MODIFICATION OF FILTER RESPONSES BY THE GENERALIZED BILINEAR TRANSFORMATIONS AND THE INVERSE BILINEAR TRANSFORMATIONS.

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Abstract

In this paper, modification of filter responses by the application of a generalized bilinear transformation and the inverse bilinear transformation has been investigated. The responses of low-pass, high-pass, band-pass and band-elimination filters are considered.

Introduction

In a previous paper [1], the generalized bilinear transformation

$$s = k\frac{z + a_0}{z + b_0} \qquad ..(1)$$

has been studied and some of its properties have been obtained. In particular, the study has been focused on the determination of conditions of stability of the transfer function, when the transformation (1) is applied. It has been shown that the conditions for stability for k > 0 are:

$$|a_0| \le 1 \qquad ..(2a)$$

$$|b_0| \le 1 \qquad ..(2b)$$

and

$$a_0 b_0 < 0 \qquad ..(2c)$$

In this paper, we study the effects of the applications of these generalized bilinear transformations coupled with the inverse bilinear transformation

$$z = \frac{1+S}{1-S} \qquad ..(3)$$

(S is used in order to distinguish the new analog transfer function from the starting analog transfer function).

Transformations Applied to an All-Pole Analog Transfer Function

A convenient starting point is an all-pole analog transfer function $H_a(s)$. This can always be realized as a singly-terminated or doubly-terminated low-pass ladder network [2] with inductors in the series arm and capacitors in the shunt arm. The other filters can be obtained by the application of suitable frequency transformations to this starting filter [2]. This could also be considered as the starting point for the design of 1-D wave digital and other filters [3]. Any other positive value of k will either increase or decrease the bandwidth of the pass-band.

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(a) Application of low-pass transformation:

For this case, we have $b_0 = 1$ and a_0 can be changed, subject to the condition of (2a). Without loss of generality, we assume $k = 1$. The digital transfer function is readily obtained. For the resulting digital transfer function, if the inverse bilinear transformation (3) is applied, it can be shown that the new analog transformation is obtained as

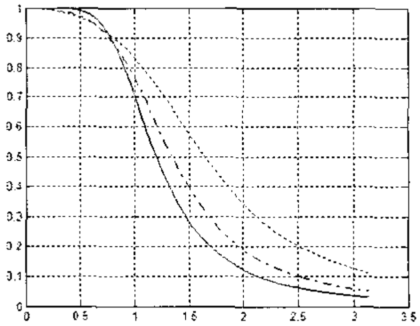
$$s \rightarrow \frac{(1+a_0)}{2}\left[S+\frac{1-a_0}{1+a_0}\right] \qquad ..(4)$$

This clearly shows that the resulting low-pass transfer function in S is always stable. Also, an inductor of value L in the starting filter is replaced by a series combination of an inductor of value $\{L(1+a_0)/2\}$ and a resistor of value $\{L(1-a_0)/2\}$ in the resulting filter. Also, a capacitor C in the starting filter is replaced by a parallel combination of a capacitor of value $\{C(1+a_0)/2\}$ and a resistor of value $2/\{C(1-a_0)\}$. The frequency response of the resulting function is obtained by the substitution of $s = (0.5)[(1-a_0)+j\omega(1+a_0)]$.

Numerical Example 1 : We start with an all-pole third–order Butterworth filter, whose transfer function in the analog domain is given by

$$H_{3a}(s) = \frac{1}{s^3+2s^2+2s+1} \qquad ..(5)$$

Fig.1 shows the magnitude responses of the modification for different values of a_0.



a_0 = - 1 ___ a_0 = - 0.8 _._._
a_0 = - 0.5 ....

Fig.1: Modification of the filter response due to low-pass transformation.

(b) Application of high-pass transformation:

For this case, we have $b_0 = -1$ and a_0 can be changed, subject to the condition of (2a). Without loss of generality, we assume $k = 1$. The digital transfer function is readily obtained. For the resulting digital transfer function, if the inverse bilinear transformation (3) is applied, it can be shown that the new analog transformation is obtained as

$$s \rightarrow \frac{(1-a_0)}{2S}\left[S+\frac{1+a_0}{1-a_0}\right] \qquad ..(6)$$

This clearly shows that the resulting high-pass transfer function in S is always stable. Also, an inductor of value L in the starting filter is replaced by a series combination of a capacitor of value $2/\{L(1+a_0)\}$ and a resistor of value $\{L(1-a_0)/2\}$ in the transformed filter. A capacitor of value C is replaced by a parallel combination of an inductor of value $2/\{C(1+a_0)\}$ and a resistor of value $2/\{C(1-a_0)\}$. Also, the frequency response of the resulting function is obtained by the substitution of s = $\left[\left(\frac{1-a_0}{2}\right)+\frac{(1+a_0)}{j(2\omega)}\right]$.

