GENERATION OF A CLASS OF TWO-DIMENSIONAL (2-D) TRANSFER FUNCTIONS YIELDING VARIABLE MAGNITUDE AND CONTOUR CHARACTERISTICS

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ABSTRACT

The properties of a two-dimensional (2-D) discrete transfer function with the degree of each variable being unity are discussed. The coefficients of the denominator polynomial contain a parameter k (having real values) whose bounds are determined by stability considerations. These bounds are obtained by testing the overall polynomial at only four points $z_1 = \pm 1$ and $z_2 = \pm 1$. A suitable numerator polynomial can be associated to get the overall transfer function. By varying the values of k, different magnitude and contour characteristics are obtained. Such structures can be cascaded so that the magnitude responses can be varied.

1. INTRODUCTION

There has been much research done on two-dimensional (2-D) IIR filter design [1]. The classical least-squares minimization technique whose objective function includes a penalty function term to ensure stability is formulated in [2]. A minimax criterion that is solved by a linear programming approach is discussed in [3]. The McClellan transformation can also be applied to the numerator and denominator of a one-dimensional (1-D) filter [1]. Another method is to design an analog filter and apply the bilinear transformation [4]. The singular value decomposition has been applied in [5]. Obtaining desired magnitude and group delay characteristics are discussed in [6][7][8][9].

A popular design method for variable digital filters is based on frequency transformations [10]. A more recent approach is to represent the transfer function as a multidimensional polynomial of frequencies thereby making the frequency response variable [11]. This design technique is time consuming due to the many coefficients that need to be optimized and no guarantee of stability is achieved [12]. The technique in [12] reduces the design complexity, guarantees stability and achieves approximately linear phase by decomposing the complex frequency response specifications into the product of two parts. The first part corresponds to the frequency responses of constant (not variable) 2-D filters and the second part corresponds to the desired values of 1-D polynomials which are relatively easier to approximate.

In 2-D digital filter design, it is highly desirable to be able to adjust a single parameter so as to obtain variable magnitude characteristics in that the 2-D magnitude response and the corresponding contour plots can be varied. In this paper, we present an approach that involves adjustment of a single scalar parameter to achieve variable magnitude characteristics of 2-D IIR filters. Stability is guaranteed by deriving bounds on the scalar parameter. One of the possible approaches is to use a multiplier k in the feedback path [13] which can be varied, subject to the constraints imposed on it due to the stability considerations. As has been shown in [13], the complexity of the determination of the limits of k increases with the degree of each of the variables. To a certain extent, this difficulty can be overcome by employing the graphical technique [14]. An effective alternative approach to the graphical technique, which is described in this paper, is to design a 2-D filter so that the degree of each variable is unity (the overall degree being two) and to cascade several such sections. The advantage of this method is that the stability of each section can be ensured independently. The overall response is the product of the responses of each individual section.

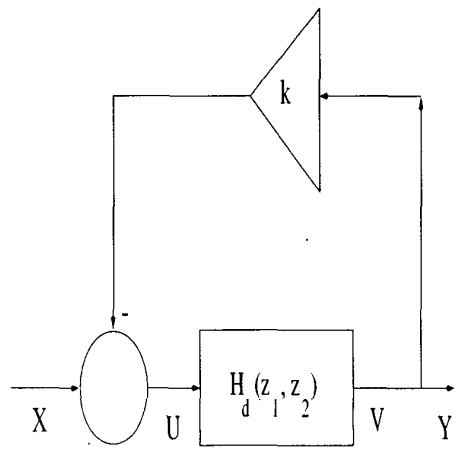

Figure 1: Basic structure considered

2. STRUCTURE AND STABILITY CONSIDERATIONS

Figure 1 shows the basic structure considered. The transfer function of the generating filter is given by

$$H_d(z_1, z_2) = \frac{V(z_1, z_2)}{U(z_1, z_2)} = \frac{N_d(z_1, z_2)}{D_d(z_1, z_2)} \quad (1)$$

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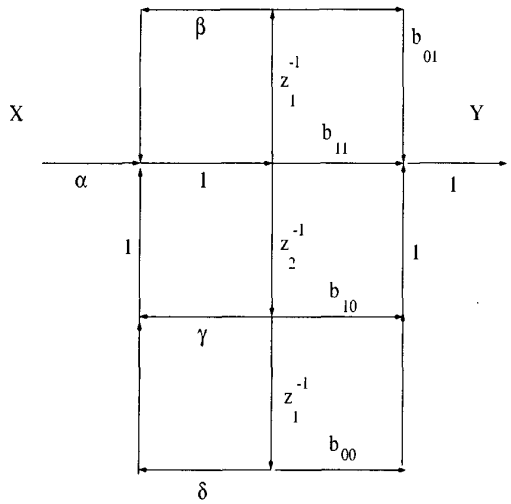

