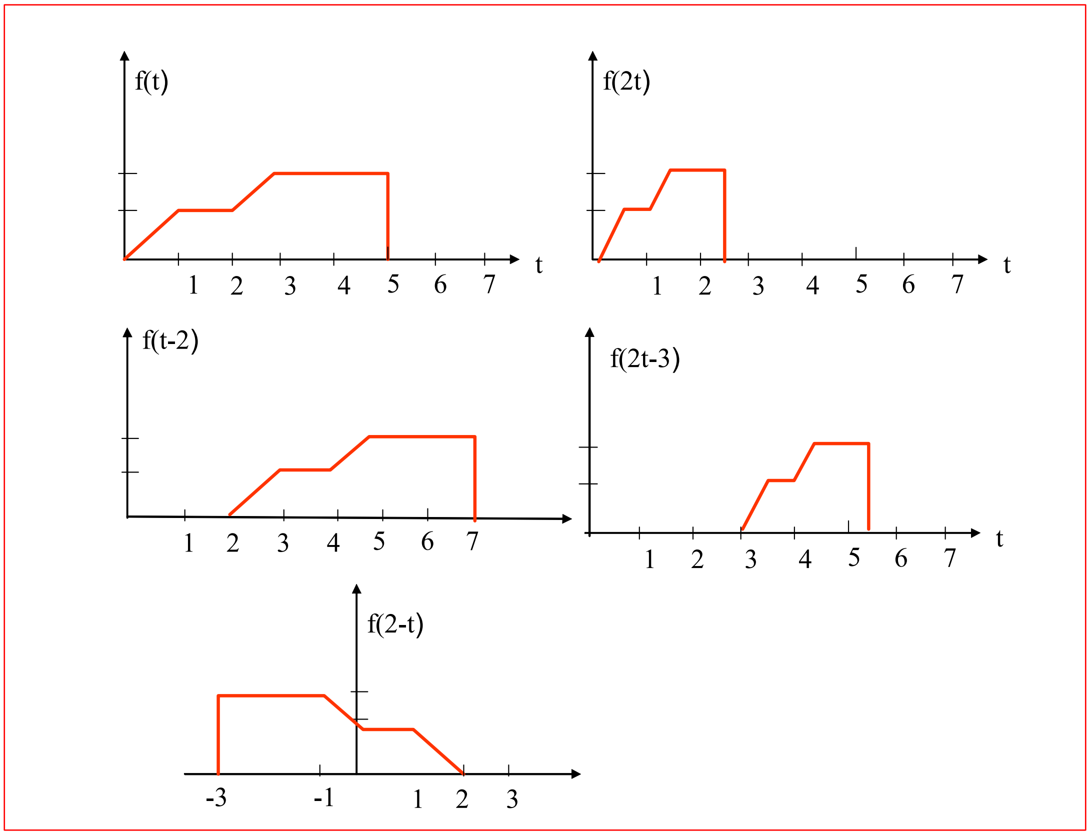
Problem 2-1 Given the signal f(t) shown below, Sketch f(t-3), f(2-t), f(2t) and f(2t-3).



Problem 2–2 A digital audio system has an ADC that samples at *48 KHz* and yields a *20 bit* sample when commanded.

a) How many *bits* would be required on a compact disc to fill *80 minutes*?

b) What dynamic range can be expected from this system?

c) If the ADC has a ± 10 volt supply voltage, what would be the smallest value of voltage that could be accurately represented?

d) What is the SQNR for this system?

e) If the ADC were corrupted by noise and had a constant "noise floor" of .6 volts, what would be the effective number of bits for the ADC?

a) 20 bits/sample * 48000 samples/sec * 60 seconds per minute * 80 minutes

= 4.608 gigibits

b) $\text{Dynamic Range (dB)} = 20 \log (2^{N})$

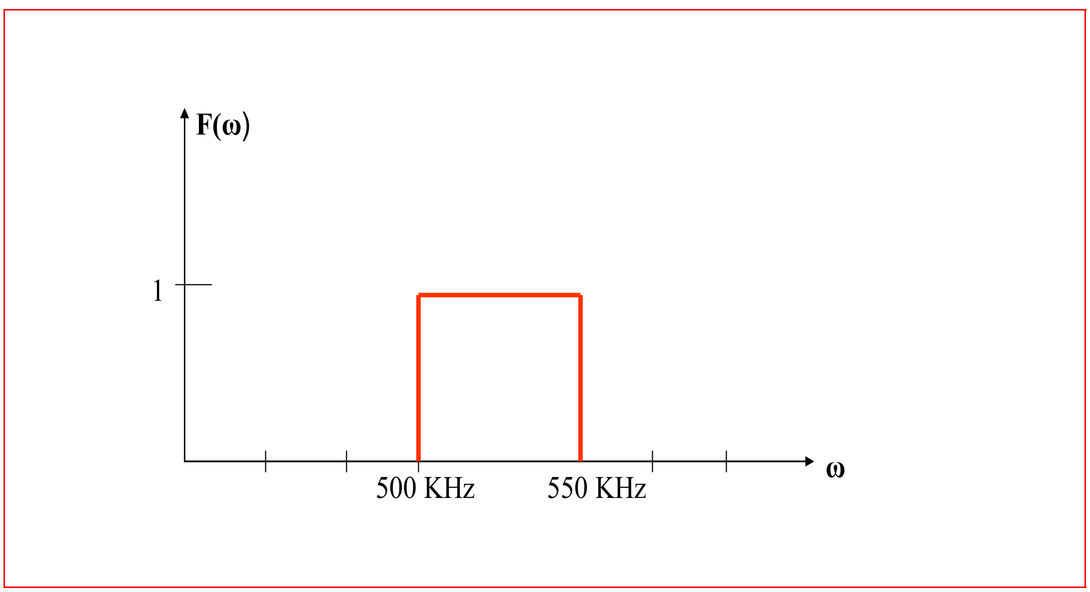
Therefore, $\text{Dynamic Range (dB)} = 20 \log (2^{20}) = 20*6 = \underline{120\text{ db}}$

c) $q = \frac{voltage\ range}{2^{N}-1} = \frac{10V-(-10V)}{2^{20}-1} = \frac{20}{1{,}048{,}576-1}\ V = \underline{19.07}\ \mu\text{Volts}$

d) $\text{SQNR (dB)} = 6.02N + 1.763 = 6.02(20) + 1.763 = \underline{122.16\text{ db}}$

e) The smallest interval, q, we can represent is 0.6 V given the noise floor. So, we can represent values from .6 to 10V with a resolution of no greater than 0.6 V. This can be represented by $\frac{10}{.6}$ = 16.67 levels. Rounding down we call it 16 levels. To represent 16 discrete levels we need 4 bits. So, the effective number of bits of this A/D converter with the high noise floor is 4 bits.

Problem 2-3 A signal has the following frequency characteristics:



a) Employing Shannon's Sampling Theorem, what is the minimum rate that you can sample at and still recover the signal?

b) Describe the method you would use to recover the signal.

c) Show an alternative method for sampling and recovering this signal that uses the smallest sampling rate given the bandwidth of the signal

Hint: use a different filter than a lowpass filter for recovery.

