecker delta function. When modes $m$ and $n$ are considered to be the same guided mode, the orthogonality relation requires that the amplitude coefficient $A$ satisfies the following equation:

$$A = \sqrt{\frac{4 \omega \mu_0 P}{\beta (2d + 1/\alpha_c + 1/\alpha_s)}}. \tag{8}$$

B. Substrate Radiation Modes

For $k_c n_f < |\beta| < k_c n_s$, the solutions of the scalar Helmholtz equation show standing wave behavior in the substrate region in addition to the film region. The fields decay exponentially in the cover region as in the case of the guided modes. As the range of allowed $\beta$ values vanishes for $n_c = n_s$, substrate radiation modes are not supported by the symmetric slab waveguides. Matching the tangential field components $E_y$ and $H_x$ at $x = d$ and $x = -d$ yields

$$E_y = \begin{cases} [B \cos(\kappa_f d) + C \sin(\kappa_f d)] \cdot \exp[-\alpha_c(x - d)], & \text{for } x \geq d \\ B \cos(\kappa_f x) + C \sin(\kappa_f x), & \text{for } |x| \leq d \\ [B \cos(\kappa_f d) - C \sin(\kappa_f d)] \cdot \exp[\alpha_s(x + d)], & \text{for } x \leq -d \end{cases} \tag{9}$$

where

$$\kappa_s = \sqrt{\beta^2 - k_s^2 n_s^2}$$

$$\kappa_f = \sqrt{k_c^2 n_f^2 - \beta^2}$$

$$\alpha_c = \sqrt{\beta^2 - k_c^2 n_c^2}$$

$$B = \frac{\kappa_f \cos(\kappa_f d) + \alpha_c \sin(\kappa_f d)}{\kappa_f \sin(\kappa_f d) - \alpha_c \cos(\kappa_f d)} C$$

and

$$H = \frac{\kappa_f}{\kappa_s} C \left[ B \sin(\kappa_f d) + \cos(\kappa_f d) \right].$$

Here, $B$ and $H$ are functions of $C$ and the amplitude coefficient $C$ will be related to the power $P$ by the use of the orthogonality relationship.

Unlike the case of the guided modes, matching the boundary conditions does not result in an eigenvalue equation and there are no other restrictions on the $\beta$ values in the case of the substrate radiation modes. Hence, any $\beta$ value in the given continuum range is acceptable and corresponds to a substrate.
radiation mode. Since the integral in the orthogonality relation for the guided modes does not result in a finite value when the substrate radiation modes are substituted, the orthogonality relation needs to be redefined by the use of Dirac delta function for this case

$$\int_{-\infty}^{\infty} E_y(\beta_m)E^*_y(\beta_n) \, d\beta = \frac{2\omega \mu_0}{\beta_m} P\delta(\beta_m - \beta_n), \quad (10)$$

When the same substrate radiation mode is substituted for both mode $m$ and mode $n$, the orthogonality relation requires that the amplitude coefficient $C$ satisfies (11), shown at the bottom of the page.

**C. True Radiation Modes**

The values of $\beta$ in the range $0 \leq |\beta| \leq k_n a_c$ yield true radiation modes which show standing wave behavior in all three regions. Matching the tangential components at $x = d$ and $x = -d$ results in

$$E_y = \begin{cases} 
F_i [\cos(\kappa_f d) + G_i \sin(\kappa_f d)] 
& \cdot \cos[\kappa_s(x-d)], \\
+F_i [\cos(\kappa_f x) + G_i \sin(\kappa_f x)] 
& \cdot \sin[\kappa_s(x-d)], \\
F_i [\cos(\kappa_f d) - G_i \sin(\kappa_f d)] 
& \cdot \cos[\kappa_s(x+d)], \\
+F_i [\cos(\kappa_f x) - G_i \sin(\kappa_f x)] 
& \cdot \sin[\kappa_s(x+d)], 
\end{cases} \quad (12)$$

