A STUDY OF THE EFFECTS OF A DOUBLE 1-D BILINEAR TRANSFORMATION ON THE FREQUENCY RESPONSES OF SOME BASIC FILTERS

C.S.Gargour[1], V.Ramachandran[2] and Ravi P.Ramachandran[3]

[1] École de Technologie Supérieure, 1100 Notre-Dame West, Montreal, QC, Canada, H3C 1K3.

[2] Department of Electrical and Computer Engineering, Concordia University, Montreal, QC, Canada, H3G 1M8.

[3] Department of Electrical and Computer Engineering, Rowan University, Glassboro, New Jersey, U.S.A., 08012.

Abstract

In this paper, the effects of the successive application of a generalized bilinear transformation and of the inverse bilinear transformation to a starting analog low-pass filter and an all-pass filter have been studied.

Introduction

The generalized bilinear transformation

$$s = k\frac{z + a_0}{z + b_0} \qquad (1)$$

has been considered in [1] and some of its properties have been studied. Among these, the conditions by which should be satisfied by k, a_0 and b_0 in order that the discrete transfer function resulting from the application of this transformation to a stable analog transfer function, is always stable, have been obtained. Specifically, for k > 0. they are

$$|a_0| \le 1, \qquad (2a)$$

$$|b_0| \le 1, \qquad (2b)$$

and

$$a_0 b_0 < 0 \qquad (2c)$$

For the resulting discrete transfer function so obtained, one can now apply the familiar inverse transformation

$$z = \frac{1+S}{1-S} \qquad (3)$$

so that a new analog filter transfer function is obtained. (The capital letter S is used to distinguish it from that of (1) only). By this process, one gets a different type of filter whose frequency response is modified as compared to that of the starting filter. Some cases are discussed below:

Modification of Transfer functions

Any transfer function can be written as a product of several functions, each being of order one or two. Therefore, one can study the effects of these transformations on such lower order blocks. The overall modified transfer function is the product of these modified lower order blocks.

(a) First-order transfer function

We shall first study the starting first-order analog transfer function given by

$$H_1(s) = \frac{d_1 s + d_0}{c_1 s + c_0} \qquad (4)$$

with $c_1 > 0$ and $c_0 > 0$, because of stability considerations. Application of transformation given by (1) results in

$$H_{d1}(z) = \frac{z(d_1k + d_0) + (d_1ka_0 + d_0b_0)}{z(c_1k + c_0) + (c_1ka_0 + c_1b_0)} \qquad (5)$$

When (5) is subjected to transformation (3), one gets

$$H_{a1}(S) = \frac{S[d_1k(1-a_0) + d_0(1-b_0)] + [d_1k(1+a_0) + d_0(1+b_0)]}{D_{a1}(S)} \qquad (6a)$$

where $D_{a1}(S)$ =

$$S[c_1k(1-a_0) + c_0(1-b_0)] + [c_1k(1+a_0) + c_0(1+b_0)] \qquad (6b)$$

Particular cases can now be considered.

Case (i) : Let $d_1 = 0$ and $d_0 = c_0$, giving in (4), an all-pole filter with response at zero frequency being unity. We get

$$H_{a1}(S) = \frac{S[c_0(1-b_0)] + [c_0(1+b_0)]}{D_{a1}(S)} \qquad (7)$$

Case (ii) : Let $d_1 = -c_1$ and $d_0 = c_0$ giving in (4) an all-pass function. We get

$$H_{a1}(S) = \frac{S[-c_1k(1-a_0) + c_0(1-b_0)] + [-c_1k(1+a_0) + c_0(1+b_0)]}{D_{a1}(S)} \qquad (8)$$

It is noted that (8) is not an all-pass transfer function in general.

(b) Second-order transfer function

A general second-order analog transfer function is given by

$$H_2(s) = \frac{d_2s + d_1s + d_0}{c_2s + c_1s + c_0} \qquad (9)$$

with $c_2 > 0$, $c_1 > 0$ and $c_0 > 0$ because of stability considerations. When transformation (1) is applied to (9), we get the modified transfer function as

$$H_{d2}(z) = \frac{z^2(d_2k^2 + d_1k + d_0) + z(2k^2d_2a_0 + ka_0d_1 + kb_0d_1 + 2b_0d_0) + (k^2a_0^2d_2 + ka_0b_0d_1 + d_0b_0^2)}{D_{d2}(z)} \qquad (10a)$$

where $D_{d2}(z)$ =

$$z^2(c_2k^2 + c_1k + c_0) + z(2k^2c_2a_0 + ka_0c_1 + kb_0c_1 + 2b_0c_0) + (k^2a_0^2c_2 + ka_0b_0c_1 + c_0b_0^2) \qquad (10b)$$