Figure 2: Signal flow graph for Type 3 filter, $\alpha = 1/(1 + kb_{11})$, $\beta = -(c_{01} + kb_{01})/(1 + kb_{11})$, $\gamma = -(c_{10} + kb_{10})/(1 + kb_{11})$ and $\delta = -(c_{00} + kb_{00})/(1 + kb_{11})$

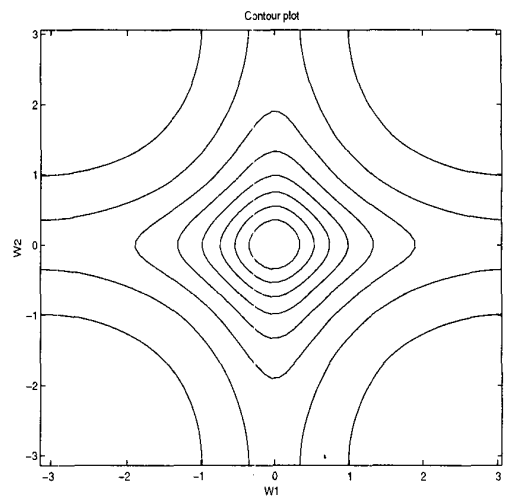

Figure 3: Contour plot of transfer function (Eq. (11)) with $k = 0$

Analysis of the structure yields

$$H(z_1, z_2) = \frac{Y(z_1, z_2)}{X(z_1, z_2)} = \frac{N_d(z_1, z_2)}{D_d(z_1, z_2) + kN_d(z_1, z_2)} \quad (2)$$

The degree in each variable is unity and hence,

$$\begin{aligned} d(z_1, z_2) &= D_d(z_1, z_2) + kN_d(z_1, z_2) \\ &= a_{11} z_1 z_2 + a_{10} z_1 + a_{01} z_2 + a_{00} \end{aligned} \quad (3)$$

Theorem 1: In order that $H(z_1, z_2)$ is always stable, the conditions to be satisfied are given by $d(1, 1) > 0$, $d(-1, -1) > 0$, $d(1, -1) < 0$ and $d(-1, 1) < 0$. The proof is omitted due to space considerations. This theorem clearly shows that stability conditions need be tested only at the points $z_1 = \pm 1$ and $z_2 = \pm 1$ and no other points need be tested on the unit bidisc.

3. CLASSIFICATION AND REALIZATIONS

Depending on where k is manifested in the overall denominator, different classifications are possible. This manifestation depends on the nature of $N_d(z_1, z_2)$. For each of these cases, the bounds on k for stability can be obtained using Theorem 1. Also, for each type, separate signal-flow graphs can be given leading to possible implementations without delay-free loops. Throughout the paper, it is assumed that the starting or generating filter $H_d(z_1, z_2)$ is stable and it does not contain any non-essential singularities of the second kind [15]. Also, without loss of generality, the form of D_d is $D_d(z_1, z_2) = z_1 z_2 + c_{10} z_1 + c_{01} z_2 + c_{00}$.

Type 1: This is the simplest type in which k occurs in only one type of term. Further subclassification is possible into three subclasses. In the first subclass, **Type 1(a)**, $N_d(z_1, z_2) = b_{00}$ and

$$H(z_1, z_2) = \frac{b_{00}}{D_d(z_1, z_2) + kb_{00}} \quad (4)$$

In the second subclass, **Type 1(b)**, k occurs only in the first order terms or terms having the factors z_1 and z_2. Then, $N_d(z_1, z_2) = b_{10} z_1 + b_{01} z_2$ and

$$H(z_1, z_2) = \frac{b_{10} z_1 + b_{01} z_2}{D_d(z_1, z_2) + k(b_{10} z_1 + b_{10} z_2)} \quad (5)$$

In the third subclass, **Type 1(c)**, k occurs only in the $z_1 z_2$ term, $N_d(z_1, z_2) = b_{11} z_1 z_2$ and

$$H(z_1, z_2) = \frac{b_{11} z_1 z_2}{D_d(z_1, z_2) + kb_{11} z_1 z_2} \quad (6)$$