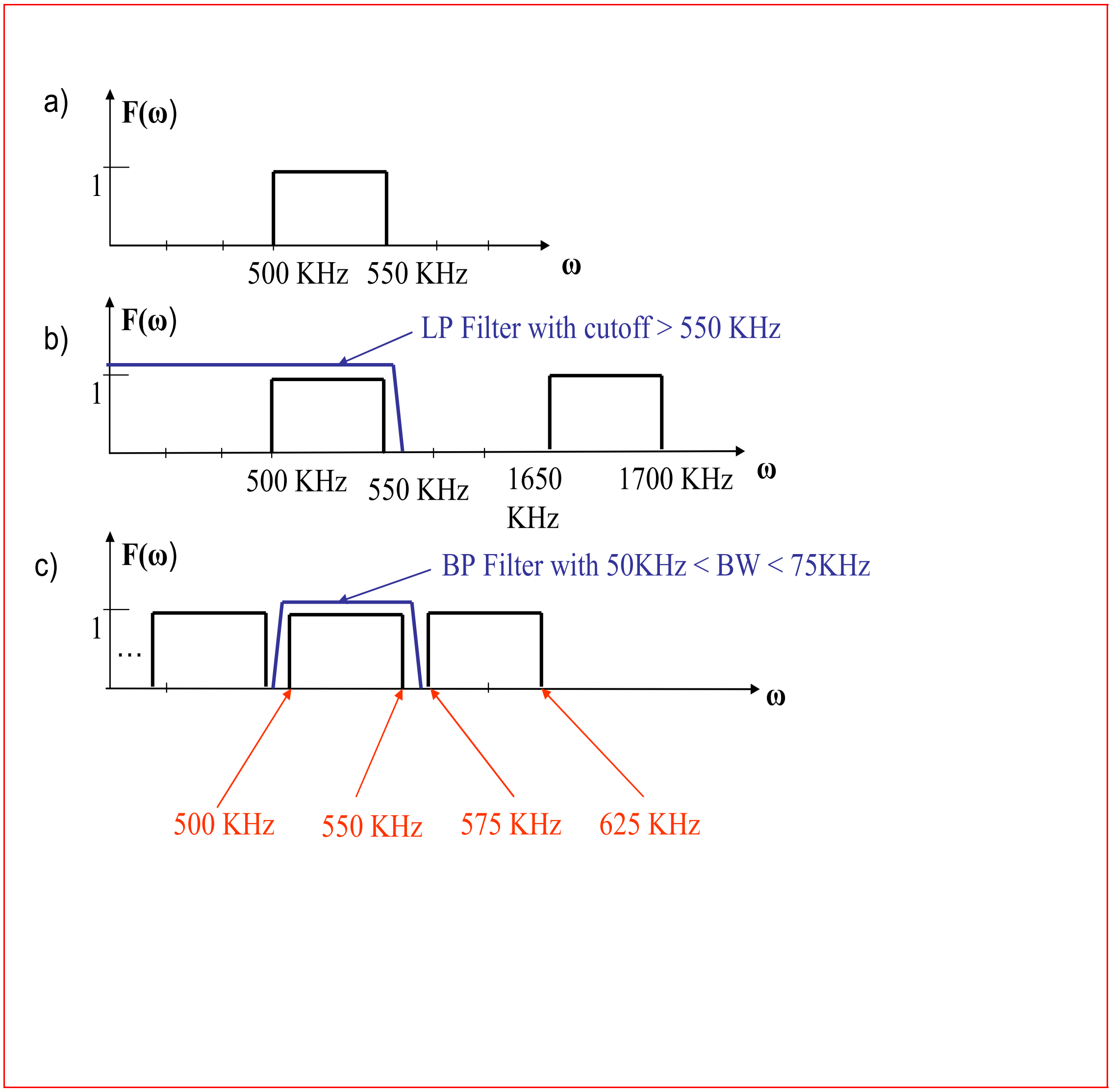
d) Describe why the method in (c) is more efficient for recovery of the signal?

a) The minimum frequency that you can sample and recover the signal in a) below would be two times the highest frequency of interest, which in this problem is 550 KHz. Therefore, the sampling frequency should be 1.1 MHz or greater.

b) A lowpass filter with a cutoff frequency of at least 550 KHz is required to recover the original signal. This is shown in b) below.

c) The 50 KHz bandwidth signal could be sampled at a rate greater than 50 KHz. We will use 75 KHz. This will produce non-overlapping replicas of the original signal spaced apart by 25 KHz. Now, we can recover the signal with a bandpass filter with a 50 KHz > BW > 75 KHz. This is shown in c) below.

d) This "bandpass sampling" enables us to use a much smaller sampling rate than standard Nyquist sampling. The larger the ratio of the frequencies of interest to the bandwidth of the signal, the greater the efficiency gained from this scheme.



Problem 2–4 For each of the following sinusoids, determine whether the sinusoid is periodic or aperiodic. If the signal is periodic, *how many samples* over *how many periods* need to be collected to show periodicity?

a) $y1 = \sin(\frac{\pi}{2}n)$

b) $y1 = \sin(\frac{3}{4}n)$

c) $y1 = \sin(\frac{4}{7}n)$

d) $y1 = \sin(\frac{3\pi}{7}n)$

a) The ratio of $\frac{2\pi}{\Omega}$ is rational, therefore, periodic. The ratio 4/1 implies that there are 4 sampling instances in 1 period.

b) Again, the ratio of $\frac{2\pi}{\Omega}$ is not rational, therefore, not periodic.

c) Once again, the ratio of $\frac{2\pi}{\Omega}$ is not rational, therefore, not periodic.

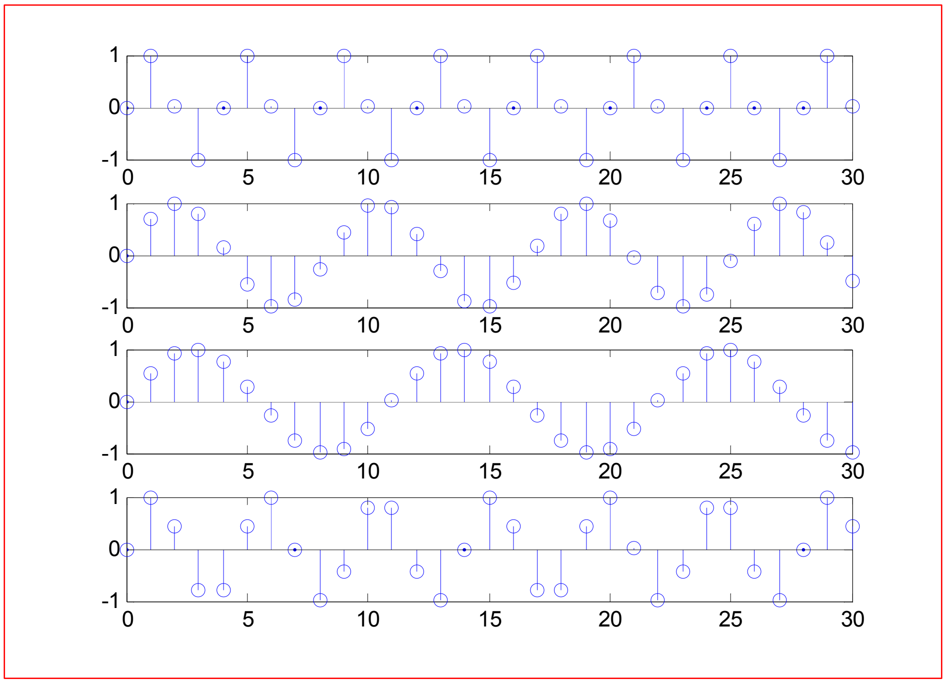
d) The ratio of $\frac{2\pi}{\Omega}$ is rational, therefore, it is periodic. The ratio 14/3 implies that there are 14 sampling instances in 3 periods.

Problem 2-5 For each sinusoid in Problem 2-4, write a MATLAB program to plot each waveform. Then, indicate the period for those sinusoids that are periodic.

```
>>  n=0:1:30;                          % sinusoid a) periodic
>>  y1=sin(pi/2*n);
>> subplot(4,1,1);stem(n,y1);hold
>>  n=0:1:30;                          % sinusoid b) non-periodic
>>  y1=sin((3/4)*n);
>> subplot(4,1,2);stem(n,y1)
>>  n=0:1:30;                          % sinusoid c) non-periodic
>>  y1=sin((4/7)*n);
```

```
>> subplot(4,1,3);stem(n,y1)

>>  n=0:1:30;                    % sinusoid d) periodic

>>  y1=sin((3*pi/7)*n);

>> subplot(4,1,4);stem(n,y1)
```

This produces the following plot:



Given that **P** is the number of periods and **Q** is the number of samples. The number of samples per repetition can be calculated by the expression $\frac{P}{Q}=\frac{2\pi}{\Omega}$.

For sinusoid a) where $\Omega=\frac{\pi}{2}$, this translates to $\frac{2\pi}{\frac{\pi}{2}}$ or **4 samples per period** .

For sinusoids b) and c) the ratio of $\frac{P}{Q}$ are irrational, therefore they are not periodic.

For sinusoid d) this is $\frac{2\pi}{\frac{3\pi}{7}} = \textbf{14 samples per 3 periods}$.