where

$$\kappa_s = \sqrt{\kappa_o^2 - \beta^2}$$

$$\kappa_f = \sqrt{\kappa_o^2 - \beta^2}$$

$$\kappa_c = \sqrt{\kappa_o^2 - \beta^2}$$

$$K = F_i \frac{\kappa_f}{\kappa_c} [G_i \cos(\kappa_f d) - \sin(\kappa_f d)]$$

and

$$L = F_i \frac{\kappa_f}{\kappa_s} [\sin(\kappa_f d) + G_i \cos(\kappa_f d)].$$

Like the substrate radiation mode case, matching the boundary conditions does not yield an eigenvalue equation. Therefore, any value of $\beta$ in the continuum range is acceptable and corresponds to a true radiation mode. However, unlike the substrate radiation mode case, matching the boundary conditions does not yield enough equations to solve for all the unknowns. In other words, the coefficient $F_i$ and $G_i$ are still undetermined after matching the boundary conditions. While one can be determined by the use of the orthogonality relation, the other one can be chosen arbitrarily. This arbitrariness is related to the arbitrary phases of the two plane waves generating the radiation modes [3]. Two true radiation modes need to be considered for each $\beta$ value in the continuum to have a complete orthogonal set of modes [5]. The orthogonality requires that the following equation in terms of the coefficients $G_k$ and $G_l$ of the two true radiation modes is satisfied:

$$2 \cos^2(\kappa_f d) + \kappa_f^2 \sin^2(\kappa_f d) \left( \frac{1}{\kappa_o^2} + \frac{1}{\kappa_c^2} \right)$$

$$+ G_k G_l \left[ \frac{2 \sin^2(\kappa_f d) + \kappa_f \cos(\kappa_f d)(1 + \frac{1}{\kappa_c^2})}{\kappa_f^2 \sin(\kappa_f d) \cos(\kappa_f d) \left( \frac{1}{\kappa_o^2} + \frac{1}{\kappa_c^2} \right)} \right] = \Omega. \quad (13)$$

$G_k$ and $G_l$ can be chosen arbitrarily to satisfy this equation. One arbitrary solution to (13) is [5]

$$G_k = -G_l = \sqrt{\frac{2 \cos^2(\kappa_f d) + \kappa_f^2 \sin^2(\kappa_f d) \left( \frac{1}{\kappa_o^2} + \frac{1}{\kappa_c^2} \right)}{2 \sin^2(\kappa_f d) + \kappa_f \cos(\kappa_f d)(1 + \frac{1}{\kappa_c^2})}}. \quad (14)$$

Although the choice of the coefficients $G_k$ and $G_l$ are arbitrary, it should be noted here that this choice does not limit our ability to represent any kind of mode shape satisfying Maxwell’s equations in terms of the orthogonal set of modes chosen.

When the same true radiation mode is plugged into the orthogonality equation (10) as mode $m$ and $n$, the coefficient $F_i$ needs to satisfy the following relation in terms of $G_i$ as shown in (15) at the bottom of the page. Using these relations, all the coefficients of the true radiation modes can be determined.

We have completed the summary of three groups of modes (guided modes, substrate radiation modes and true radiation modes), and stated that any two modes in a given group (e.g., guided modes) are orthogonal to each other. In addition, any mode in any group is orthogonal to every mode in the other two groups. The three groups of modes summarized and the evanescent radiation modes form a complete orthogonal set of modes. Evanescent radiation modes are also described by (12), but they have imaginary values of $\beta$ and correspond to values of $\kappa_s$ in the range $k_o a_c < |\kappa_s| < \infty$. They do not carry power away from the discontinuity but are
important in determining the electromagnetic wave shape around the discontinuity. Inclusion of evanescent radiation modes may affect the results of the analysis through modifying the coefficients of each mode. Imaginary β values and κs approaching infinity complicate the analysis substantially, and therefore, as in previous work, the evanescent radiation modes will be ignored.

III. STEP DISCONTINUITY

It is assumed that a number of TE modes are incident on the step discontinuity shown in Fig. 2. The incident guided modes, the reflected guided modes, and the continuum of backward-traveling radiation modes exist on the left portion of the waveguide, while the transmitted guided modes and the continuum of forward traveling radiation modes exist on the right-hand portion of the waveguide.

The normalized field distributions of the incident guided, transmitted guided, reflected radiation and transmitted radiation modes are \( E_y^{(i)} \), \( E_y^{(t)} \), \( E_y^{(r)}(\kappa_s) \), and \( E_y^{(t)}(\kappa_s) \), respectively. The propagation constants, \( \beta_1, \beta_2, \) and \( \beta(\kappa_s) \), correspond to the guided modes on the left and right, and that of a given radiation mode, respectively.