Type 2: In this type, k occurs in two types of terms. There are three subclasses. In the first subclass, **Type 2(a)**, k occurs in the constant and first order terms, $N_d(z_1, z_2) = b_{10} z_1 + b_{01} z_2 + b_{00}$ and

$$H(z_1, z_2) = \frac{b_{10} z_1 + b_{01} z_2 + b_{00}}{D_d(z_1, z_2) + k(b_{10} z_1 + b_{01} z_2 + b_{00})} \quad (7)$$

In the second subclass, **Type 2(b)**, k occurs in the constant and $z_1 z_2$ terms, $N_d(z_1, z_2) = b_{11} z_1 z_2 + b_{00}$ and

$$H(z_1, z_2) = \frac{b_{11} z_1 z_2 + b_{00}}{D_d(z_1, z_2) + k(b_{11} z_1 z_2 + b_{00})} \quad (8)$$

In the third subclass, **Type 2(c)**, k occurs in the first order and $z_1 z_2$ terms, $N_d(z_1, z_2) = b_{11} z_1 z_2 + b_{10} z_1 + b_{01} z_2$ and

$$H(z_1, z_2) = \frac{b_{11} z_1 z_2 + b_{10} z_1 + b_{01} z_2}{D_d(z_1, z_2) + k(b_{11} z_1 z_2 + b_{10} z_1 + b_{01} z_2)} \quad (9)$$

Type 3: In this type, k occurs in all three types of terms. Accordingly, the only possible transfer function is

$$H(z_1, z_2) = \frac{b_{11} z_1 z_2 + b_{10} z_1 + b_{01} z_2 + b_{00}}{D_d(z_1, z_2) + k(b_{11} z_1 z_2 + b_{10} z_1 + b_{01} z_2 + b_{00})} \quad (10)$$

resulting from $N_d(z_1, z_2) = b_{11} z_1 z_2 + b_{10} z_1 + b_{01} z_2 + b_{00}$. A signal flow graph for the Type 3 filter is given in Fig. 2. The Type

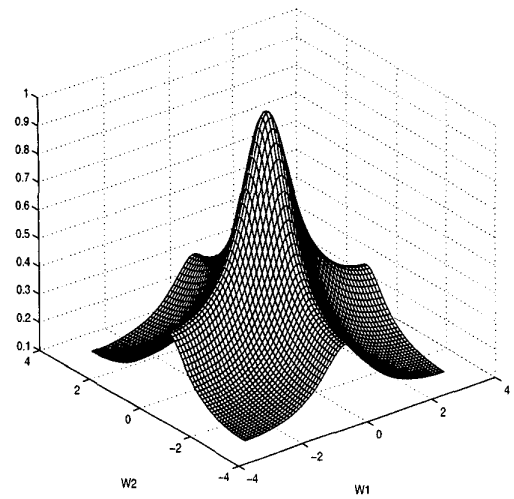

Figure 4: Magnitude response plot of transfer function (Eq. (11)) with $k = 0$

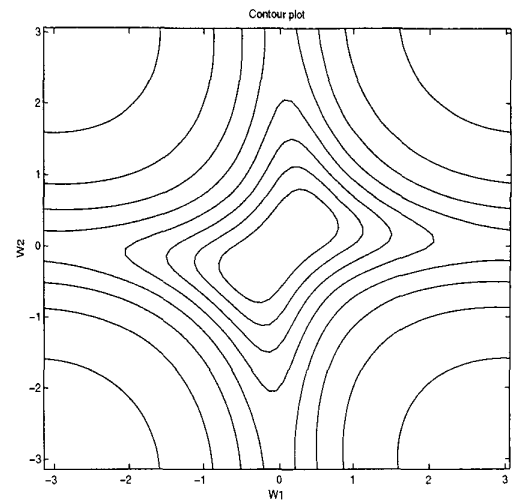

Figure 5: Contour plot of transfer function (Eq. (11)) with $k = 0.1$

1 and Type 2 filters are special cases of the Type 3 filter in that certain terms in $N_d(z_1, z_2)$ are zero. Simpler signal flow graphs can be obtained for Type 1 and Type 2 filters but are not shown due to space constraints.