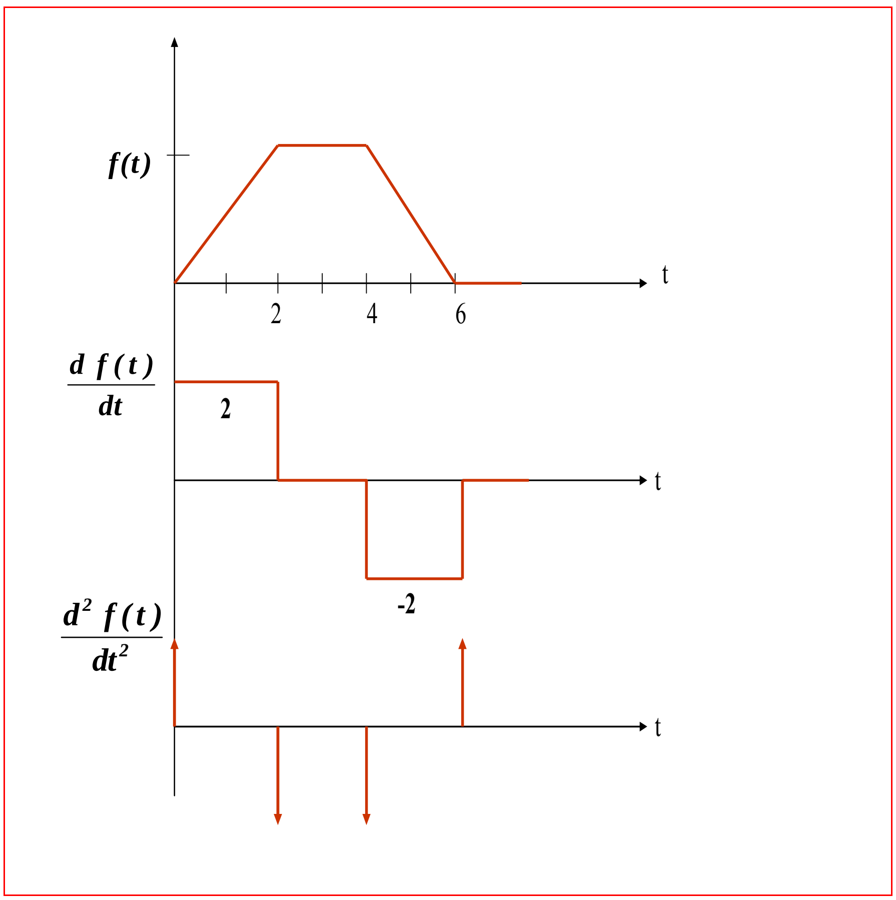
Problem 2–6 A signal f(t) is described by the following relationship:

$$f(t) = u(t)r(2t) - u(t-2)r(2t-2) - u(t-4)r(2t-4) + u(t-6)r(2t-6)$$

a) Draw the resulting waveform, f(t).

b) Draw the waveform of $\frac{d}{dt}f(t)$.

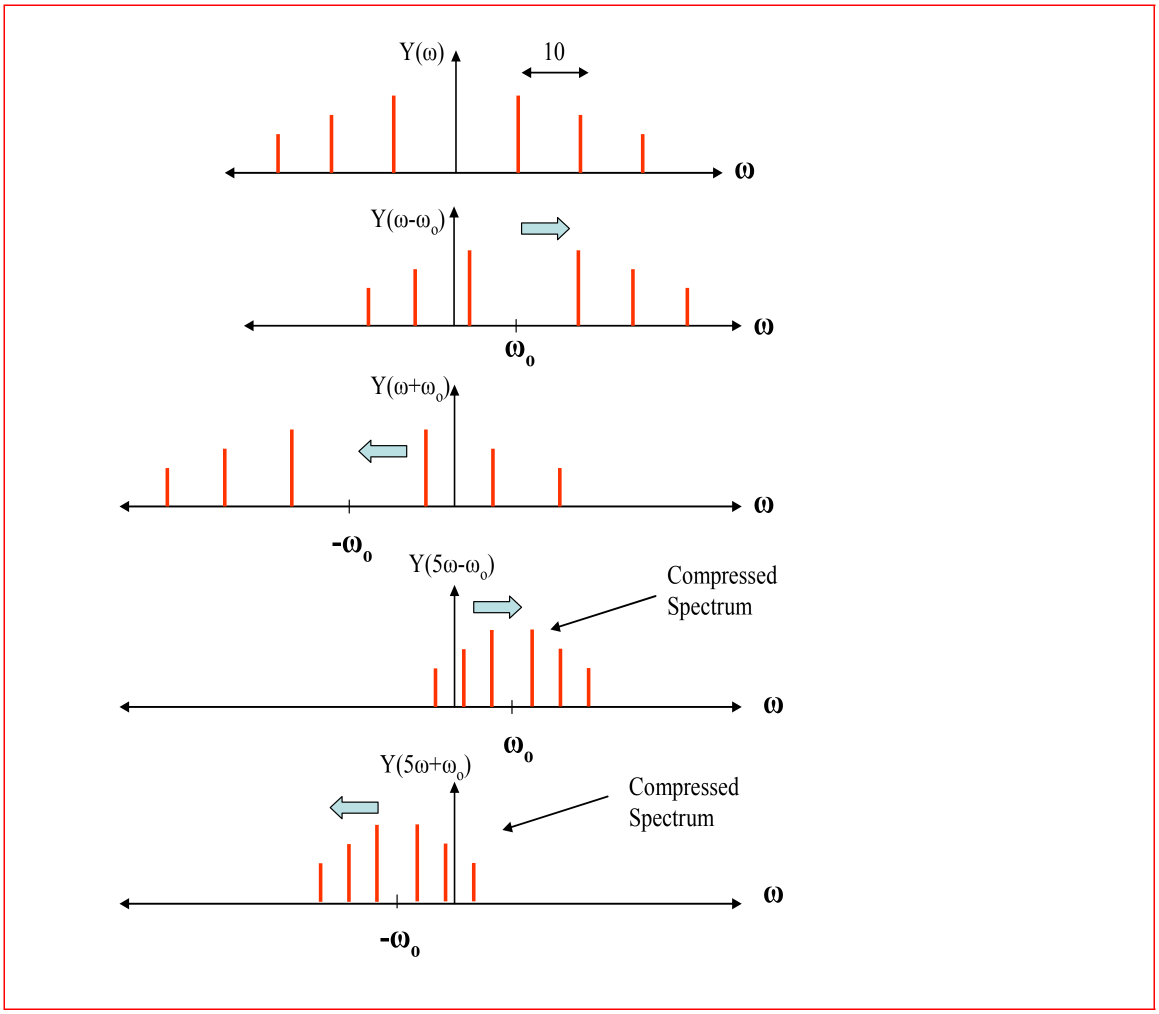
c) Draw the waveform of $\frac{d^2}{dt^2}f(t)$.



Problem 2-7 Given the spectrum, $Y(\omega)$, described by:

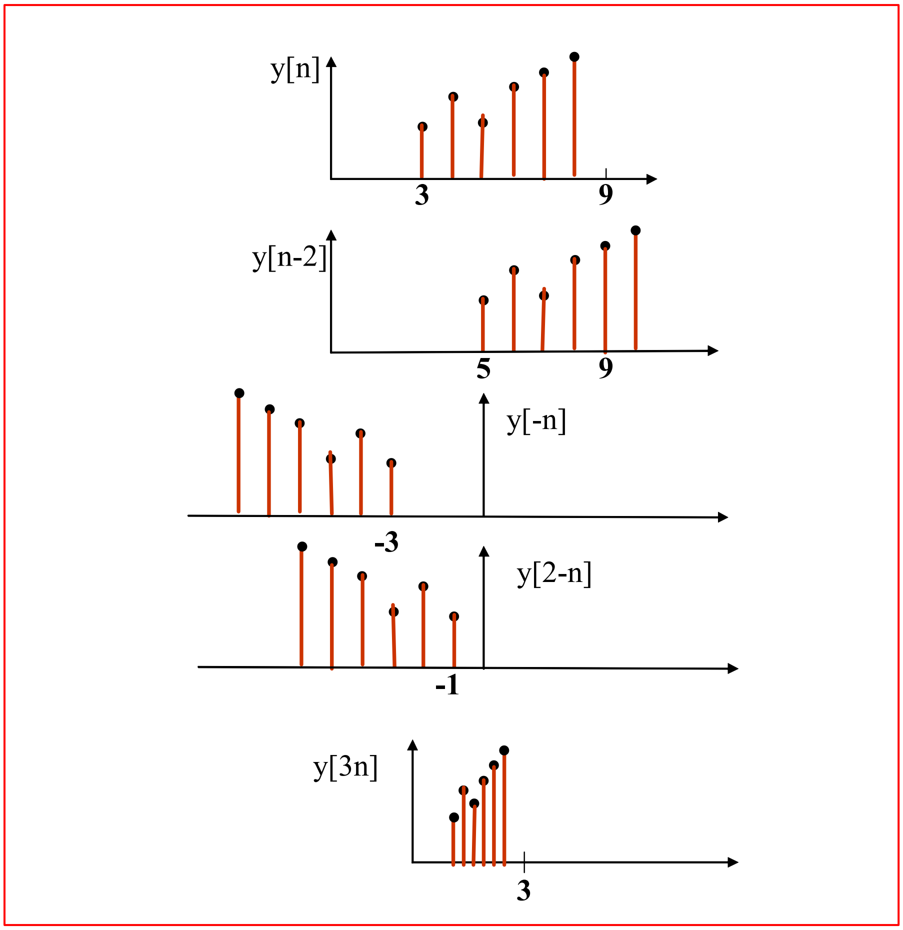
$$Y(\omega) = \delta(\omega-10) + .8\delta(\omega-20) + .6\delta(\omega-30) - \delta(\omega+10) - .8\delta(\omega+20) + .6\delta(\omega+30)$$

