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Experimental verification of the end-effect Weibull model as a predictor of the tensile strength of Kevlar-29 (poly *p*-phenyleneterephthalamide) fibres at different gauge lengths

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Abstract. This paper describes the first application of the four-parameter end-effect Weibull model as a predictor of tensile failure frequency at a gauge length different from that used to generate the model parameters. As-received Kevlar-29 fibres were tensile tested at gauge lengths of 10, 25 and 40 mm. The resulting failure data were used to determine four empirical constants using a maximum-likelihood regression. The model and parameters were used to predict failure frequency at a gauge length outside the initial range (5 mm). The results show that the end-effect Weibull model accurately represents the data from which its parameters are evaluated and that the model may be applied as an effective predictor for gauge lengths beyond the original testing range.

1. Introduction

High-performance polymers such as Kevlar (poly *p*-phenyleneterephthalamide), shown in figure 1, possess exceptional tensile strengths that make them ideal materials for use in such diverse products as brake pads, bullet resistant vests, kayaks, marine ropes and tennis rackets [1]. However, accurately quantifying and predicting the tensile strength of these polymers at different gauge lengths is a complex undertaking.

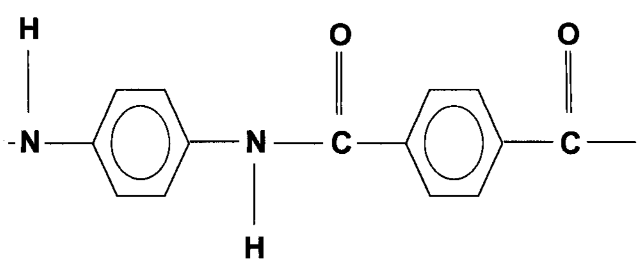

Figure 1. The repeat unit of Kevlar.

Tensile failure is inherently probabilistic. Microscopic flaws and crystalline misalignments cause the fibres to fail below their intrinsic tensile strength [2]. As a result, the actual strength at which a given fibre will fail depends on both the frequency and severity of these flaws. Therefore, statistical analysis of tensile failure data should provide insight into the distribution of flaws within the fibre.

Because tensile failure data do not conform to rigid statistical distributions, a flexible distribution that can be altered by the data themselves is required. The Weibull distribution is the preferred form for analysing most failure data [3–8]. However, even the flexible Weibull distribution cannot separate the failures that result from true flaws from those that are artefacts of the tensile test itself. These 'clamp effects' are analysed in detail by Phoenix and Sexsmith [9]. In building upon this work, Stoner *et al* [10] developed the end-effect Weibull model, in which distinct Weibull distributions were used to characterize failures from true flaws and from artefacts in carbon fibres. This model was subsequently used to trace the propagation of flaws from precursor fibres into carbon fibres [11].

Although the end-effect Weibull model accurately represented the data upon which it was based, no attempts were made to use the Weibull parameters as predictors of tensile failure at other gauge lengths. This paper describes the application of the end-effect Weibull model to predict tensile strengths of Kevlar fibres at gauge lengths other than those used to evaluate the Weibull parameters.

2. Theory

A polymer fibre may be viewed as a chain of interlocking links, with flaws acting as 'weak links' in the chain [12]. Like any other chain, the fibre will fail at its weakest link. If a simple Weibull distribution is applied to the failure data, the Weibull cumulative distribution function takes the form

$$F_i = 1 - \exp\left(-\frac{\sigma}{\sigma_0}\right)^m \tag{1}$$

where F_i is the probability of any link failing at or below the applied stress level (σ), σ_0 is a scale factor that corresponds to the severity of flaws, and m is the shape factor that relates to the distribution of flaws within the fiber. Because a link must either survive or fail, the probability for survival (S) of a link (i) is given by

$$S_i = 1 - F_i. \tag{2}$$

Substituting equation (1) into equation (2) provides

$$S_i = \exp\left[-\left(\frac{\sigma}{\sigma_0}\right)^m\right]. \tag{3}$$

The fibre will fail if any individual link fails, thus the probability of a fibre of gauge length (L) surviving is the product of the survival probabilities (S_i) for each link in that length of fibre

