# Analysis of Recoil Compressive Failure in High Performance Polymers Using Two- and Four-Parameter Weibull Models

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*Abstract:* In this study, large amounts of recoil compressive failure data were gathered for Kevlar-29 fibers. Once the dependence of failure frequency on stress level had been determined experimentally, two-parameter and four-parameter Weibull models were used to represent the data. An analysis of these results shows that each model represents the failure well, but that the four-parameter model offers little improvement over the two-parameter Weibull model. Deconstruction of the four-parameter model indicated that a single failure mechanism dominated and that this mechanism could be well represented by the two-parameter Weibull model. These results imply that while recoil compressive failure is more complicated than tensile failure, recoil failure is dominated by a single flaw distribution and can be represented by a more simple model than tensile failure.

*Key Words:* Weibull, recoil failure, compression

## 1. INTRODUCTION

The applicability of high performance polymers in structural applications is limited by their comparatively poor compressive strengths. The nearly perfect axial orientation formed during the dry-jet wet-spinning of high performance polymers provides the fibers with exceptional tensile strength [1], but also dramatically reduces the ability of the fibers to distribute a compressive load [2]. Kevlar (poly p-phenylene terephthalamide (PPTA)) fiber-reinforced composites typically possess compressive strengths more than five times less than their tensile strengths [3–5]. The repeat unit of Kevlar is shown in figure 1.

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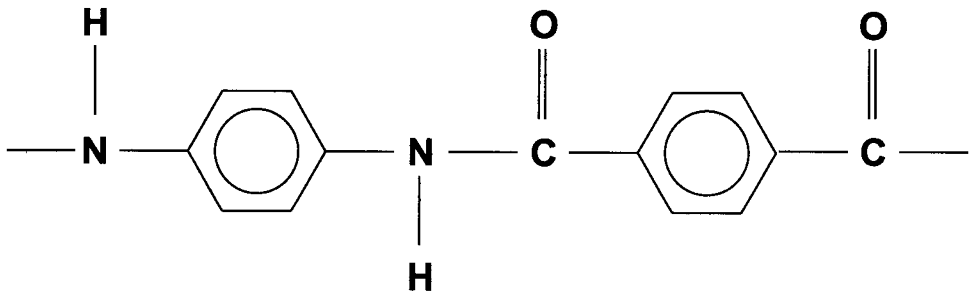

Figure 1. The repeat unit of Kevlar.

McGarry and Moalli [6] analyzed the compressive failure of PPTA. Using scanning electron microscopy, they found that the primary mode of compressive failure in poly p-phenylene benzobisoxazole (PBO) and PPTA fibers is the 'kink band' phenomenon. This failure mode involves initiation of a fiber kink defect at the fiber surface. From the surface, the kink band defect propagates across the cross section of the polymer fiber, causing failure.

Because compressive failure in high performance fiber-reinforced composites results from the low compressive strengths of the fibers [7–10], accurate and simple measurement of compressive failure data in fibers is essential.

Compressive testing of composites is well established by the American Society for Testing and Materials (ASTM) [11], but these tests involve many steps, large quantities of fiber, and are influenced by the quality of bonding between the fiber and matrix. Therefore, it would be advantageous to directly measure the compressive strengths of the fibers themselves. For polymeric fibers, the fiber compressive strengths estimated from composite compression tests agree most accurately with those obtained by the recoil test [12].

In the recoil test, fibers are placed under a static tensile load. Next, the fiber is ruptured using either an electric discharge or surgical scissors. The stored energy travels down both halves of the fiber in the form of a recoil compressive wave. The two fiber halves either survive this recoil wave or they experience a secondary failure. Figure 2 depicts recoil compressive failures and survivals. This analysis makes several assumptions [13] about the fiber. Specifically, the fiber obeys Hook's law for linearly elastic materials, is rigidly clamped at each end, has no initial velocity, and has a uniform initial stress along its length at failure. From these assumptions, the axial stress history following tensile failure can be represented by the solution of the Fourier series shown in equation (1)

$$\frac{\sigma(x,t)}{\sigma_n} = \sum_{m=0}^{\infty} \frac{4}{(2m+1)\pi} \sin\frac{(2m+1)\pi}{2} \cos\frac{(2m+1)\pi x}{2L} \cos\frac{(2m+1)(E/\rho)^{0.5} t}{2L} \tag{1}$$