The unknown magnitude coefficients, \( a_m \), \( c_m \), and \( q_m(\kappa_s) \) can be calculated by requiring that the total tangential components of the electric and magnetic fields, \( E_y \) and \( H_z \), are continuous at the step discontinuity. For the single-mode case, the continuity requirement results in the following equations [4]–[6]:

\[
(1 + a_1)E_y^{(i)} + \int_{0}^{\kappa_{m}n_s} q_m(\kappa_s)E_y^{(r)}(\kappa_s) d\kappa_s = c_1E_y^{(t)} + \int_{0}^{\kappa_{m}n_s} q_m(\kappa_s)E_x^{(t)}(\kappa_s) d\kappa_s \tag{16}
\]

\[
(1 - a_1)\beta_1E_y^{(i)} - \int_{0}^{\kappa_{m}n_s} q_m(\kappa_s)\beta(\kappa_s)E_y^{(r)}(\kappa_s) d\kappa_s = c_1\beta_1E_y^{(t)} + \int_{0}^{\kappa_{m}n_s} q_m(\kappa_s)\beta(\kappa_s)E_x^{(t)}(\kappa_s) d\kappa_s \tag{17}
\]

In the multimode case, the following equations should be employed to accommodate the additional guided modes on both waveguide sections. It is assumed that \( M \) modes exist on the left section and \( L \) modes exist on the right section without introducing any loss of generality:

\[
\sum_{m=1}^{M} (1 + a_m)E_y^{(i)} + \int_{0}^{\kappa_{m}n_s} q_m(\kappa_s)E_y^{(r)}(\kappa_s) d\kappa_s = \sum_{l=1}^{L} c_lE_y^{(t)} + \int_{0}^{\kappa_{l}n_s} q_l(\kappa_s)E_x^{(t)}(\kappa_s) d\kappa_s \tag{18}
\]

\[
\sum_{m=1}^{M} (1 - a_m)\beta_1E_y^{(i)} - \int_{0}^{\kappa_{m}n_s} q_m(\kappa_s)\beta(\kappa_s)E_y^{(r)}(\kappa_s) d\kappa_s = \sum_{l=1}^{L} c_l\beta_1E_y^{(t)} + \int_{0}^{\kappa_{l}n_s} q_l(\kappa_s)\beta(\kappa_s)E_x^{(t)}(\kappa_s) d\kappa_s \tag{19}
\]

To determine the unknown amplitude coefficients \( a_m \) for \( m = 1 \) to \( M \), \( c_l \) for \( l = 1 \) to \( L \), and the amplitude functions \( q_m(\kappa_s) \) and \( q_l(\kappa_s) \) for the range \( \kappa_s = 0 \) to \( \kappa_s = k_0n_s \), these two sets of equations have to be solved. Since an exact analytical solution is not possible, it is necessary to make some approximations to solve the sets of equations given. In the exact numerical method, it is chosen to discretize the problem instead of making some physical approximations [6].

Suchoski and Ramaswamy [6] used the exact numerical method to calculate the power loss at a step discontinuity for the single-mode case. We apply the method to the multimode case. In the method, to obtain the linearly independent equations necessary to determine the unknown coefficients, the continuity equations, (18) and (19), are multiplied with different modes and integrated over \( x \) from \( -\infty \) to \( +\infty \). After the equations are simplified using the orthogonality relations, the integrals over \( \kappa_s \) are evaluated numerically by using Simpson’s Rule.

First, we will multiply both sides of (18) by each incident guided mode and by \( N - 1 \) reflected radiation modes corresponding to the discretized values of \( \kappa_s(\kappa_s = k_0n_s/N, 2k_0n_s/N, \cdots, (N - 1)k_0n_s/N) \). It will be clear later why the two end values of \( \kappa_s \) are not used.

By multiplying both sides of (18) by \( E_y^{(i)} \) for \( k = 1, 2, \cdots, M \) and integrating over \( x \) from \( -\infty \) to \( +\infty \) and considering the orthogonality relationships \( \int_{-\infty}^{\infty} E_y^{(i)}E_y^{(t)} dx = 0 \) for \( k = 1, 2, \cdots, M \), the following equation set can be obtained:

\[
\sum_{l=1}^{L} c_l \int_{-\infty}^{\infty} E_y^{(i)}E_y^{(t)} dx + \int_{0}^{\kappa_{m}n_s} q_m(\kappa_s) \int_{-\infty}^{\infty} E_y^{(i)}E_y^{(t)}(\kappa_s) dx d\kappa_s \tag{20}
\]