$$S = \prod_{i=1}^{L} S_i. \tag{4}$$

If each link is assumed to survive or fail independently of its neighbours, equation (4) can be rewritten as

$$S = (S_i)^L. \tag{5}$$

Substituting equation (3) into equation (5) provides

$$S = \exp\left[-L\left(\frac{\sigma}{\sigma_0}\right)^m\right] \tag{6}$$

and, correspondingly

$$F = 1 - \exp\left[-L\left(\frac{\sigma}{\sigma_0}\right)^m\right] \tag{7}$$

where L is the length of fibre being tested.

Previous research [9–11] has shown that the simple Weibull model cannot account for failure artefacts introduced during tensile testing by the non-uniform stress distribution that occurs at the fibre–glue interface where the fibre is mounted to the testing tab. Detailed descriptions of the end-effect model and the nature of end effects have been published previously by Stoner *et al* [10]. Although all fibres are subject to end effects, longer fibres are more likely to contain a fatal flaw, making end-effect failure less likely.

End effects do not constitute a specific flaw population, but the flexible nature of the Weibull distribution enables it to be used to characterize the failure resulting from end effects. However, the functional forms of the distribution must be slightly different from those presented previously. Because only the segments of the fibre near the glue spot are subject to end effects, there can be no gauge length dependence in the end-effect survival function. That is, gauge length would be completely irrelevant if all failures resulted from end effects. If the total number of chain links subject to end effects is taken to be δ, then the probability of a fibre surviving end effects alone at a given stress level becomes

$$S_E = \exp\left[-\delta\left(\frac{\sigma}{\sigma_0}\right)^m\right]. \tag{8}$$

The inclusion of the δ term would seem to increase the number of adjustable parameters from two to three. However, neither δ nor σ_0 can have gauge length dependence provided that $L \gg \delta$. Thus, they can be combined into a single empirical parameter called σ_{0E}.

Obviously, a fibre must survive both end effects and its flaw population to avoid failing, so the overall survival relationship becomes

$$S = S_E S_F. \tag{9}$$

In this development, F and E are used to distinguish failure resulting from flaws (F) from those caused by end effects (E). As in the simple Weibull model, the probability of a fibre surviving its inherent flaws is given by

$$S_F = \exp\left[-L\left(\frac{\sigma}{\sigma_{0F}}\right)^{m_F}\right]. \tag{10}$$

It is worth observing that the length of fibre subject to end effects (δ) may also contain flaws. Thus, L continues to be the correct gauge length in equation (10), rather than $L - \delta$. The total survival relationship becomes

$$S = \exp\left[-L\left(\frac{\sigma}{\sigma_{0F}}\right)^{m_F} - \left(\frac{\sigma}{\sigma_{0E}}\right)^{m_E}\right] \tag{11}$$

while the probability of failure becomes

$$F = 1 - \exp\left[-L\left(\frac{\sigma}{\sigma_{0F}}\right)^{m_F} - \left(\frac{\sigma}{\sigma_{0E}}\right)^{m_E}\right]. \tag{12}$$

Equation (12) can be used to evaluate the four empirical Weibull parameters (m_F, m_E, σ_{0E}, σ_{0F}). However, this evaluation is more complex than it was for the simple Weibull model because the equation cannot be linearized. The parameters in these nonlinear equations were evaluated using maximum-likelihood estimation. Essentially, maximum-likelihood theory evaluates the probability that a given set of tensile strengths would have resulted from a specific