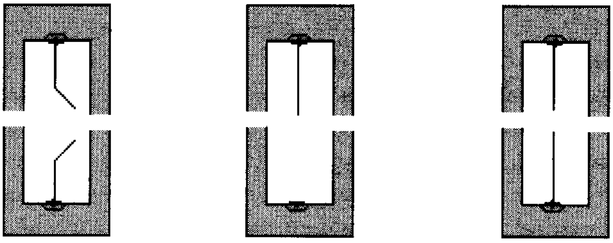


Figure 2. Potential results of the single filament recoil (recoil compressive) test.

where $x$ is the fiber coordinate from a clamped end, $\sigma/\sigma_n$ is the normalized stress (stress divided by maximum stress tested), $L$ is the length of the broken fiber end, $m$ is an integer between zero and infinity, $t$ is time, $E$ is Young's modulus, and $\rho$ is density.

Each test results in two data points (the top and bottom halves of the original fiber which either survive or fail during the recoil). Unlike tensile testing, recoil compressive tests do not provide the exact compressive strength of a filament. Because failure is artificially induced, the resulting recoil compressive wave may be significantly above or significantly below the intrinsic compressive strength of the fiber. If the fiber ends experience a secondary failure, there is no way to ascertain how much stress above the intrinsic compressive strength the recoil wave imparted on the fiber. Similarly, there is no way to know how much the intrinsic compressive strength of the fiber exceeded the stress imposed by the recoil compressive wave that resulted in two survivals.

Several methods have been proposed to evaluate recoil test data. In the first method used by Allen [13], Wang et al. [14], and Crasto and Kumar [15], the data are arranged in order of ascending load levels. A stress range is identified over which the data change from all survivals to all failures. The mean of the two endpoints of this range is presented as the average recoil compressive strength of the fiber set. In essence, this approach disregards the majority of data points and characterizes the compressive properties of the entire fiber batch based on two points. Additionally, the next fiber tested could always radically change the estimated mean strength if it were to become the highest survivor or lowest failure. Newell and Gustafson [16] proposed a statistical interpretation in which a moving average was used to minimize data loss and the impact of outliers.

Hayes [17] proposed the use of a Weibull model to represent the recoil compressive failure data. This is consistent with the work of Creighton and Sutcliffe [18] who used the Weibull distribution to model the waviness of carbon fibers in composites and to relate this waviness to the compressive strengths of the fibers. It is also consistent with the work of Creasy [19] who used a Weibull distribution to model the compressive failure of high performance polymer composites.

Although these efforts represent significant progress in the interpretation of recoil compressive failure data, they are not complete. Many researchers have shown that a two-parameter Weibull model cannot fully capture the complexities of tensile failure [20, 21]. Because the mechanism of recoil compressive failure is more complex than tensile failure, it is possible that a more rigorous model will be required for recoil compressive failure as well.

## 2. THEORETICAL DETAILS

A polymer fiber may be viewed as a chain of interlocking links, with kink bands acting as "weak links" in the chain [22]. Like any other chain, the fiber will fail at its weakest link. Failure data do not conform well to rigid statistical distributions. Thus, a model that could enable the shape of the distribution to be altered by the data itself is required. The Weibull distribution can adopt a variety of shapes, as shown in figure 3, and is the preferred model for considering failure data [23, 24]. If a simple Weibull distribution is applied to the failure data, the Weibull cumulative distribution function takes the form

$$F(\sigma)_i = 1 - \exp\left(-\frac{\sigma}{\sigma_0}\right)^m \tag{2}$$

where $F_i$ is the probability of any link failing at or below the applied stress level $(\sigma)$, $\sigma_0$ is a scale factor that corresponds to the severity of flaws, and $m$ is the shape factor that relates to the distribution of kinks within the fiber. Because a link must either survive or fail, the probability for survival $(S)$ of a link $(i)$ is given by

$$S_i = 1 - F_i. \tag{3}$$

Substituting equation (2) into equation (3) provides

$$S_i = \exp\left(-\left(\frac{\sigma}{\sigma_0}\right)^m\right). \tag{4}$$

The fiber will fail if any individual link fails, thus the probability of a fiber of gauge length $(L)$ surviving is the product of the survival probabilities $(S_i)$ for each link in that length of fiber:

$$S = \prod_{i=1}^{L} S_i. \tag{5}$$

If each link is assumed to survive or fail independently of its neighbors, equation (5) can be rewritten as