for \( k = 1, 2, \cdots, M \).
Multiplying (18) by $E_y^{(p)}(k_{nj})$ for $j = 1, 2, \cdots, N - 1$ and integrating over $x$ from $\infty$ to $\infty$, and considering the orthogonality relationships \[ \int_{-\infty}^{\infty} E_y^{(p)}(k_{ij}) E_y^{(p)}(k_{nj}) \, dx = (2\omega \mu_0/\beta_{k_{ij}}) \delta(k_{ij} - k_{nj}) \] and \[ \int_{-\infty}^{\infty} E_y^{(r)}(k_{ij}) E_y^{(r)}(k_{nj}) \, dx = 0 \] for $j = 1, 2, \cdots, N - 1$ yields the following equation set:

\[ q_r(k_{sj}) \frac{2\omega \mu_0}{\beta(k_{sj})} = \sum_{i=1}^{L} a_{ij} \int_{-\infty}^{\infty} E_{yj}^{(i)} E_y^{(r)}(k_{sj}) \, dx + \int_{0}^{k_{ij} n_s} q_r(k_{sj}) \int_{0}^{k_{ij} n_s} \left[ \int_{-\infty}^{\infty} E_y^{(r)}(k_{ij}) E_y^{(r)}(k_{nj}) \, dx \right] \, dk_{sj} \tag{21} \]

for $j = 1, 2, \cdots, N - 1$.

Next, we will multiply both sides of (19) by each transmitted guided mode and by $N - 1$ forward radiation modes corresponding to discretized values of $k_{s}$ ($k_{s} = k_{0} n_s/\mathcal{N}, 2k_{0} n_s/\mathcal{N}, \cdots, (N-1)k_{0} n_s/\mathcal{N}$).

Multiplying both sides of (19) by $E_y^{(t)}$ for $p = 1, 2, \cdots, L$ and integrating over $x$ from $\infty$ to $\infty$, and considering the orthogonality relationships \[ \int_{-\infty}^{\infty} E_y^{(t)}(k_{ij}) E_y^{(t)}(k_{nj}) \, dx = (2\omega \mu_0/\beta_{p}) \delta(p - p) \] for $p = 1, 2, \cdots, L$ and \[ \int_{-\infty}^{\infty} E_y^{(r)}(k_{ij}) E_y^{(r)}(k_{nj}) \, dx = 0 \] for $p = 1, 2, \cdots, L$, the following equation set can be obtained:

\[ \sum_{m=1}^{M} (1 - a_{rm}) \beta_{2m} \int_{-\infty}^{\infty} E_{ym}^{(i)} E_y^{(t)} \, dx - \int_{0}^{k_{0} n_s} q_r(k_{sj}) \int_{0}^{k_{0} n_s} \left[ \int_{-\infty}^{\infty} E_y^{(r)}(k_{ij}) E_y^{(r)}(k_{nj}) \, dx \right] \, dk_{sj} = 2\omega \mu_0 \epsilon_{tp} \tag{22} \]

for $p = 1, 2, \cdots, L$.

Finally, multiplying (19) by $E_y^{(t)}(k_{sj})$ for $j = 1, 2, \cdots, N - 1$ and integrating over $x$ from $\infty$ to $\infty$, and considering the orthogonality relationships \[ \int_{-\infty}^{\infty} E_y^{(r)}(k_{ij}) E_y^{(t)}(k_{sj}) \, dx = (2\omega \mu_0/\beta_{k_{ij}}) \delta(k_{ij} - k_{sj}) \] and \[ \int_{-\infty}^{\infty} E_y^{(r)}(k_{ij}) E_y^{(r)}(k_{nj}) \, dx = 0 \] for $j = 1, 2, \cdots, N - 1$ yields

\[ \sum_{m=1}^{M} (1 - a_{rm}) \beta_{2m} \int_{-\infty}^{\infty} E_{ym}^{(i)} E_y^{(t)}(k_{sj}) \, dx + \int_{0}^{k_{0} n_s} q_r(k_{sj}) \beta(k_{sj}) \int_{0}^{k_{0} n_s} \left[ \int_{-\infty}^{\infty} E_y^{(r)}(k_{ij}) E_y^{(t)}(k_{sj}) \, dx \right] \, dk_{sj} = 2\omega \mu_0 \epsilon_{tp} \tag{23} \]

for $j = 1, 2, \cdots, N - 1$.