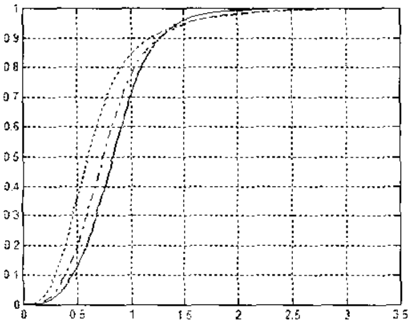
<u>Numerical Example 2</u> : We start with the same transfer function given by (5). The high-pass transformation is applied. Fig.2 gives the modification of the responses for different values of a_0.

(c) Application of band-pass transformation

We consider one possibility given by

$$s \to k_1 \frac{z-a_1}{z+1} + k_2 \frac{z+a_2}{z-1} \qquad ..(7)$$

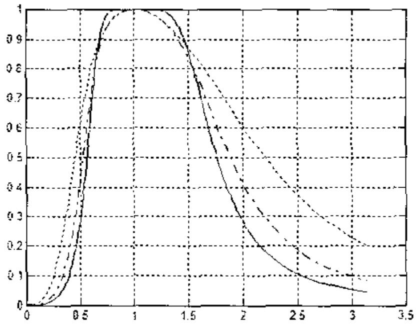
This can be considered as a combination of a low-pass and a high-pass filter. There are other possibilities and are not considered here.



a_0 = - 1 ___ a_0 = - 0.8 _._._
a_0 = - 0.5 ....

Fig.2: Modification of the filter response due to high-pass transformation.

When condition (2a) is applied, it is seen that the resulting transfer function is stable. It can be readily shown that, in the resulting transfer function in S, an inductor of value L in the starting filter is replaced by a series combination of an inductor $\{k_1L(1+a_1)/2\}$, a resistance of $\{L[(1-a_1)k_1+(1-a_2)k_2]\}/2$ and a capacitor of $2/[k_2L(1+a_2)]$. A capacitor of value C in the starting filter is replaced by a parallel combination of a capacitor of $k_1C(1+a_1)/2$, a resistor of $2/C\{(1-a_1)k_1+(1-a_2)k_2\}$ and an inductor of $2/\{kC(1+a_2)\}$. The frequency response is readily obtained. Fig.3 shows the different responses, the starting point being the transfer function given in (5).



a_0 = - 1 ___ a_0 = - 0.8 _._._
a_0 = - 0.5 ....

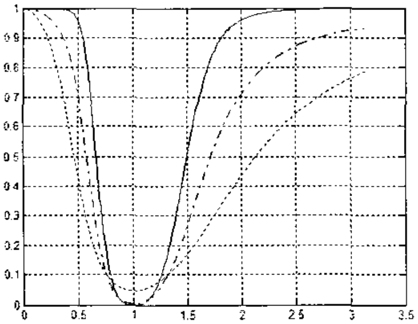
Fig.3: Modification of the filter response due to the band-pass transformation considered.

(d) Application of band-elimination transformation

We consider one possibility given by

$$s \to 1/(k_1 \frac{z-a_1}{z+1} + k_2 \frac{z+a_2}{z-1}) \qquad ..(8)$$

This can be considered as the reciprocal of the transformation considered in the previous case. There are other possibilities and are not considered here. When condition (2a) is applied, it is seen that the resulting transfer function is stable. It can be readily shown that, in the resulting transfer function in S, an inductor of value L in the starting filter is replaced by a parallel combination of a capacitor $\{k_1(1 + a_1)\}/2L$, a resistance of $2L/\{(1-a_1)k_1 + (1-a_2)k_2\}$ and an inductor of value $2L/\{k_2(1 + a_2)\}$. A capacitor of value C in the starting filter is replaced by a series combination of an inductor of $\{k_1(1+a_1)\}/2C$, a resistor of $\{(1-a_1)k_1+(1-a_2)k_2\}/2C$ and a capacitor of $2C/\{k_2(1+a_2)\}$. The frequency response is readily obtained. Fig.4 shows the different responses, the starting point being the transfer function given in (5).



a_0 = - 1 ___ a_0 = - 0.8 _._._
a_0 = - 0.5 ....

Fig.4: Modification of the filter response due to the band-elimination transformation considered.

Summary and Discussions

It is amply demonstrated that a filter response can be modified by the application of a suitable general bilinear transformation, providing the designer with the modified digital transfer function. The cases considered are the low-pass, high-pass, band-pass and the band-elimination transformations. These are applied to a starting singly or doubly terminated analog low-pass reactance network. The modified digital transfer function can be realized suitably. It is also shown that this modified digital filter has an analog counterpart where an inductance is replaced by a series combination of an inductor and a resistor, and a capacitor is replaced by a parallel combination of a capacitor and a resistor. Even though a Butterworth filter response is considered as the starting point, any other filter can be subjected to this treatment in order to modify the response. In all cases, a large family of responses will be possible.

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