$$S = (S_i)^L. \tag{6}$$

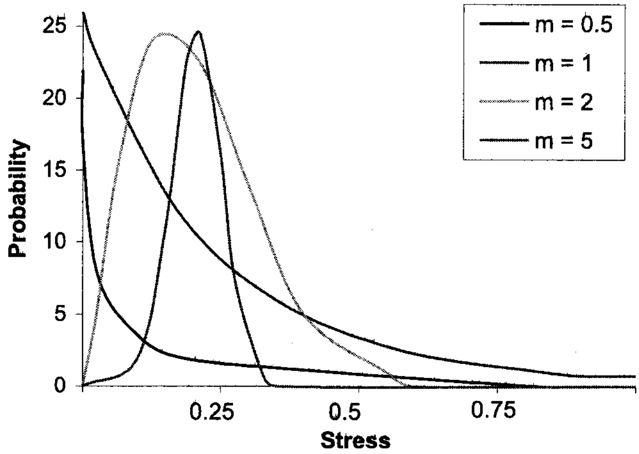


Figure 3. Various possible shapes of the two-parameter Weibull distribution.

Substituting equation (3) into equation (5) provides

$$S = \exp\left(-L\left(\frac{\sigma}{\sigma_0}\right)^m\right) \tag{7}$$

and, correspondingly,

$$F(\sigma) = 1 - \exp\left(-L\left(\frac{\sigma}{\sigma_0}\right)^m\right) \tag{8}$$

where $L$ is the length of fiber being tested. The Weibull probability density function may be obtained by differentiating $F$ with respect to $\sigma$:

$$f = \frac{dF}{d\sigma} = \frac{Lm\sigma^{m-1}}{\sigma_0^m}\exp\left[-L\left(\frac{\sigma}{\sigma_0}\right)^m\right]. \tag{9}$$

The mean stress value can be obtained from

$$\bar{\sigma} = \int_0^\infty \sigma f(\sigma)\, d\sigma. \tag{10}$$

Substituting equation (9) into equation (10) provides

$$\bar{\sigma} = \sigma_0 \Gamma \left(1 + \frac{1}{m}\right) \tag{11}$$

where $\Gamma$ is the statistical gamma function.

If kink band formation alone essentially causes all of the failures, this model should provide an accurate representation of the recoil failure data. However, if another mechanism (for example, sheer failure, testing artifacts, etc) contributed to the failure, this secondary mechanism would not be accounted for by the two-parameter Weibull model. Stoner et al. [20] proposed four-parameter Weibull models that could account for these additional mechanisms in tensile failure. If a second independent distribution exists, failures from that distribution can be represented as

$$S_2 = \exp\left(-L\left(\frac{\sigma}{\sigma_{02}}\right)^{m_2}\right). \tag{12}$$

Because the fiber must survive both distributions, the overall survival becomes

$$S = \exp\left(-L\left(\frac{\sigma}{\sigma_{01}}\right)^{m_1} - L\left(\frac{\sigma}{\sigma_{02}}\right)^{m_2}\right) \tag{13}$$

while the probability of failure becomes

$$F(\sigma) = 1 - \exp\left(-L\left(\frac{\sigma}{\sigma_{01}}\right)^{m_1} - L\left(\frac{\sigma}{\sigma_{02}}\right)^{m_2}\right). \tag{14}$$

Equation (14) can be used to evaluate the four empirical Weibull parameters ($m_1$, $m_2$, $\sigma_{01}$, $\sigma_{02}$). However, this evaluation is more complex than it was for the simple Weibull model because the equation cannot be linearized.

The parameters in these non-linear equations were evaluated using a maximum likelihood estimation. Essentially, the maximum likelihood theory evaluates the probability that a given set of tensile strengths would result from a specific set of Weibull parameters. The observed failure distributions and equation (14) are provided, while the parameters are systematically varied to maximize the likelihood that the observed failure distribution results from the given parameters. Derivations of the equations used in maximum likelihood evaluation can be found in the previously mentioned work of Stoner et al. [20] or in the book by Everitt and Hand [25].

## 3. EXPERIMENTAL DETAILS

Fiber samples of Kevlar-29 were mounted on paper tensile testing tabs and secured with Epoxy 220. Fiber diameters were determined by laser diffraction. The tab was then secured in the locking grips of a modified Instron Ultimate Testing Machine. After a static tensile load was applied to the fiber, the fiber was cut in the center using sharpened surgical scissors. The filament pieces were examined under a magnifying lens for classification as survivals (1) or failures (0). A total of 1253 Kevlar-29 fibers were tested at stress levels