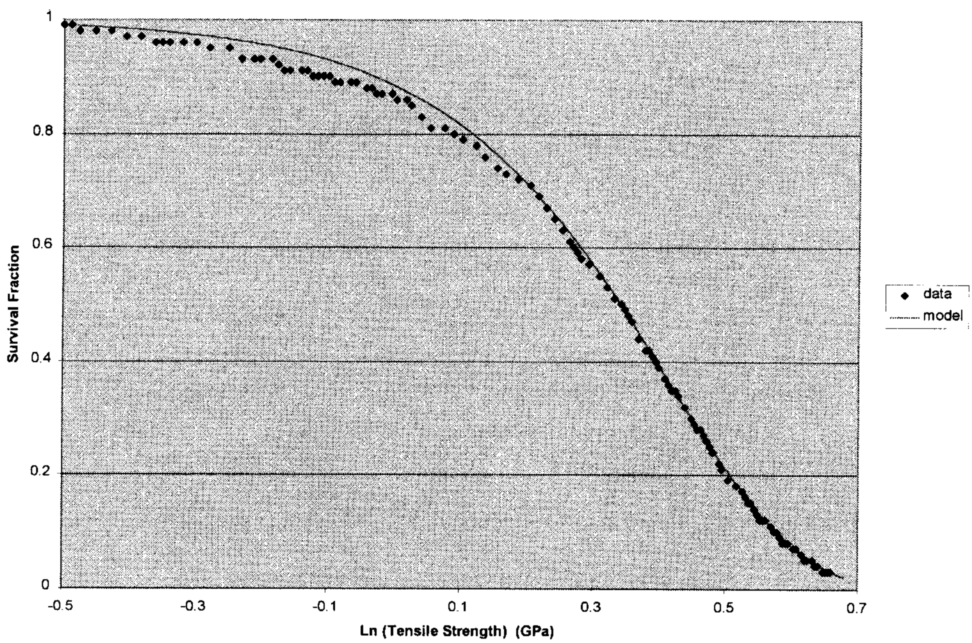

Figure 2. Comparison of failure data with the end-effect Weibull model at 10 mm gauge length.

set of Weibull parameters. The observed failure distributions and equation (12) are provided, while the parameters are systematically varied to maximize the likelihood that the observed failure distribution resulted from the given parameters. Derivations of the equations used in maximum-likelihood evaluation can be found in the previously mentioned work of Stoner *et al* [10] or in the book by Everitt and Hand [13].

3. Experimental procedure

As-received Kevlar-29 samples were obtained from the E I DuPont de Nemours Corporation. The single filament testing procedure used in this study followed the procedure described in ASTM Standard D-3379-75 [14]. Fibres were mounted on testing tabs using Hughes Epoxy 220 and placed in a 65 °C drying oven for 24 h. Fibre diameters were measured using a laser diffraction technique with a 0.95 mW helium–neon red laser. This technique has been found to be accurate to within 0.1 μm. A minimum of 150 samples were tested at each of three gauge lengths (10, 25 and 40 mm).

The tensile failure data obtained from these three gauge lengths were used to determine the four Weibull parameters. Next, 175 filaments were tested using a 5 mm gauge length. The results of this test were compared with predictions made by applying the Weibull parameters determined from the 10, 25 and 40 mm tests.

4. Results and discussion

Table 1 summarizes the results of the maximum-likelihood analysis for the 10, 25 and 40 mm samples. Figures 2 through 4 show that the end-effect model provided an excellent representation of the data used in determining the empirical Weibull parameters.