Sketch $Y(\omega)$, $Y(\omega-\omega_o)$, $Y(\omega+\omega_o)$, $Y(5\omega-\omega_o)$ and $Y(5\omega+\omega_o)$.

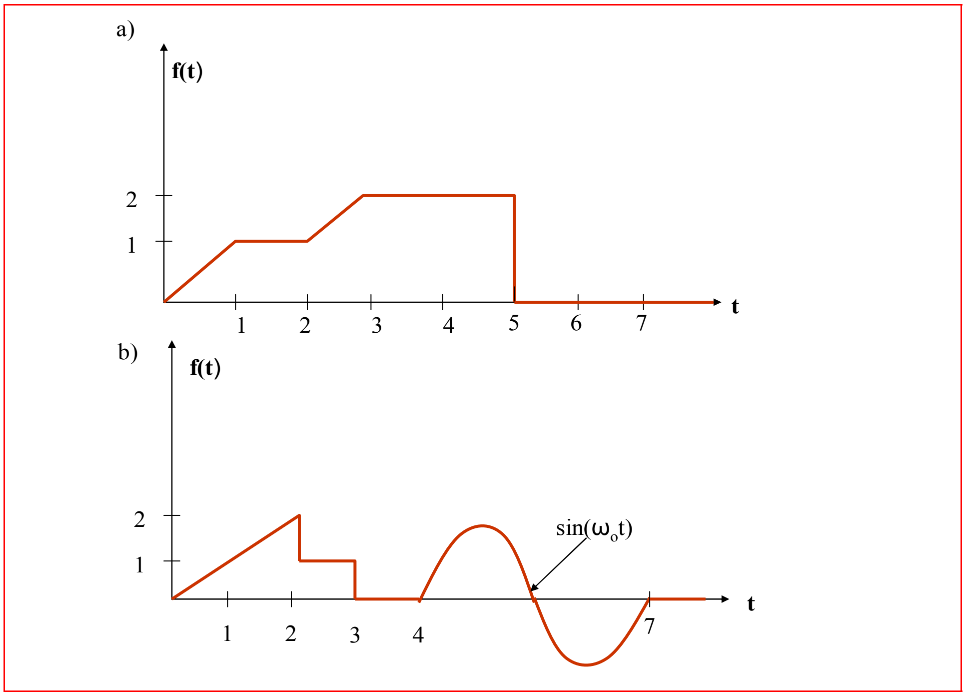


Problem 2–8 Given the signal y[n] shown in the following.

Sketch y[n-2], y[2-n], y[-n], and y[3n].



Problem 2-9 Write a mathematical representation to describe the following signals:



a) $f(t) = r(t) - r(t-1) + r(t-2) - r(t-3) - 2u(t-5)$

b) $r(t) - r(t-2) - u(t-2) - u(t-3) + u(t-4)\sin(\omega_o(t-4)) - u(t-7)\sin(\omega_o(t-7))$

Problem 2–10 Given a signal with two-sided spectrum $F(\omega)$, with the following characteristics:

$|F(\omega)| = 1$, for $-110 \le \omega \le -100$ and $100 \le \omega \le 110$

$|F(\omega)| = .5$, for $-130 \le \omega \le -120$ and $120 \le \omega \le 130$

$|F(\omega)| = 0$, elsewhere

a) Draw the spectrum of $|F(\omega)|$.

b) Draw the spectrum if $|F(\omega)|$ is sampled at 500 radians per second.

c) Draw the spectrum if $|F(\omega)|$ is sampled at 190 radians per second.

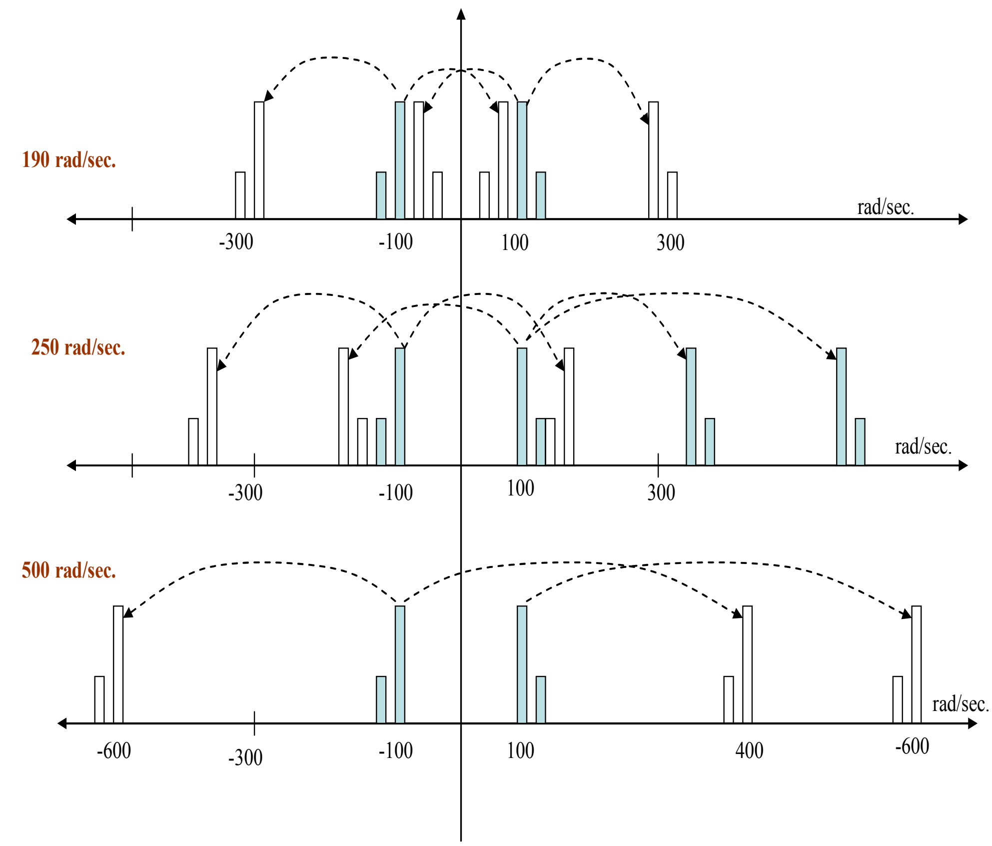
d) Draw the spectrum if $|F(\omega)|$ is sampled at 250 radians per second.

e) Which spectra are recoverable? Explain why or why not.