For $H_{d2}(z)$, we can apply the general bilinear transformation given in (2). The resulting transfer function is given by

$$H_{a2}(S) = \frac{N_{a2}(S)}{D_{a2}(S)} \qquad (11a)$$

where $N_{a2}(S)$ =

$$S^2\{k^2d_2(1-a_0)^2 + kd_1(1-a_0)(1-b_0) + d_0(1-b_0)^2\} + S\{k^2d_2(1-a_0^2) + kd_1(1-a_0b_0) + d_0(1-b_0^2)\} + \{k^2d_2(1+a_0)^2 + kd_1(1+a_0)(1+b_0) + d_0(1+b_0^2)\} \qquad (11b)$$

and $D_{a2}(S)$ =

$$S^2\{k^2c_2(1-a_0)^2 + kc_1(1-a_0)(1-b_0) + c_0(1-b_0)^2\} + S\{k^2c_2(1-a_0^2) + kc_1(1-a_0b_0) + c_0(1-b_0^2)\} + \{k^2c_2(1+a_0)^2 + kc_1(1+a_0)(1+b_0) + c_0(1+b_0^2)\} \qquad (11c)$$

As before, we can examine some particular cases.

Case(i) :Let $d_2 = 0$, $d_1 = 0$ and $d_0 = c_0$, giving in (9) an all-pole filter with unity response at zero frequency. We obtain

$$H_{a2}(S) = \frac{c_0}{D_{a2}(S)}\left\{ \begin{matrix} S^2(1-b)^2 + S(1-b_0^2) + \\ (1+b_0)^2 \end{matrix} \right\} \qquad (12)$$

Case(ii) : Let $d_2 = c_2$, $d_1 = -c_1$ and $d_0 = c_0$, giving in (9) an all-pass transfer function. We obtain

$$H_{a2}(S) = \frac{N_{b2}(S)}{D_{a2}(S)} \qquad (13a)$$

where $N_{b2}(S) =$
$$S^2\{k^2c_2(1-a_0)^2 - kc_1(1-a_0)(1-b_0) + c_0(1-b_0)^2\} + S\{k^2c_2(1-a_0^2) - kc_1(1-a_0b_0) + c_0(1-b_0^2)\} + \{k^2c_2(1+a_0)^2 - kc_1(1+a_0)(1+b_0) + c_0(1+b_0^2)\} \qquad (13b)$$

It is noted that (13a) is not an all-pass function in general.

Numerical Examples

In each of the following examples, three cases are considered and they are: (a) $b_0 = 1$, $k = 1$ and $a_0 = -1$(given by ____ curves), (b) $b_0 = 1$, $k = 1$ and $a_0 = 0.25$ (given by ……. curves) and (c) $b_0 = 1$, $k = 1$ and $a_0 = -0.25$ (given by _ _ _ curves)

Example 1 [2}: We consider a fourth-order all-pole Butterworth filter. Fig.1 shows the curves of the frequency responses for the three cases.

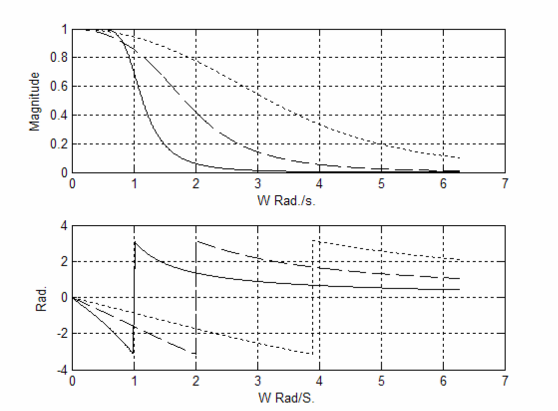

Fig.1

Example 2 [2] : We consider a fourth-order all-pass transfer function, whose denominator corresponds to the fourth order Butterworth polynomial. Fig.2 shows the corresponding curves of the frequency responses (magnitude and phase) responses for the three cases.