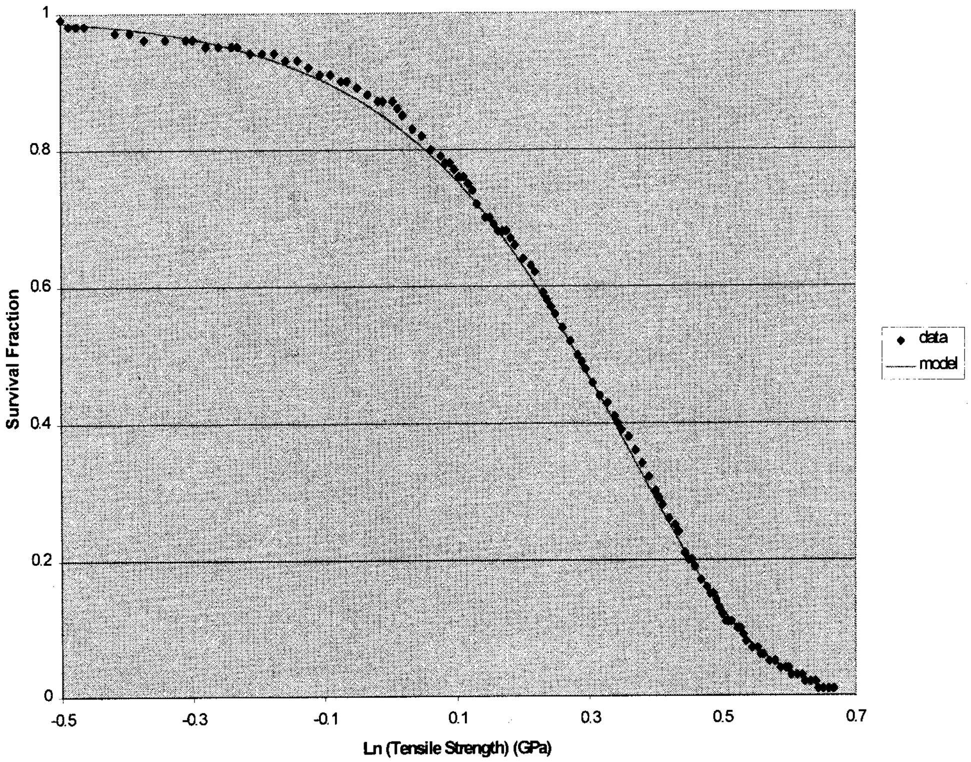

Figure 3. Comparison of failure data with the end-effect Weibull model at 25 mm gauge length.

Table 1. Summary of Weibull parameters determined from 10, 25 and 40 mm trials.

Parameter	Value
M_F	4.6091
σ_{0F}	3.4452
M_E	5.2261
σ_{0E}	1.5880

More significantly, figure 5 shows that the same values of the empirical parameters provided a highly accurate prediction, within 5% of observed failure values for the 5 mm tests. The model can also be deconstructed to separate the influence of true flaws from end-effects, since independent Weibull shape and scale parameters are determined for the flaw and end-effect distributions. As expected, end effects resulted in a relatively small number of failures for large gauge length samples and substantially more as gauge length decreased. These results compare favourably with those obtained using the more complicated and data intensive stochastic model proposed by Wagner [15], without requiring consideration of diameter variations between individual filaments.

These results have two significant consequences. First, they provide experimental validation of the end-effect Weibull model as evidenced by the ability of the model to predict failures at a gauge length outside its initial range, without needing to consider the physics of the 'clamp effects'. Although it can be correctly observed that the addition of a fourth parameter should inherently improve the accuracy of the model, previous studies [10, 11] have