Below is a table description of the sampled frequency components:

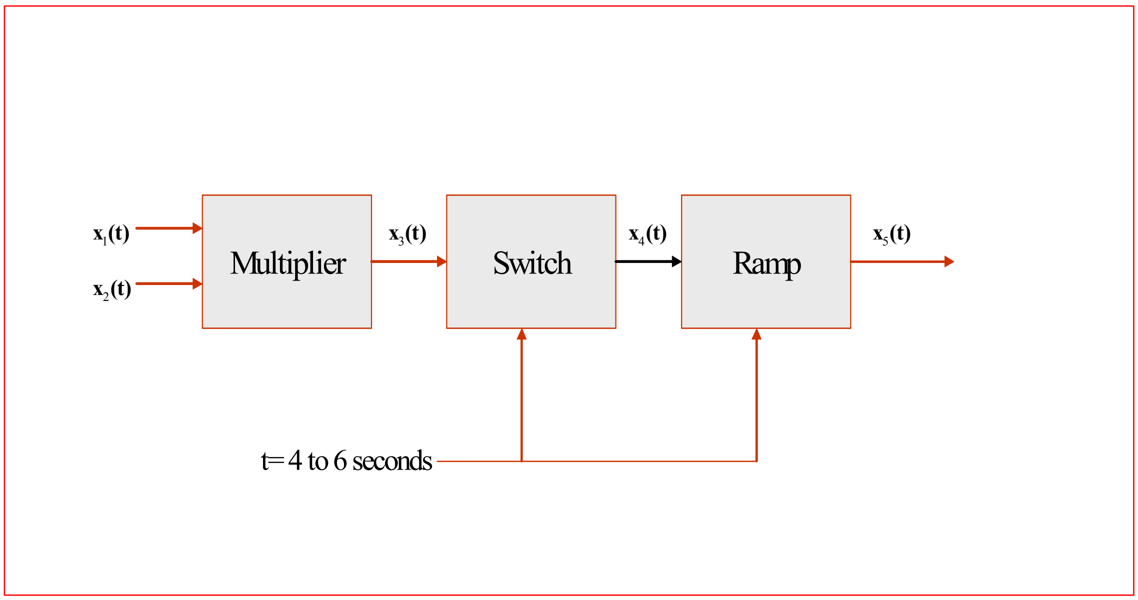
Frequency Components Unsampled	**-130 to -120 r/s**	**-110 to -100 r/s**	**100 to 110 r/s**	**120 to 130 r/s**
Sampled at 500 r/s $f_o \pm nf_s$	-630 to -620 rad/s and 370 to 380 rad/s	-610 to -600 rad/s and 390 to 400 rad/s	-400 to -390 rad/s and 600 to 610 rad/s	-380 to -370 rad/s and 620 to 630 rad/s
Sampled at 250r/s $f_o \pm nf_s$	-380 to -370 rad/s and 120 to 130 rad/s	-360 to -350 rad/s and 140 to 150 rad/s	-150 to -140 rad/s and 350 to 360 rad/s	-130 to -120 rad/s and 370 to 380 rad/s
Sampled at 190r/s $f_o \pm nf_s$	-320 to -310 r/s and 60 to 70 rad/s	-300 to -290 r/s and 80 to 90 rad/s	-90 to -80 r/s and 290 to 300 rad/s	-70 to -60 r/s and 310 to 320 rad/s

On the following page is sketches of the spectrums described in the problem after sampling at 190, 250 and 500 rad/s. The original spectrum is indicated in **blue**. The arrows show the translation to the sampled versions of the original spectrum.



e) The spectrums sampled at 190 rad/s and 500 rad/s are recoverable with a sharp bandpass filter. The situation where the sampling is at 250 rad/s would require a bandpass filter with a zero transition region which is, of course, impossible. Therefore, the second situation has a spectrum that is not recoverable.

Problem 2-11 The system shown below multiplies two sinusoids, $\mathbf{x_1(t)}$ and $\mathbf{x_2(t)}$, of different frequency together. The resultant signal $\mathbf{x_3(t)}$ runs into a switch (like a unit step function) and turned on at t=4 seconds, then turned off at t=6 seconds. The resultant signal will be called $\mathbf{x_4(t)}$. That signal is run into a block that ramps up the signal at a 0.5 volts/second rate from t=4.1 seconds to t=6 seconds. The resulting output signal is $\mathbf{x_5(t)}$.



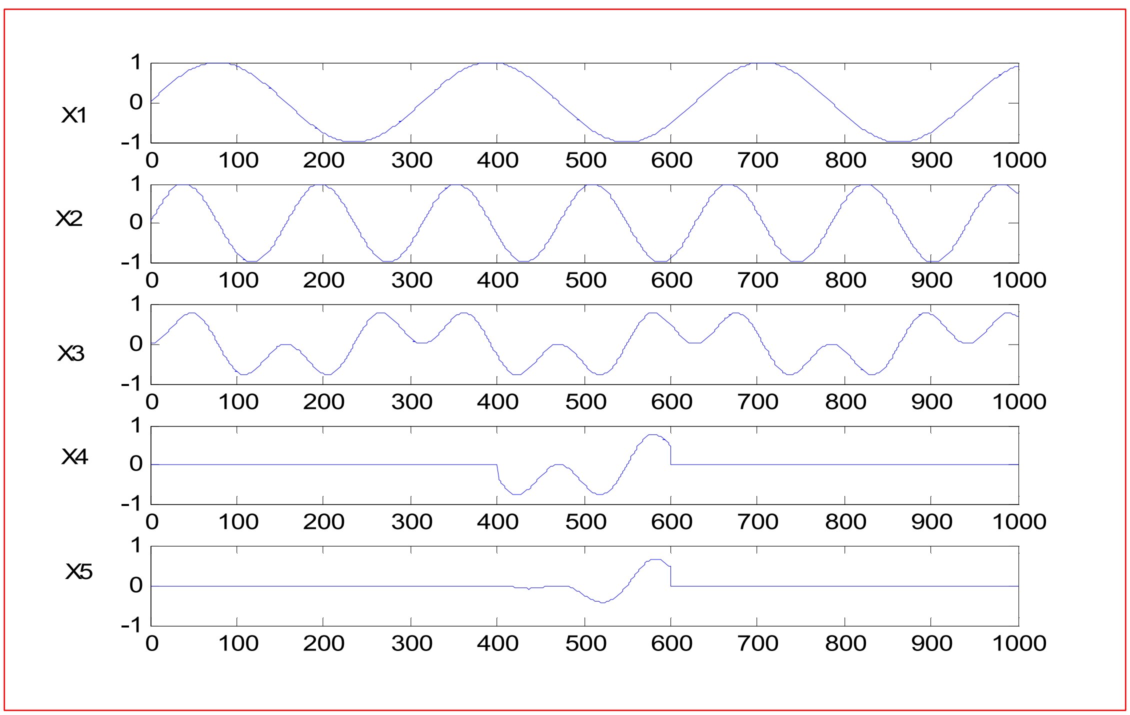
a) Given an $\mathbf{x_1(t)}$ and $\mathbf{x_2(t)}$ a 2 radians per second sinusoid and 4 radians per second sinusoid respectively, write an expression that describes the process used to determine $\mathbf{x_5(t)}$.

b) Write a program to plot $\mathbf{x_1(t)}$ to $\mathbf{x_5(t)}$.Use 100 pts /second to approximate the continuous functions for about 10 seconds.

a) $x_5 = x_1(t) * x_2(t) * .5r(t-4.1) * (u(t-2)-u(t-4))$

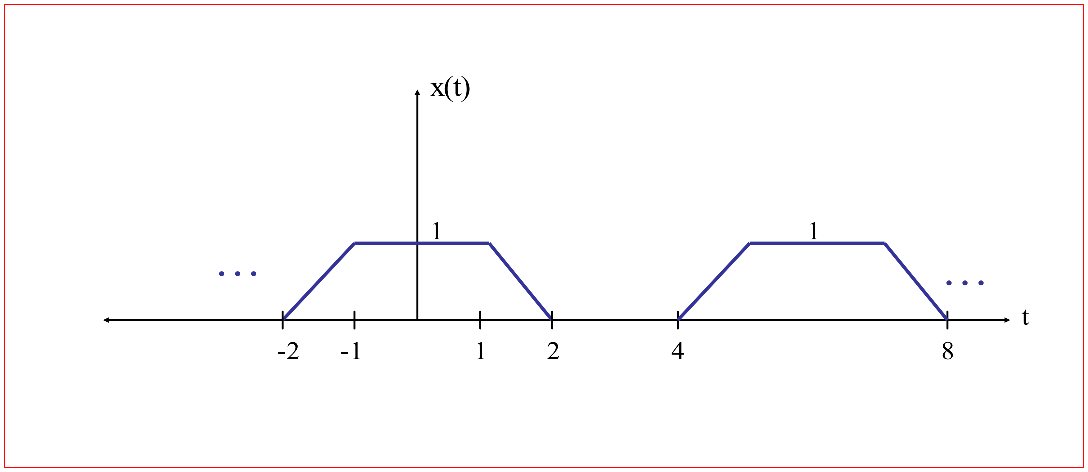
b) >> n=1:1000;   % define 10 secs. of points

```
>> x1=sin(2*n*.01);              % define x1
>> x2=sin(4*n*.01);              % define x2
>> mult=x1.*x2;                  % derive x3
>> x3=mult;                      % x3 is the output of the multiplier
>> sw=[zeros(1,400) ones(1,200) zeros(1,400)];      % derive the switching element
>> x4=x3.*sw;                    % apply the switch to the output of mult
>> n2=1:190;                     % define the ramp function time
>> ramp=[zeros(1,410) .005.*n2 zeros(1,400)];   % define the ramp function time
>> x5=x4.*ramp;                  % apply the ramp to x4
>> subplot(5,1,1);plot(n,x1);
>> subplot(5,1,2);plot(n,x2);
>> subplot(5,1,3);plot(n,x3);   % plots of x1, x2, x3, x4 and x5
>> subplot(5,1,4);plot(n,x4);
>>subplot(5,1,5);plot(n,x5);
```