Now, the integrals over $k_{sj}$ in (20)–(23) will be evaluated using Simpson’s rule with $N$ panels. Considering that no radiation mode exists for $k_{sj} = 0$ and that no power can be coupled to a radiation mode traveling perpendicular to the slab waveguide yields $q_r(0) = q_r(k_{0} n_s) = q_r(0) = q_r(k_{0} n_s) = 0$ [6]. Then, applying Simpson’s rule approximation to (20)–(23) yields (24)–(27)

\[ \frac{2\omega \mu_0}{\beta_{k_{ij}}} (1 + a_{ij}) \]

\[ = \sum_{i=1}^{L} a_{ij} \int_{-\infty}^{\infty} E_{yj}^{(i)} E_y^{(t)} \, dx + \int_{0}^{k_{0} n_s} q_r(k_{sj}) \int_{0}^{k_{0} n_s} \left[ \int_{-\infty}^{\infty} E_y^{(r)}(k_{ij}) E_y^{(t)}(k_{sj}) \, dx \right] \, dk_{sj} + \frac{2\Delta k_{s}}{3} \sum_{i=2, i\text{ even}}^{N-1} q_r(k_{sj}) \int_{0}^{k_{0} n_s} \left[ \int_{-\infty}^{\infty} E_y^{(r)}(k_{ij}) E_y^{(t)}(k_{sj}) \, dx \right] \, dk_{sj} \tag{24} \]

for $k = 1, 2, \cdots, M$

\[ q_r(k_{sj}) \frac{2\omega \mu_0}{\beta(k_{sj})} \]

\[ = \sum_{i=1}^{L} a_{ij} \int_{-\infty}^{\infty} E_{yj}^{(i)} E_y^{(r)}(k_{sj}) \, dx + \int_{0}^{k_{0} n_s} q_r(k_{sj}) \int_{0}^{k_{0} n_s} \left[ \int_{-\infty}^{\infty} E_y^{(r)}(k_{ij}) E_y^{(t)}(k_{sj}) \, dx \right] \, dk_{sj} + \frac{2\Delta k_{s}}{3} \sum_{i=2, i\text{ even}}^{N-1} q_r(k_{sj}) \int_{0}^{k_{0} n_s} \left[ \int_{-\infty}^{\infty} E_y^{(r)}(k_{ij}) E_y^{(r)}(k_{sj}) \, dx \right] \, dk_{sj} \tag{25} \]

for $j = 1, 2, \cdots, N - 1$

\[ \sum_{m=1}^{M} (1 - a_{rm}) \beta_{2m} \int_{-\infty}^{\infty} E_{ym}^{(i)} E_y^{(t)} \, dx + \frac{4\Delta k_{s}}{3} \sum_{i=1}^{N-1} q_r(k_{sj}) \int_{0}^{k_{0} n_s} \left[ \int_{-\infty}^{\infty} E_y^{(r)}(k_{ij}) E_y^{(t)}(k_{sj}) \, dx \right] \, dk_{sj} \tag{26} \]

for $p = 1, 2, \cdots, L$

\[ \sum_{m=1}^{M} (1 - a_{rm}) \beta_{2m} \int_{-\infty}^{\infty} E_{ym}^{(i)} E_y^{(t)}(k_{sj}) \, dx + \frac{4\Delta k_{s}}{3} \sum_{i=1}^{N-2} q_r(k_{sj}) \int_{0}^{k_{0} n_s} \left[ \int_{-\infty}^{\infty} E_y^{(r)}(k_{ij}) E_y^{(t)}(k_{sj}) \, dx \right] \, dk_{sj} \tag{27} \]

for $j = 1, 2, \cdots, N - 1$.

In (24)–(27), $\Delta k_{s} = k_{0} n_s/N$. Closed forms of integrals in (24)–(27) can be obtained by substituting the corresponding
mode shapes for the modes in the integrals. We now have
$M+L+2N-2$ linearly independent equations in terms of
the unknown amplitude coefficients. Since $q_0(0) = q_0(k_0n_a) = q_0(k_0n_b) = 0$, we only need to calculate $M+L+2N-2$ coefficients. Hence, $M+L+2N-2$ equations generated
are enough to calculate the unknown amplitude coefficients.

These equations can be put in the form

$$[A]x = b$$

(28)

where $A$ is an $(M+L+2N-2)$ by $(M+L+2N-2)$ matrix, $b$
is the column vector of constants, and $x$ is the column vector
of unknown amplitude coefficients. Once the matrix system
is solved and the amplitude coefficients are calculated, the
amount of power coupled to each reflected and transmitted
guided mode and reflected and transmitted radiation power
can be determined.