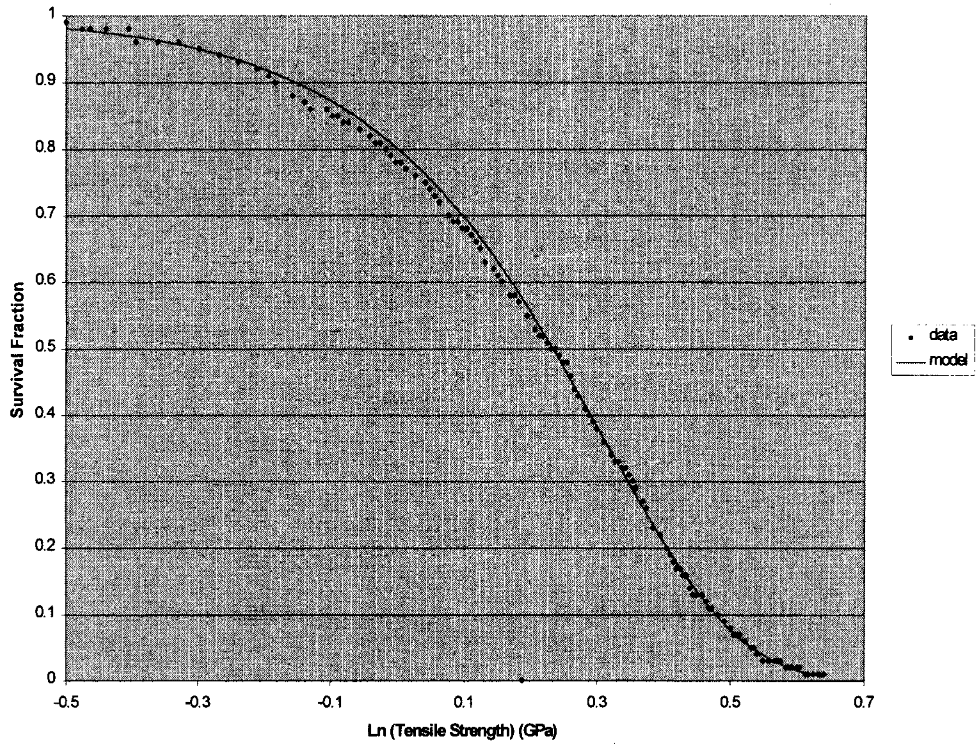

Figure 4. Comparison of failure data with the end-effect Weibull model at 40 mm gauge length.

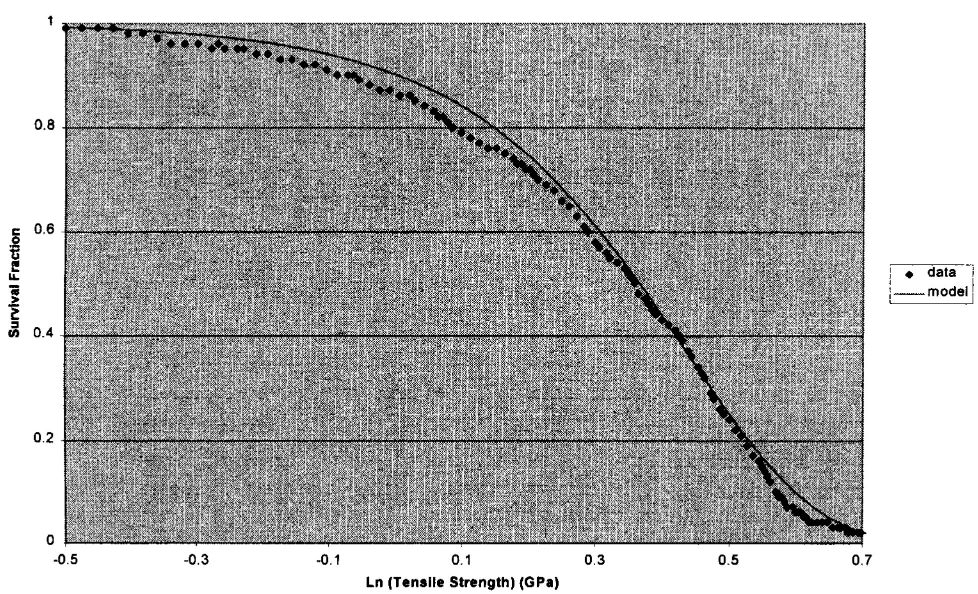

Figure 5. Comparison of end-effect failure model prediction with tensile failure data at a 5 mm gauge length.