Problem 2–12 Given the time-domain waveform, x(t), in the following figure:

Write a piece-wise algebraic expression that describes the calculation of the Fourier coefficient a_n. (Do not solve the integral)



$$a_n = \frac{2}{T}\int_0^1 f(t)\cos(n\omega t)dt + \frac{2}{T}\int_1^2 f(t)\cos(n\omega t)dt$$

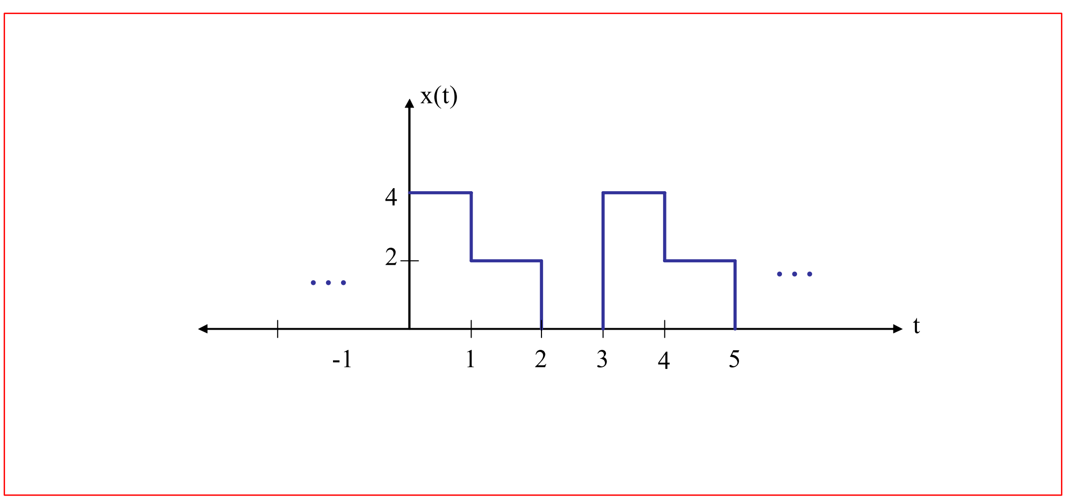
The period T=3, so we have:

$$a_n = \frac{2}{T}\int_0^1 (1)\cos(n\omega t)dt + \frac{2}{T}\int_1^2 (2-t)\cos(n\omega t)dt$$

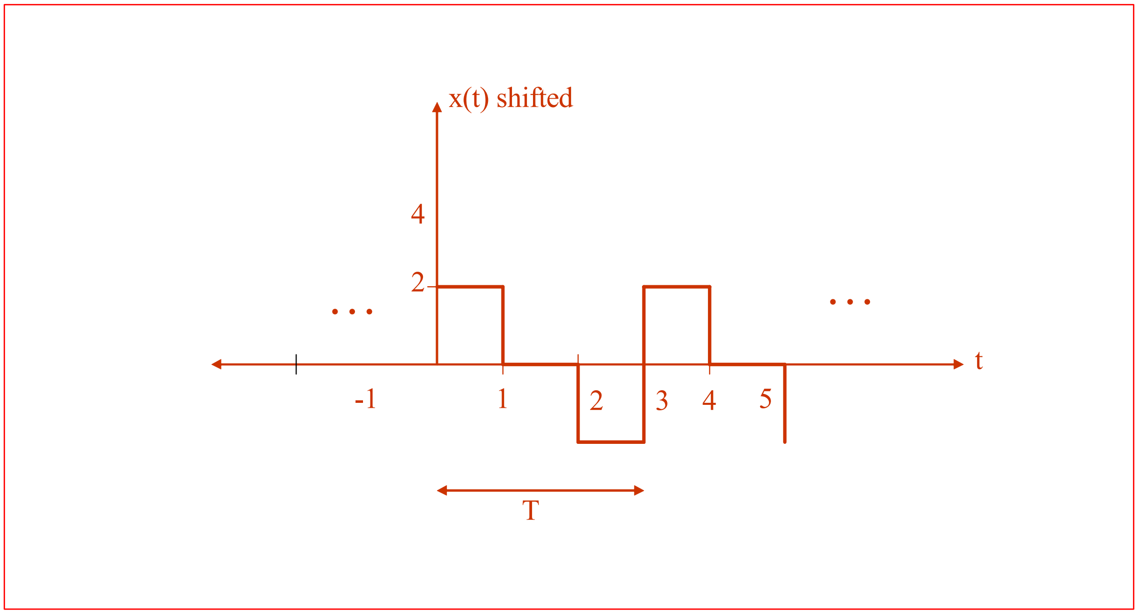
So,

$$a_n = \frac{6}{n^2\pi^2}\left[\cos\left(\frac{n\pi}{3}\right) - \cos\left(\frac{2n\pi}{3}\right)\right]$$

Problem 2-13 Repeat Problem 2-12 for the waveform in the figure below for the b_n coefficient.



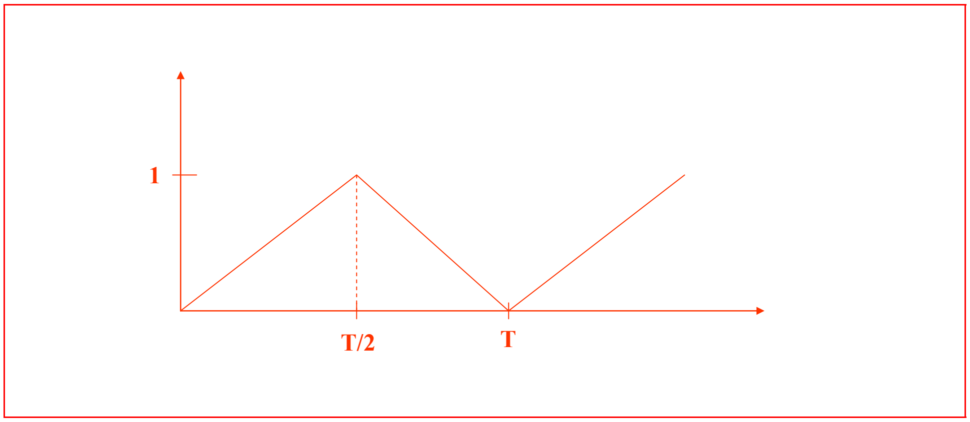
Given that T=3, we will shift the waveform, x(t), by -2 units in the y direction to determine the symmetry. This only effects the d.c. term of the Fourier series.



$$\mathbf{b_n} = \frac{2}{3}\int_0^1 2\sin\left(\frac{2n\pi}{3}t\right)dt + 0 + \frac{2}{3}\int_2^3 (-2)\sin\left(\frac{2n\pi}{3}t\right)dt$$

Problem 2–14 For the following mathematical expression, draw the corresponding waveform:

$$f(t) = \frac{2}{T}r(t) - \frac{4}{T}u(t-\frac{T}{2})r(t-\frac{T}{2}) + \frac{2}{T}u(t-T)r(t-T)$$



Problem 2–15 For Problem 2–13, write a MATLAB program to plot the first 6 Fourier coefficients of b_n .

```
>> n=1:6;                                          % first 6 coefficients
>> bn=(2./n*pi).*(2-cos(2*n*pi/3)-cos(4*n*pi/3))   %  expression for bn
bn =
1.9099    0.9549         0    0.4775    0.3820         0
```

