Lect.: 11  Title: Conservation of Mass

Class L.O.:

1. Able to explain in words and mathematically the conservation of mass.
2. Able to use conservation of mass to account for all mass flows crossing a control surface.

Class M.Map:

QUESTION? – How do you handle a situation where the mass inside the control volume is changing.

Topic 1:  Control Volume and Surfaces in Review

Review:

- CV = is the volume of interest. (period)
- The control surface (CS) is closed and forms the control volume.
  - Arbitrary
  - Fixed or moving
  - Rigid or deforming

- Open Control Volumes
  - Mass flows across the control surface
  - With mass comes energy -- naturally

⇒ General Diagram, Piston-Cylinder, Steam Generator, Gen. Mass Cons.
QUESTION? How do write down the conservation of mass?

Topic 2: Conservation of Mass - Expressions

➔ In words

Mass inside a control volume changes with time when we add some mass and / or we take some out.

➔ In math

\[
\frac{dm_{C.V.}}{dt} = \sum \dot{m}_i - \sum \dot{m}_e
\]

Rate of Change = + in - out

or

\[
\frac{dm_{C.V.}}{dt} - \sum \dot{m}_i + \sum \dot{m}_e = 0
\]

This is also called the “continuity equation”.

Consider the case of different components or different states of matter

\[
m_{C.V.} = \int \rho dV = \int (1 / \nu) dV = m_A + m_B + m_C + ...
\]
Diving in closer to the control surface

**Conservation of Mass in Pipe**

\[ \dot{V} = \int \vec{V} \cdot \vec{n} \, dA = \int V_{local} \cdot \cos \theta \, dA \]

\[ \dot{V} = V_{avg} \cdot A \]

\[ \dot{m} = \rho_{avg} \dot{V} = \dot{V} / \nu = \int \rho \vec{V} \cdot \vec{n} \, dA = \int \rho V_{local} \, dA \]

\[ \dot{m} = \rho V_{avg} A \]

**#1 Wind Energy**

A windmill takes a fraction of the wind kinetic energy out as power on a shaft. In what manner does the temperature and wind velocity influence the power? Hint: write the power as mass flow rate times specific work.

The work as a fraction \( f \) of the flow of kinetic energy becomes

\[ \dot{W} = \dot{m}w = \dot{m} \frac{1}{2} v_{in}^2 = \rho A v_{in} \frac{1}{2} v_{in}^2 \]

so the power is proportional to the velocity cubed. The temperature enters through the density, so assuming air as ideal gas

\[ \rho = 1/\nu = P/RT \]

and the power is inversely proportional to temperature.

**#2 Tank Mass after elapsed time – Constant Flows**
\[
\frac{dm_{CV}}{dt} = \dot{m}_1 + \dot{m}_2 - \dot{m}_3
\]

\[
\frac{dm_{CV}}{dt} = [0.8 + 1.3 - 2.6] \frac{kg}{s}
\]

\[
\int_{0}^{3600} \frac{dm_{CV}}{dt} dt = [m_{CV}|_{final} - m_{CV}|_{initial}] kg
\]

\[
m_{CV}|_{final} = \int_{0}^{3600} \frac{dm_{CV}}{dt} dt + m_{CV}|_{initial}
\]

\[
m_{CV}|_{final} = [-0.5(3600) - (-0.5)(0)] + 3000
\]

\[
m_{CV}|_{final} = -1800 + 3000 = 1200 kg
\]

or simply

\[
m_{cv,final} = 3000 kg - 0.5 \frac{kg}{s} \times 3600 s = 1200 kg
\]

#3 Tank Mass after elapsed time – Changing CV mass

Water flows into a tank at a constant flow rate of 14 kg/s.

Water exits the tank with a mass flow rate proportional to the height of liquid inside: \( m_{dot\_e} = 15 \text{ L} \), where L is the instantaneous liquid height.

The base of the tank is 0.3 m\(^2\) (A) and the density of water 1000 kg/m\(^3\).

If the tank is initially empty – plot water height vs time.
\[
\frac{dm_{cv}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_e \\
\dot{m}_e = 15L(t) \\
\dot{m}_{in} = 14 \\
\frac{dm_{cv}}{dt} = 14 - 15L \\
\frac{d(\rho LA)}{dt} = 14 - 15L \\
\frac{dL}{dt} + \frac{15}{\rho A} L = \frac{14}{\rho A} \\
\tau = \frac{15}{\rho A} \\
\frac{dL}{dt} + \tau L = \frac{14}{15} \\
e^{\tau t} \frac{dL}{dt} + e^{\tau t} \tau L = e^{\tau t} \frac{14}{15} \tau \\
\frac{d}{dt} \left( e^{\tau t} L \right) = e^{\tau t} \frac{14}{15} \tau \\
\int_0^t \frac{d}{dt} \left( e^{\tau t} L \right) dt = \int_0^t e^{\tau t} \frac{14}{15} \tau dt \\
e^{\tau t} L = \frac{14}{15} \tau e^{\tau t} + C \\
L = \frac{14}{15} \frac{C}{e^{\tau t}} \\
L = \frac{14}{15} + Ce^{-\tau t} \\
at t = 0, L = 0 \rightarrow C = -\left(\frac{14}{15}\right) \\
L = \frac{14}{15} \left(1 - e^{-\tau t}\right)
\]

Watch your units.

14 = units = kg/s
15 = units = kg/(s-m)

\( \tau \) = time constant with units of 1/s

\( \Rightarrow \) Introduce an integrating factor

\( \Rightarrow \) integral form used to find function of L(t) at ANY time.

A review of these steps found here

http://en.wikibooks.org/wiki/Differential_Equations/First_Order_Linear_1
QUESTION? Is mass conserved when energy levels are changed?

Topic 3: Mass – Energy Physics (Sect. 5.9)

If we return to physics does the mass change if the energy content changes?

\[ E = mc^2 \]

\[ m = \frac{E}{c^2} \]

Consider rigid vessel post combustion

1kg of Gasoline /Air mix after Combustion must be cooled to return to original state

\[ Q_1^2 = U_2 - U_1 + W_1 \]

\[ Q_1^2 = U_2 - U_1 = -2900kJ \]

\[ dm = \frac{dE}{c^2} = \frac{dU}{c^2} = \frac{-2900e3}{2.9979e8} = -3.23e-11kg \]

Conclusions:

- Mass change associated with energy changes is too small to detect by any standard means
- We can treat conservation of mass and conservation of energy as two separate laws without significant error with processes that do not involve nuclear reactions.

Home Work
Please see the WIP.

http://en.wikipedia.org/wiki/Mass-energy_equivalence