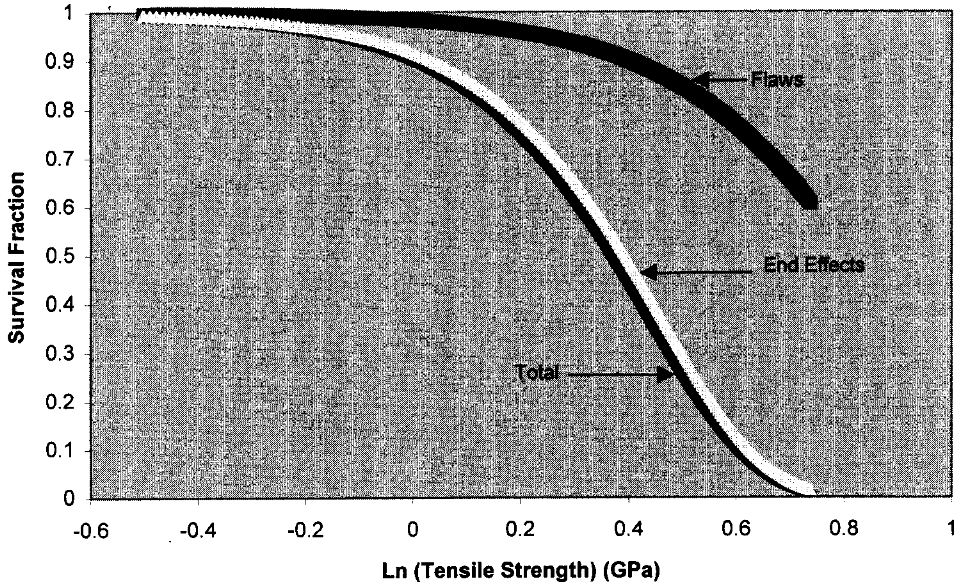

Figure 6. Relative contributions of end effects and flaws at 5 mm gauge length.

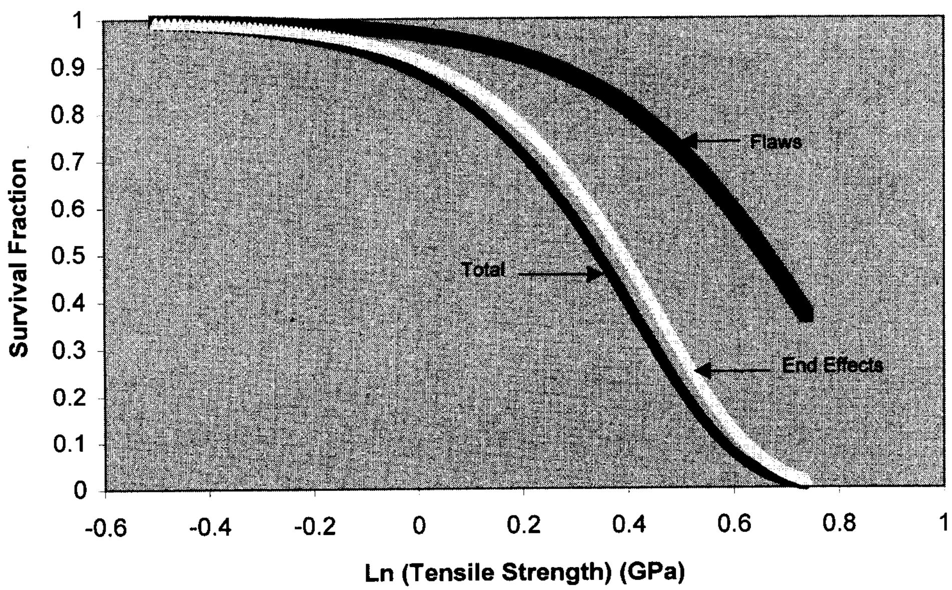

Figure 7. Relative contributions of end effects and flaws at 10 mm gauge length.

demonstrated that the end-effect model provides superior representation of experimental data when compared with other four-parameter models in which both failure distributions are gauge length dependent. Secondly, the model shows promise as offering a means of predicting the functional tensile strength of fibres in composites at a wide variety of gauge lengths with relatively minimal amounts of required data.