Problem 2–16 For the square wave of Example 2–10:

a) Run the square wave program of Example 2–10 for 4, 10, 50, 100, and 200 terms. Use the MATLAB "hold" command to plot the approximations on the same graph.

b) Using MATLAB or a similar software tool, determine the amount of overshoot for each of the situations. Tabulate the percentage of overshoot given by:

$$\text{Percentage of overshoot} = \frac{\text{Amount of Overshoot}}{\text{Height of the Ideal Squarewave}} \text{ times } 100\ \%$$

c) Describe how the overshoot varies versus the number of terms.

d) Describe the frequency and shape of the "rippling" that takes place near the discontinuity.

e) Given your observations in a) through d), state the Gibbs Phenomena in your own words.

a) and b):

We will use the custom m-file **square_fourier** with the syntax:

square_fourier (N) and set the parameters to:

1 Hz Square wave

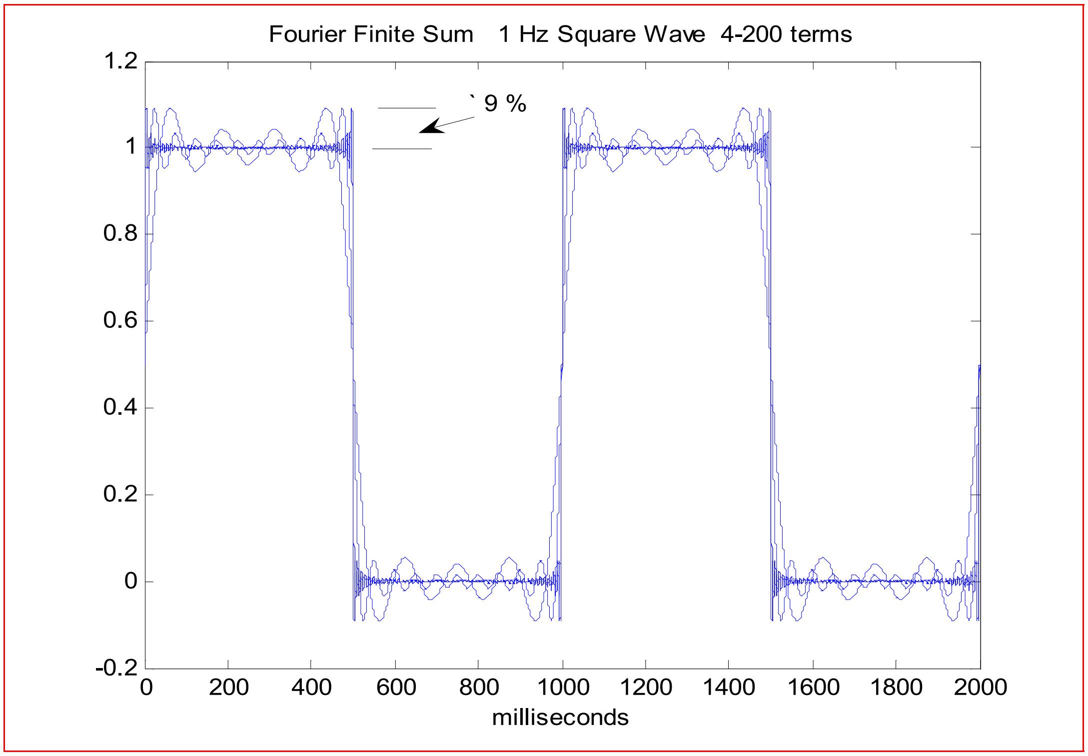
Start Time = 0 ms

End time = 2000 ms

Increment = .1 ms

N= 4, 10, 50, 100, and 200

A **hold** statement is placed after the N=4 loop is executed. Subsequent loops will plot on top of the N=4 results. The resulting graph is shown below:



c) The overshoot is the same for any number of terms, ~ 9%.

d) The rippling or oscillations becomes more compressed in the area of the discontinuity as the number of term increase.

e) The Gibb's phenomenon observes that as the number of terms in the Fourier Series is increased, the error remains the same (at ~ 9%) and the ripples or fluctuations become more compressed near the area of the discontinuity.

Problem 2–17 For a square wave with amplitude of 1 (Peak amplitude =0.5), an average value 0, and with a period of $T = 1/f_0$:

a) Determine the Fourier coefficients, a_0, a_n, and b_n of the square wave.

b) Put the solution in the form:

$$s(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos(n\omega_0 t + b_n \sin(n\omega_0 t) \right]$$

c) Explain what happens to the Fourier series expansion relative to the one described in Example 2–9. Why?

a) The $\mathbf{a_n}$ coefficients are all zero, including $\mathbf{a_0}$.

$\mathbf{b_n}$ is equal to: $\frac{1}{n\pi}[-(-1)^n + 1]$.

b) $s(t) = 0 + \frac{2}{\pi}\sum_{n=0}^{\infty}\left[\frac{\sin(2\pi f_0(2n+1)t}{2n+1}\right]$

c) The $\mathbf{a_n}$ coefficients for **n** greater than zero are exactly the same as in Example 2-9 because the period and the shape of the function remained the same. However, the a_0 coefficient became **0**. The $\mathbf{a_0}$ coefficient is zero because the signal in this problem is bipolar and has an average value (d.c.) of **0**.

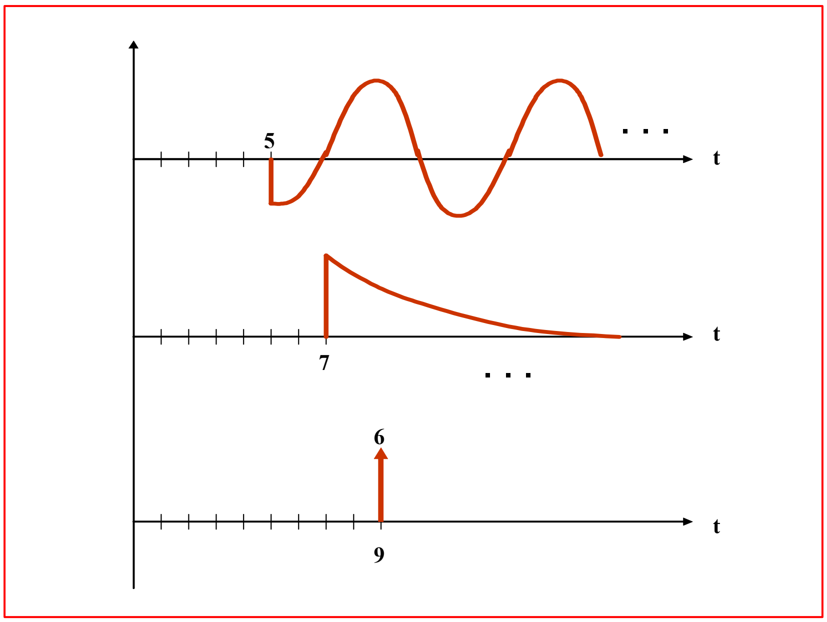
Problem 2–18 For the signal, $x(t) = u(t-5)\sin(2t) + u(t-7)e^{-4(t-2)} + 6\delta(t-9)$:

a) Show the piece-wise construction of this signal by sketching each term individually.

b) Write the equation for a discrete-time equivalent; call it x[n].

c) Create and execute a MATLAB program that plots x[n] using a "stem plot" *Note*: x[n] should be sampled at a rate adequate for accurate approximation of x(t).

a)

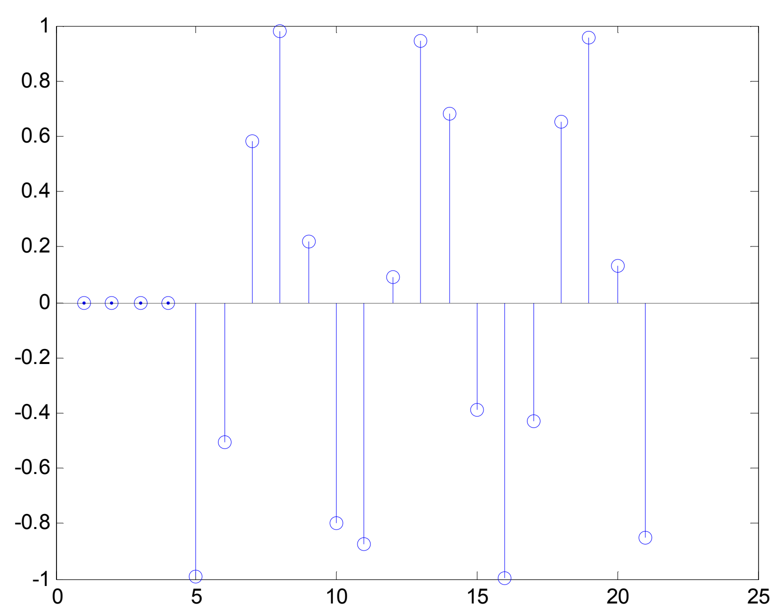


b) $x[n] = u\left(n - 5T\right)\sin(2nT) + u\left(nT - 7\right)e^{-4(nT-2)} + 6\delta(nT - 9)$

c)