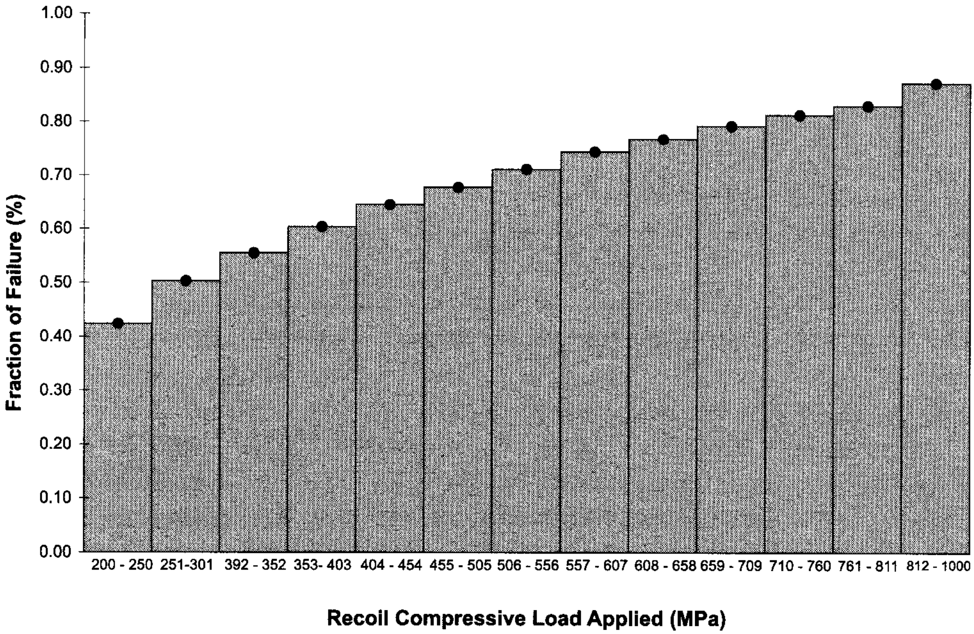


Figure 4. Histogram of recoil failure data.

ranging from 200 to 1000 MPa to provide a rigorous experimental determination of the frequency of failure $(F(\sigma))$.

## 4. RESULTS AND DISCUSSION

The failure data were classified into thirteen bins, as shown in figure 4. For example, if 100 fibers are tested in the range of loads accounted for by a given bin and 42 of these fibers fail, the failure fraction $F(\sigma)$ would be 0.42. The midpoint of each of these bins was used to provide a frequency of failure at thirteen discrete points. The extreme number of data points was used to provide a thorough characterization of the failure function.

Once experimental values for $F(\sigma)$ were obtained, maximum likelihood theory provided optimized fits for the two-parameter and four-parameter Weibull models. The optimal parameters are summarized in table 1. Figure 5 shows the two-parameter and four-parameter Weibull model predictions compared with the experimental failure data. These results indicate that both the two-parameter and four-parameter models provide reasonable estimates of the failure curve. It is also apparent that the four-parameter model offers little improvement over the two-parameter model. This result implies that a single distribution is adequate to characterize the recoil compressive failure and that a single distribution of flaws is responsible for that failure. Therefore, even though recoil compressive failure is

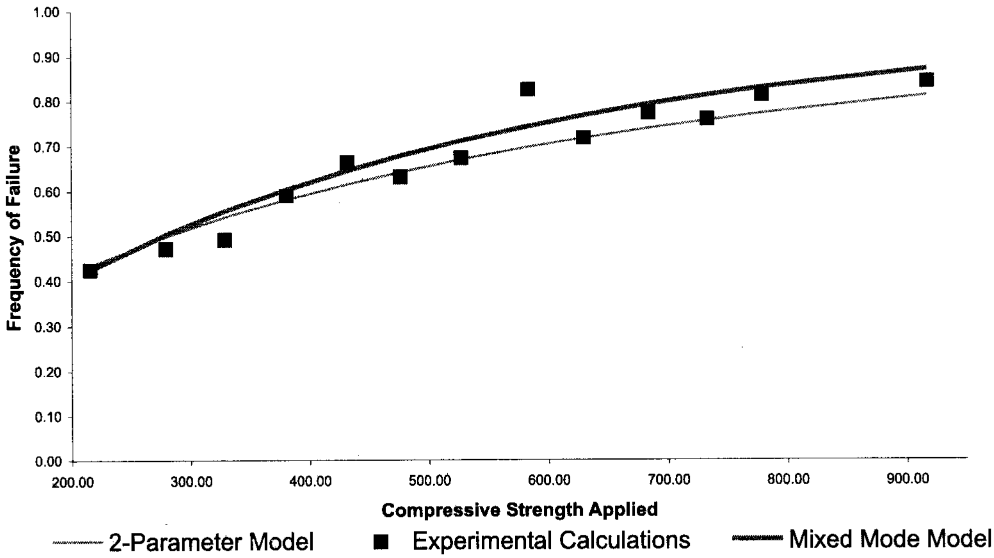


Figure 5. Comparison of two-parameter and four-parameter Weibull models.

