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THE USE OF KIRCHOFF'S CURRENT LAW AND CUT-SET EQUATIONS IN THE ANALYSIS OF BRIDGES AND TRUSSES

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Abstract - The purpose of this paper is to show that the analysis of trusses (and hence that of bridges) can be effectively carried out using the three concepts of Basic Electric Circuit Analysis, namely, the Superposition Theorem, Kirchhoff's Current Law and the Cut-set method. The curricular effect of this study is the improvement of multidisciplinary engineering education by relating the sophomore courses of Statics and Circuits and putting the courses under one common analysis framework.

Introduction

In any engineering curriculum, it is common practice to teach Statics and Basic Circuit Analysis in the sophomore year as separate subjects. In the subject of Statics [1], the analysis of bridges and trusses is taught using the two concepts based on equilibrium equations, namely (i) the algebraic sum of moments taken at a point is zero, and (ii) the algebraic sum of the various forces at any joint in each of the vertical and horizontal directions will be equal to zero.

In Basic Circuit Analysis [2], the subject matter starts with Kirchoff's Current Law (KCL). For any network, KCL states that "the vectorial sum of the various currents incident at any node is always equal to zero". Kirchhoff's Voltage Law (KVL) is not considered in this paper. In addition, when sinusoidal excitation is considered, such a current can be represented as a phasor. Also, since linear networks are considered, the principle of Superposition holds. The Superposition theorem states that "the total response in a branch is the vectorial sum of the various responses, each response being obtained when only source is considered, with all other independent sources being made equal to zero". In addition to the above, the cut-set concept is also discussed. The cut-set concept is the generalization of the KCL at a node in that the KCL holds for a surface also [3].

It is the purpose of this paper to show that the above-mentioned concepts of basic circuit analysis can be effectively used in the analysis of bridges and trusses. This common framework in the analysis of structures and circuits should be shown to the students to provide a better comprehension of the relationship between the two subjects of Statics and Circuits. For purposes of illustration, the example in [1, pp.284] will be considered in this paper with the difference that the length of each arm is arbitrary and the forces acting at junctions are independent of one another. This is as shown in Figure1.

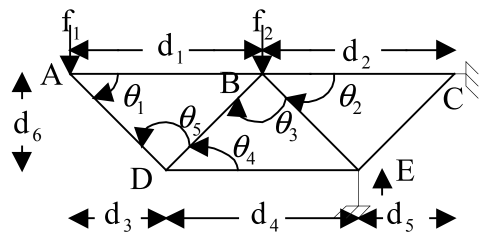

Figure 1 A general truss considered for illustration purposes.

Superposition Theorem

The general analysis requires that the moment about C is taken first. This gives

$$f_1(d_1 + d_2) + f_2d_2 - E.d_5 = 0. \tag{1}$$

The objective is to determine the vertical component E. It is readily made up of two parts E_1 (due to f_1) and E_2 (due to f_2). By making $f_2 = 0$, we get

$$E_1 = \frac{f_1\left(d_1 + d_2\right)}{d_5} \tag{2}$$

By making $f_1 = 0$, we get

$$E_2 = \frac{f_2 d_2}{d_5} \tag{3}$$

Thus,

$$E = E_1 + E_2 = \frac{f_1(d_1 + d_2) + f_2 d_2}{d_5} \tag{4}$$

This concept can be readily extended for any number of forces present. This permits us to determine the contribution of each force to E.

The above discussion makes it clear that the principle of superposition (which is the starting point and the basis for linear system theory [4]) can be readily applied here. This means that one force at a time can be considered, making all other forces equal to zero. This will enable one to determine the contribution of each force to the reactions at the hinged ends by the equilibrium equation.

In addition, the concept of scaling can be also be applied in that first, a unit force is applied and its effect can be determined. Then, the required multiplication factor is applied to obtain the actual effect. This can be illustrated as follows:

Let $f_2 = 0$ and a unit force be applied at the junction A. Then, from (2), we get

$$E_{1a} = \frac{(d_1 + d_2)}{d_5} \tag{5}$$

When a force f_1 is applied, this is multiplied by a factor f_1 so that

$$E_1 = f_1 . E_{1a} = \frac{f_1 (d_1 + d_2)}{d_5} \tag{6}$$

The factor E_{1a} is the **influence coefficient**. The effect of f_2 can be obtained in a similar fashion.

Kirchhoff's Current Law (KCL)

In order to illustrate this principle, we choose one suitable junction for the purposes of analysis. In this case, we start with the junction A, which in circuit theory terms, is a node. The corresponding force diagram, depicting both external and internal forces, is shown in Figure 2.

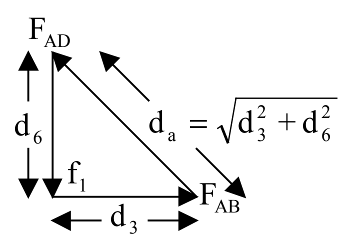

Figure 2 The force diagram for the junction A.

Analysis of the force diagram yields the equilibrium equation,

$$\frac{F_{AD}}{\sqrt{d_3^2 + d_6^2}} = \frac{F_{AB}}{d_3} = \frac{f_1}{d_6} \tag{7}$$

from which F_{AB} and F_{AD} are determined given f_1.