First Term

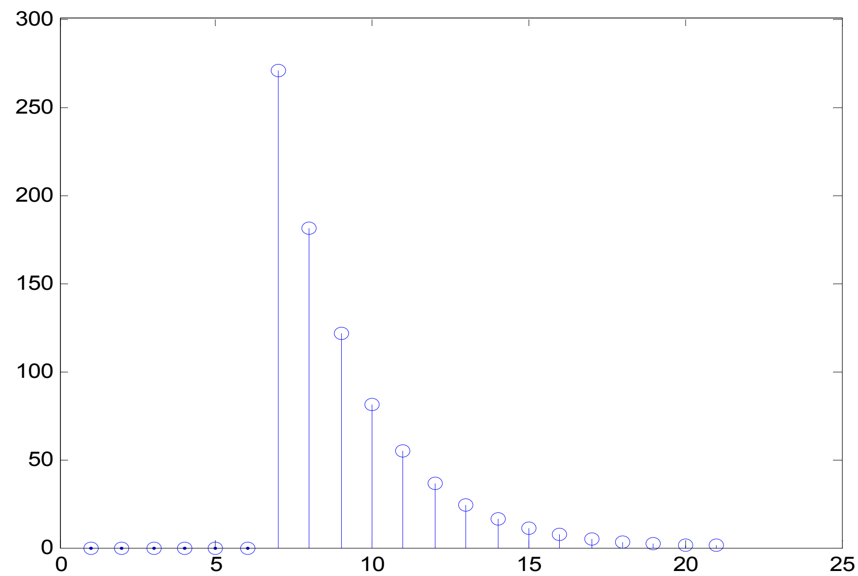
```
>> n=0:1:20;
>> T=10;                          % Pick sampling increment = 10
>> y=sin(2*n*T);                  % determine sine term
>> x=[0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1];        % determine unit step term @ n=5
>> total=x.*y;        % multiple terms together to determine the total term
>> stem(total)
```



Second Term

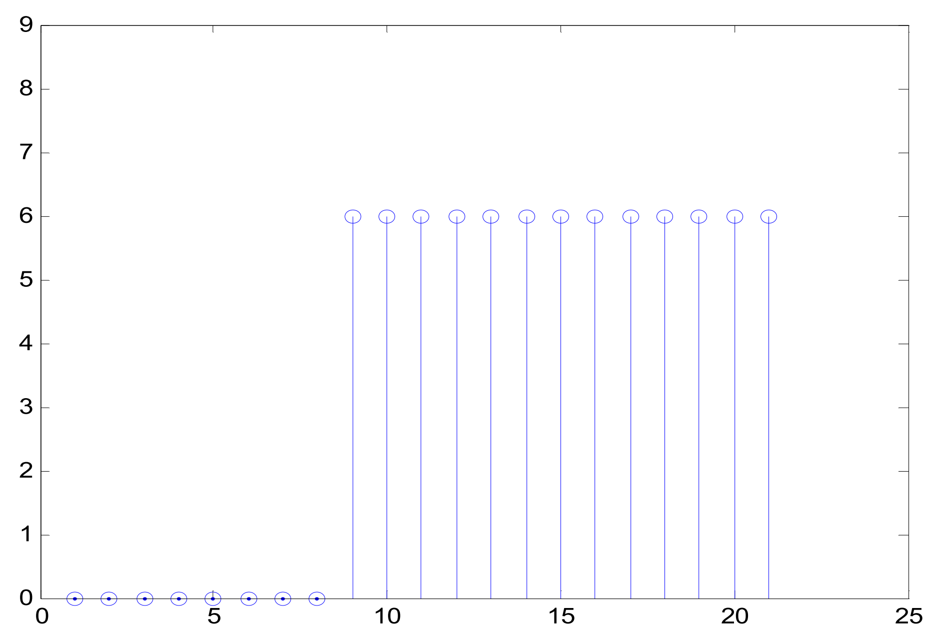
```
>> y=exp(-4*(n*T-2));
>> x=[0 0 0 0 0 0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1];
```

```
>> total=x.*y;

>> stem(total)
```



<u>Third Term</u>

```
>> x=[0 0 0 0 0 0 0 0 6 6 6 6 6 6 6 6 6 6 6 6 6];

>> stem(x)
```



Problem 2-19 A sinusoid of 10 radians per second is input to a half-wave rectifier. The half-wave rectifier maintains only positive magnitude values of the sinusoid. The negative magnitude values of the sinusoid are zero. The positive peak of the sinusoid is centered at t=0.

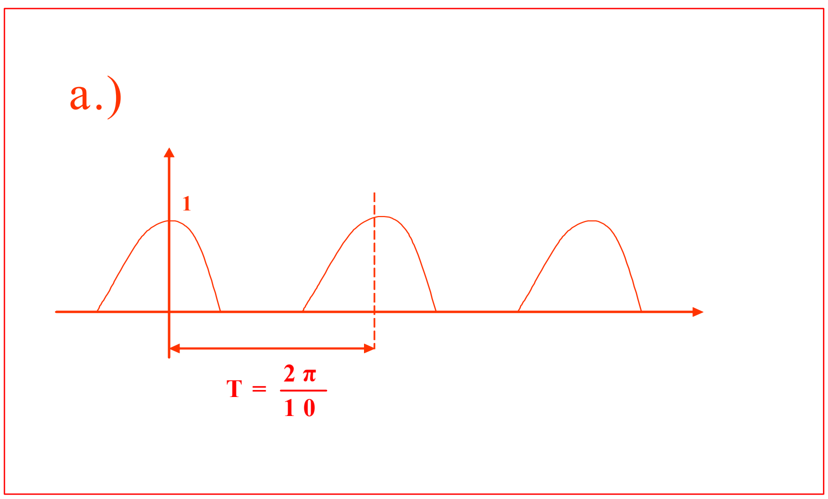
a) Draw the "rectified" sinusoid described above.

b) Determine the Fourier coefficients, a_0, a_n and b_n of the rectified sinusoid.

c) Put the solution in the form:

$$s(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)\right]$$

d) Explain what happens to the Fourier Series expansion of a sinusoid when it is run through a half-wave rectifier.



b) $b_0 = 0$, $a_0 = \frac{1}{\pi}$, $a_1 = \frac{1}{2}$, $a_n = (-1)^{\frac{n}{2}+1}\frac{2}{\pi(n^2-1)}$ for n=even

c) $\frac{1}{\pi}\left(1+\frac{\pi}{2}\cos 10\text{t}+\sum_{\text{n}=2}^{\infty}(\text{-}1)^{\frac{\text{n}}{2}+1}\frac{2}{(\text{n}^2\text{-}1)}\cos(n10t)_{\text{n=even}}\right)$

d) Because of the discontinuities created by the clipping of the sinusoid, higher-order harmonics are generated beyond the fundamental frequency of the sinusoid.

Problem 2–20 Repeat Problem 2–19 for a full-wave rectifier. Explain how each of the coefficients of the Fourier series expansion varies from those obtained in the half-wave rectifier case. Explain why certain terms vary and others don't.

$$a_n=\frac{2A}{\pi}\left(1+...(-1)^{\frac{n}{2}+1}\frac{2}{n^2-1}\cos(n\omega_1 t)\right)$$

For n=even

Only the even-numbered cosine terms are non-zero. This was the case with the half-wave rectifier except for one additional term. This additional first-order term is required to cancel out the second half cycle in the case of a half wave rectifier. The other significant difference is the magnitude term that weights the cosine terms. This makes complete sense as a full wave rectifier has twice the average voltage of a half wave rectified signal.