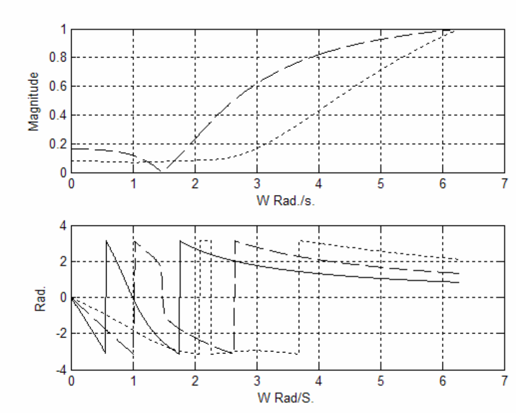

Fig.2.

Example 3 [2] : We consider a fourth-order all-pole Chebyshev filter having a ripple width of 1.25 dB in the pass-band. The corresponding analog transfer function is given by

$$H_{a4}(s) = \frac{0.2164}{s^4 + 0.8758s^3 + 1.3835s^2 + 0.6645s + 0.2499}$$

Fig.3 shows the corresponding frequency responses (magnitude and phase).

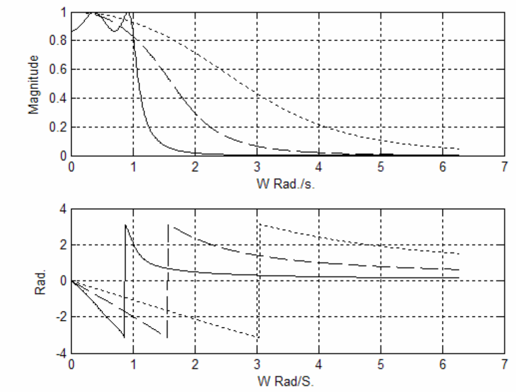

Fig.3.

Example 4 [2]: We consider a fourth-order all-pass transfer function, whose denominator corresponds to the denominator of the Chebyshev filter considered in Example 3. Fig.4 shows the corresponding frequency responses (magnitude and phase).

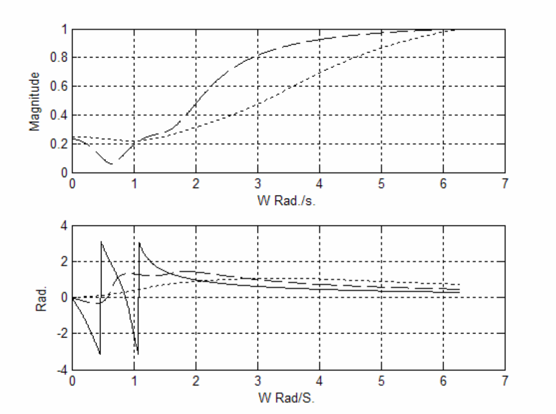

Fig.4.

Summary and Discussions

In this paper, a study has been made to investigate the effects of applying a combination of a generalized bilinear transformation coupled with the inverse bilinear transformation. In all the cases considered, the value of a_0 has been varied, while b_0 is chosen to be unity in order to preserve the low-pass nature of the starting filter. The value of k is chosen to be unity. By varying the value of k, the bandwidth of the pass-band is changed. These values ensure the stability of the overall system. It has been found that under these conditions, the resulting magnitude responses tend to become flat, while the bandwidth varies depending upon the value of a_0. It has also been found that the stop-band characteristics depend on the starting filter.

In particular, in the Butterworth low-pass filter case, the monotonic response is maintained. Also, in the Butterworth all-pass case, a notch or null is found to appear. When a Chebyshev low-pass filter is considered, the ripples tend to move towards the stop-band and the response is not equiripple in all the cases. When the Chebyshev all-pass case is considered, here also a notch or a null is found to appear.

This shows that interesting modifications of different types of filters can be obtained, which could lead to desirable forms of responses.

REFERENCES

[1] C.S.Gargour, V. Ramachandran, Ravi P.Ramachandran and F.Awad, Variable Magnitude Characteristics of 1-D IIR Filters by a Generalized Bilinear Transformation, 43rd Midwest Symposium on Circuits and Systems, Michigan State University, Michigan, U.S.A., August 8-11, 2000, (Four Pages), Session FAP-2.

[2] C.S.Gargour and V.Ramachandran, Theorie et Conception des Filtres Analogiques, Presses de l'Universite du Quebec, 1993.