4. NUMERICAL EXAMPLE

This example shows how the contour plots vary as the value of k is varied. For purposes of illustration, a Type 1 (a) filter is considered with $N_d(z_1, z_2) = 1$, and $D_d(z_1, z_2) = (z_1 - 0.5)(z_2 - 0.5)$. The transfer function is

$$H(z_1, z_2) = \frac{1}{(z_1 - 0.5)(z_2 - 0.5) + k} \qquad (11)$$

Using theorem 1, it can be determined that $-0.25 < k < 0.75$ is required for stability. Figures 3 and 4 show the contour and magnitude response plots of the transfer function in Eq. (11) when $k = 0$. Figures 5 and 6 show the contour plots of the transfer function in Eq. (11) when $k = 0.1$ and $k = -0.1$ respectively. Figures 7 and 8 show the contour plots of the transfer function in Eq. (11) when $k = 0.2$ and $k = -0.2$ respectively. There is a clear variability in the responses with varying k. The value $k = -0.2$ is close to the threshold of instability.

5. SUMMARY AND CONCLUSIONS

This paper considers the generation of 2-D discrete-domain transfer functions having the following properties: (1) the degree in each variable is unity, and (2) by changing the value of the feedback multiplier coefficient k, the magnitude and the contour characteristics can be varied. The denominator polynomial of the transfer function can be derived by a basic structure. Seven different types are possible, depending on the term(s) in which k occurs. For each type, the bounds on k to ensure stability are determined by testing the overall denominator at only four points, namely, $z_1 = \pm 1$ and $z_2 = \pm 1$. This aspect of testing stability at only four extreme points is a big advantage in terms of numerical complexity. It is also shown that by varying the multiplier value k (within the bounds of stability), the magnitude and the contour characteristics can be varied. By cascading several structures, different characteristics can be obtained.

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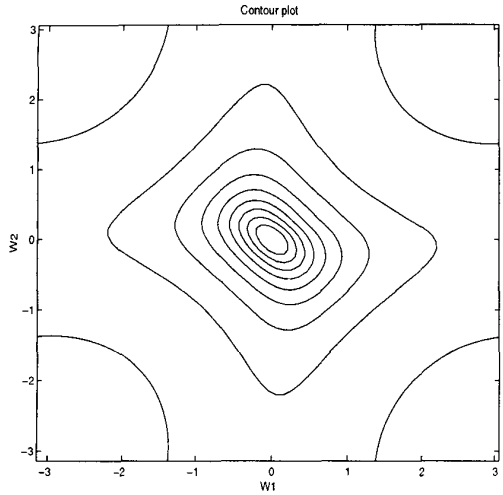

Figure 6: Contour plot of transfer function (Eq. (11)) with $k = -0.1$

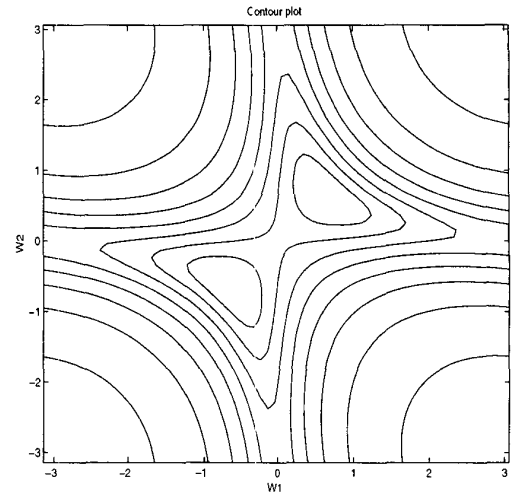

Figure 7: Contour plot of transfer function (Eq. (11)) with $k = 0.2$

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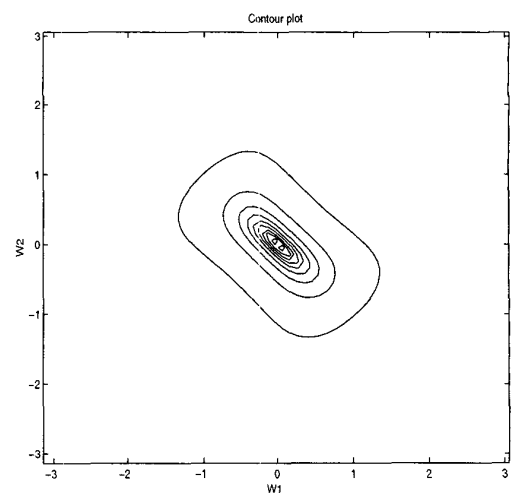

Figure 8: Contour plot of transfer function (Eq. (11)) with $k = -0.2$