To use KCL, we make use of a fundamental system modeling concept that states that a force and current are analogous through variables [4][5]. This enforces the fact that the concept of the vectorial sum of forces at a joint being zero is the application of KCL to a structure. Continuing with the example, we construct a vector phasor diagram which is identical to the shape of the truss at junction A. This is as shown in Figure 3.

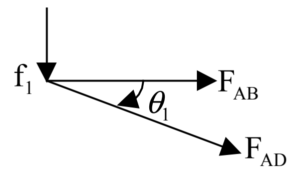

Figure 3 The vector or phasor diagram corresponding to junction A.

This diagram is made up of the various forces in each arm incident at the junction A. The forces in each arm are represented by an appropriate complex quantity. The horizontal axis becomes the real axis and the vertical axis becomes the imaginary axis. The directions of F_{AB} and F_{AD} can be chosen in an arbitrary manner initially. Application of KCL at node A gives

$$-jf_1 + F_{AB} + F_{AD}e^{-j\theta_1} = 0 \quad (8)$$

where j is a complex number ($j^2 = -1$). While writing Equation (8), the force (or equivalently, current) entering the junction (or equivalently, node) is taken to be negative and the force leaving the junction is taken to be positive. Equating the real parts, we get

$$F_{AB} + F_{AD}\cos(\theta_1) = 0 \quad (10)$$

Since

$$\cos(\theta_1) = \frac{d_3}{\sqrt{d_3^2 + d_6^2}} \quad , \quad (10a)$$

we get

$$\frac{F_{AD}}{\sqrt{d_3^2 + d_6^2}} = -\frac{F_{AB}}{d_3} \quad (10b)$$

Equation (10b) shows that the direction of one of the forces, namely either F_{AB} or F_{AD} has to be reversed. Equating the imaginary parts, we get

$$-jf_1 - j\ F_{AD}\sin(\theta_1) = 0 \quad (11a)$$

Since

$$\sin(\theta_1) = \frac{d_6}{\sqrt{d_3^2 + d_6^2}} \quad , \quad (11b)$$

we get

$$\frac{F_{AD}}{\sqrt{d_3^2 + d_6^2}} = -\frac{f_1}{d_6} \tag{12}$$

This clearly shows that the direction of F_{AD} has to be reversed, because the direction of f_1 is already given. The combination of Equations (10b) and (12) gives Equation (7).

This demonstrates clearly that KCL can be effectively used in the analysis. It is noted that this analysis gives both the magnitude and the direction of the force in each arm. It is readily seen that the horizontal component of the force is given by the real part and the vertical component of the force is given by the imaginary part.

Cut-set Equations

It is known that every node is a cut-set, in that the cut-set divides the network graph into two distinct parts [3]. Some of the cut-sets (dotted lines) are shown in Figure 4.

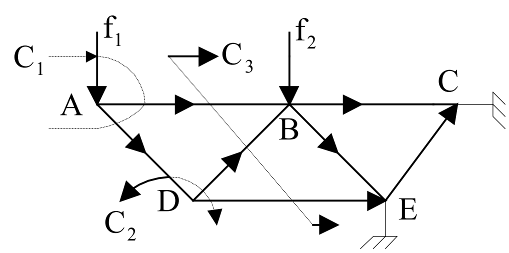

Figure 4 Some cut-sets in the graph depicting the truss.

The equation corresponding to node A is given in Equation (8). This also gives the equation corresponding to the cut-set C_1. A similar equation can be written corresponding to node D, which is as follows:

$$-F_{AD}e^{j\theta_1} + F_{DE} + F_{DB}e^{j\theta_4} = 0 \tag{13}$$

Equation (13) is also the cut-set equation corresponding to C_2. Equations (8) and (13) give the various forces along the branches AD, DE, AB and DB. Also, addition of (8) and (13) yields

$$-jf_1 + F_{AB} + F_{DB}e^{j\theta_4} + F_{DE} = 0 \tag{14}$$

This is the equation corresponding to the cut-set C_3. Equation (14) can either be used to verify the various forces already determined or used in the analysis to determine the forces. As an illustration, equating the imaginary parts in (14), we get

$$F_{DB}\sin(\theta_4) = f_1 \tag{15}$$

Since

$$\sin(\theta_4) = \frac{d_6}{\sqrt{d_6^2 + (d_1 - d_3)^2}} \quad , \tag{16}$$

we get

$$F_{DB} = \frac{f_1 . \sqrt{d_6^2 + (d_1 - d_3)^2}}{d_6} \tag{17}$$

This suffices to prove that the cut-set equations can be used effectively in the analysis. More cut-sets can be used. In fact, **Cut-set analysis** gives the same equations as the **Method of Sections**. These two methods are hence, analogous.

Similar analysis can be carried out for other nodes, thereby completely determining the various forces and their respective directions in all the branches.

Conclusions

The foregoing discussion clearly shows that the three concepts of basic circuit analysis, namely Superposition, KCL and Cut-set analysis can be effectively used in the analysis of trusses. Thus, this paper makes a thrust at multidisciplinary engineering education by showing the equivalence of structures and circuits thereby depicting the isomorphism in the analysis techniques. It is suggested that courses on Statics and Circuits can be improved and looked at under one common analysis framework.

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Biography

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Biography