Table 1. Weibull parameters from maximum likelihood regression.

| | $m_1$ | $\sigma_{01}$ (GPa) | $m_2$ | $\sigma_{01}$ (GPa) |
|---|---|---|---|---|
| Two-parameter model | 0.7428 | 0.1445 | — | — |
| Four-parameter model | 0.9083 | 1.3096 | 0.9018 | 1.087 |

conceptually more complicated than tensile failure, it can be represented by a more simple model for the high performance polymer system studied in this paper.

The four-parameter Weibull model allows for the relative contributions of end effects and flaws to be separated from the overall failure data. Because each set of Weibull parameters represents an independent flaw distribution, the failures associated with each distribution can be separated. Failures from the first distribution are reflected by $m_1$ and $\sigma_{01}$ alone. Similarly, failures caused by the second distribution are represented by $m_2$ and $\sigma_{02}$ alone. Figure 6 represents the deconstructed four-parameter model and indicates that the two supposedly independent failure mechanisms apparently follow the same distribution. This implies that a single failure mode dominates and can be thoroughly characterized by the two-parameter Weibull model. Finally, table 2 shows that the estimated means for the Kevlar-29 fibers tested in this study varied by only 40 MPa. In each case, the estimated mean fell within the range of published values (200–400 MPa) for the recoil compressive strength Kevlar-29 [12].

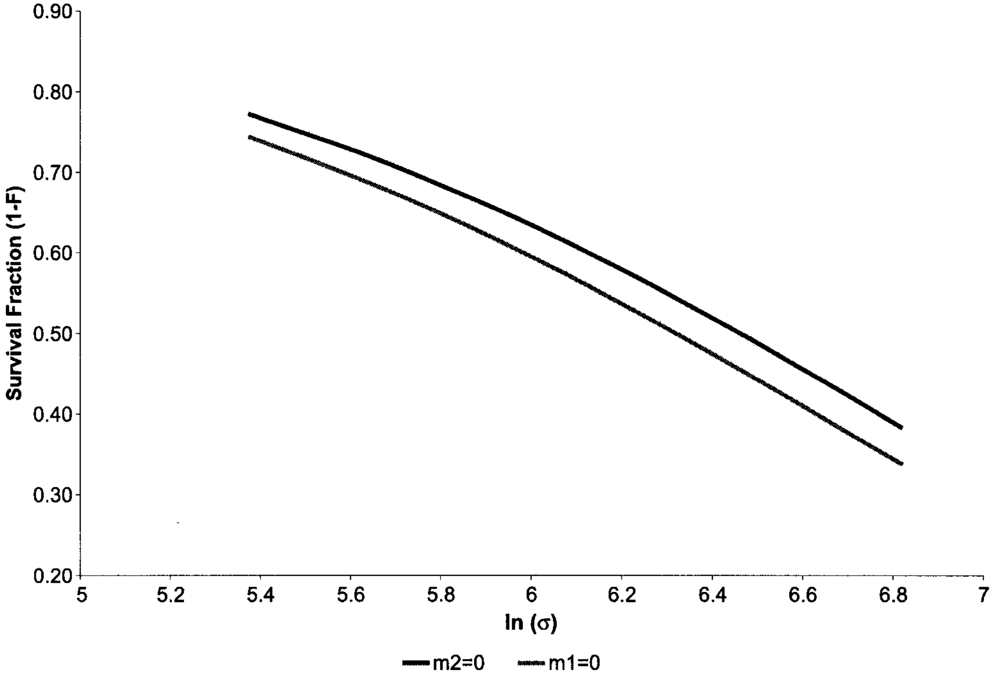


Figure 6. Deconstructed four-parameter Weibull model.

Table 2. Predicted mean recoil compressive strengths.

| Model | Mean recoil compressive strength (MPa) |
|---|---|
| Two-parameter | 285 |
| Four-parameter | 245 |

## 5. CONCLUSIONS

In this paper we have shown that the two-parameter Weibull model provides a meaningful representation of recoil compressive failure. Moreover, the four-parameter model shows little, if any, improvement over the two-parameter Weibull model. This indicates that a single failure mechanism dominates failures in recoil compression and that the two-parameter model can characterize these failures adequately.

## NOTE

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