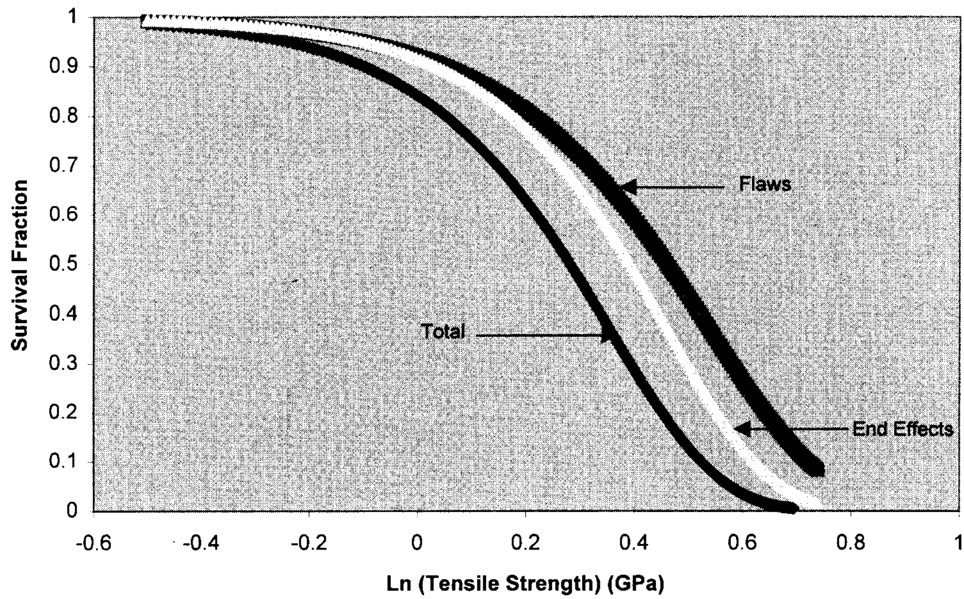

Figure 8. Relative contributions of end effects and flaws at 25 mm gauge length.

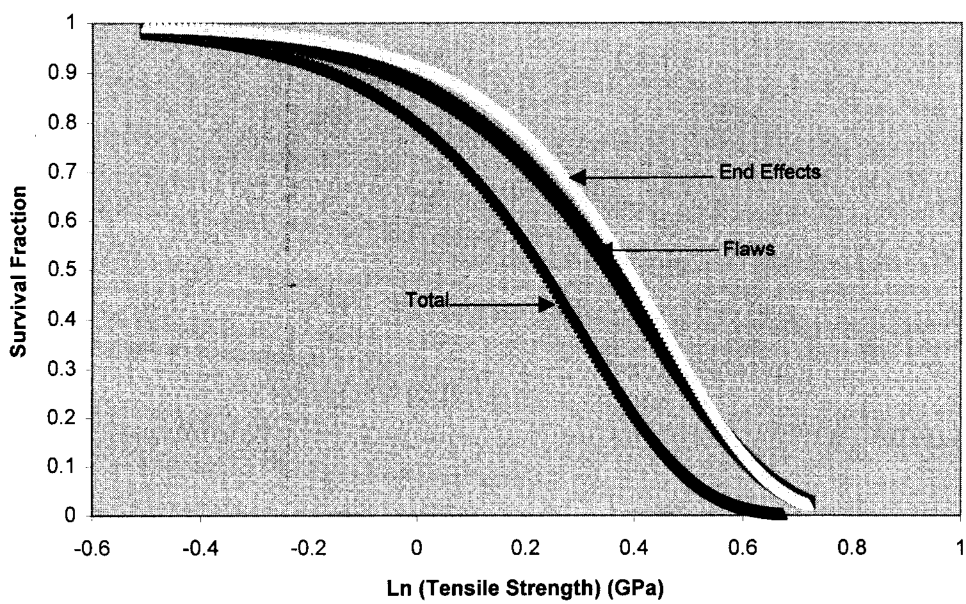

Figure 9. Relative contributions of end effects and flaws at 40 mm gauge length.

The end-effect Weibull model allows for the relative contributions of end effects and flaws to be separated from the overall failure data. Figures 6 to 9 show the relative contributions of end effects and flaws for 5, 10, 25 and 40 mm fibres respectively. As expected, the relative importance of end effects increases as gauge length decreases. Although the end effects themselves are independent of gauge length, shorter fibres are less likely to contain a severe flaw. Accordingly, they are more likely to experience end-effect failure before reaching the inherent tensile strength of the filament.

5. Conclusions

This work has shown that the end-effect Weibull model provides an exceptional fit of the tensile failure data upon which its parameters are based. More importantly, the end-effect Weibull model accurately predicted the tensile failure rates at a gauge length outside the data range used to determine the parameters. This serves to both support the validity of the end-effect model and to provide a means of predicting failure rates at a wide range of gauge lengths. Finally, the data showed that end effects became less significant as gauge length increased.

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