IV. NUMERICAL EVALUATION METHOD

In order to calculate the power loss at a step discontinuity
of a multimode waveguide with the method discussed, first,
the closed forms of the overlap integrals in (24)–(27) have
to be obtained by using the field expressions in (5), (9), and
(12). Two different closed form expressions are needed for
each overlap integral. One for the case $d_1 > d_2$, and one for
the case $d_2 > d_1$. Then, the closed form integral expressions
are substituted into the linear equations forming the
matrix equation. Matlab was utilized to solve the matrix equation

$$[A]x = b.$$ 

Once the matrix equation is solved, the amplitude
coefficients for each transmitted and reflected guided mode
and the coefficients at discrete points for reflected and transmitted
radiation modes are known. The power in each reflected and
guided mode and the total reflected and radiation mode power
can be calculated by using these coefficients.

V. RESULTS AND CONCLUSIONS

Figs. 3–5 illustrate the results obtained by using the exact
numerical technique. The results are produced for the case of
symmetrical slab waveguide sections on both sides of the step
discontinuity. The values $n_f = 1.475$ and $n_e = n_s = 1.400$
will be used. This corresponds to an $N_A$ of 0.20 and is
representative of a typical multimode fiber. We will use a core
diameter of $50 \pm 3 \mu m$. Two different wavelength values,$\lambda_1 = 1\mu m$ and $\lambda_2 = 1.55\mu m$, will be used.
In Figs. 3 and 4, the total reflected guided mode power, the total reflected and forward radiation mode power, and total power loss versus $d_1$ are plotted. This represents a multimode fiber ($d_2$) coupled to a multimode waveguide ($d_3$). Power is flowing from the fiber to the waveguide. We are assuming that the dimensions of the multimode waveguide are well controlled because of the lithographic techniques used to fabricate them. Note, however, this is not always guaranteed and is a strong function of the processing techniques used to fabricate the waveguides. For these calculations, we will assume the waveguide dimensions are fixed ($d_2$) and look at the power loss associated with a variation in $d_1$—consistent with typical manufacturing tolerances for multimode fiber. All the power components are normalized to total incident power which is the sum of the incident guided mode powers. Waveguide width $2d_2$ is held constant at 50 $\mu$m. The total power lost is regarded as the sum of the total reflected guided mode power and the total reflected and transmitted radiation mode powers. Fig. 3 is for $\lambda_1 = 1 \mu$m while Fig. 4 is for $\lambda_2 = 1.55 \mu$m. While 21 guided modes exist in the right waveguide section for the data used for Fig. 3, the right portion of the waveguide can only support 14 guided modes for the case considered for Fig. 4. The number of the guided modes in the left waveguide sections depends on $d_1$ varying.

Waveguide width $2d_2$ is varied in Fig. 5, while the waveguide width $2d_1$ is held constant at 50 $\mu$m, and $\lambda = 1.55 \mu$m is used. This case is essentially the opposite of that covered in Fig. 4. Here, the power is flowing from the multimode waveguide ($d_1$) into the multimode fiber ($d_2$). Again we consider the power loss associated with a variation in fiber core size.

Several conclusions can be drawn from the results illustrated in the figures. As in the single-mode case, the forward radiation is the largest portion of the total power loss. It can be stated that the power carried away from the step discontinuity by the forward radiation modes is much higher than the power both in the reflected guided modes and the reflected radiation modes regardless of how large the step discontinuity is by observing the actual data. When the number of the modes in both waveguide portions are equal or the number of the modes in the left portion is less, the total power loss is reasonably small. However, when the number of the modes in the left portion of the waveguide is higher than the number of the modes in the right portion of the waveguide, the power loss increases rapidly with the increasing waveguide width $2d_1$. It can also be concluded that the power loss is a strong function of the operating wavelength. Note the dramatic increase in power loss for $\lambda = 1 \mu$m (Fig. 3) as compared to $\lambda = 1.55 \mu$m (Fig. 4).

In closing, we have shown an exact numerical technique to analyze a step discontinuity in a multimode optical slab waveguide. The method used can be applied to any kind of step discontinuity regardless of the geometry mismatch, the number of the modes in both portions of the waveguide and the symmetry conditions. Furthermore, while demonstrated for slab waveguides, the same technique can be applied to three-dimensional waveguides/fibers if appropriate means are used to obtain a complete set of modes.

REFERENCES


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