A FAMILY OF LOW-PASS FILTERS HAVING PSEUDO-MONOTONIC MAGNITUDE RESPONSE APPROXIMATING LINEAR PHASE

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Abstract

A new approximation to obtain close to linear phase is presented in this paper. This is based on the rational fraction expansion of the hyberbolic cotangent function. The resulting transfer function is always of odd order. It is shown that a good approximation of the phase is obtained in the pass-band. The magnitude response at most places is monotonic in character, but does show a degree of flatness on a portion of the frequency spectrum.

Key words: linear phase, pseudo monotonic response

1. Introduction

Transfer functions approximating linear phase response have been studied, because of their extensive use in telecommunications applications. In particular, maximally flat and equidistant linear phase polynomials are discussed in [1, 2]. Bessel polynomials giving maximally flat group delay and hence linear phase are considered in [3]. A new approximation for obtaining constant group delay is discussed in [4].

In this paper, a new approximation to obtain linear phase characteristic is formulated. Along with the linear phase approximation, the magnitude response is considered and its different properties are studied.

2. The Approximation

The starting point is:

$$e^{-s} = \frac{1}{\text{Cosh(s)} + \text{Sinh(s)}} = \frac{\frac{1}{\text{Sinh(s)}}}{1 + \text{Coth(s)}} \quad ..(1)$$

The function Coth(s) can be expanded into a rational fraction expansion [5] as

$$\frac{1}{s} + \sum_{\ell=1}^{\infty} \frac{2s}{s^2 + \ell^2\pi^2} \quad ..(2)$$

This is an infinite series and has to be suitably truncated to $\ell = n$ only such that, a rational function in 's' is obtained, where the denominator is an odd polynomial and the numerator is an even one. Since (2) represents a reactance function which can always be realized by a series combination of networks each consisting of a parallel combination of an inductor and a capacitor [3], the sum of the numerator and the denominator yields a Strictly Hurwitz Polynomial (SHP). These SHPs can be used as the denominators $D_n(s)$ of a family of an all-pole transfer function $T_n(s)$. As can be readily seen, only odd orders of polynomials can be generated. Table I gives $D_n(s)$ for various orders up to n = 11. $N_n(s)$ represents the odd part of $D_n(s)$ and $M_k(s)$ gives the even part of $D_k(s)$.

Table I

n	$N_n(s)$ and $M_n(s)$
1	$N_1 = s$ $M_1 = 1$
3	$N_3 = s(s^2+\pi^2)$ $M_3 = 3s^2 + \pi^2$
5	$N_5 = s(s^2+\pi^2)(s^2+4\pi^2)$ $M_5 = 5s^4 +15\pi^2s^2+4\pi^4$
7	$N_7 = s(s^2+\pi^2)(s^2+4\pi^2)(s^2+9\pi^2)$ $M_7 = 7s^6+70\pi^2s^4 +147\pi^4s^2+36\pi^6$
9	$N_9 = s(s^2+\pi^2)(s^2+4\pi^2)(s^2+9\pi^2)(s^2+16\pi^2)$ $M_9 = 9s^8+210\pi^2s^6+1165\pi^4s^4 +2460\pi^6s^2+576\pi^8$
11	$N_{11}= s(s^2+\pi^2)(s^2+4\pi^2)(s^2+9\pi^2)(s^2+16\pi^2)(s^2+25^2)$ $M_{11} = 11s^{10}+495\pi^2s^8+7161\pi^4s^6+38225\pi^6s^4+63228\pi^8s^2+14400\pi^{10}$

Fig.1(a) gives the various phase responses up to n = 11.

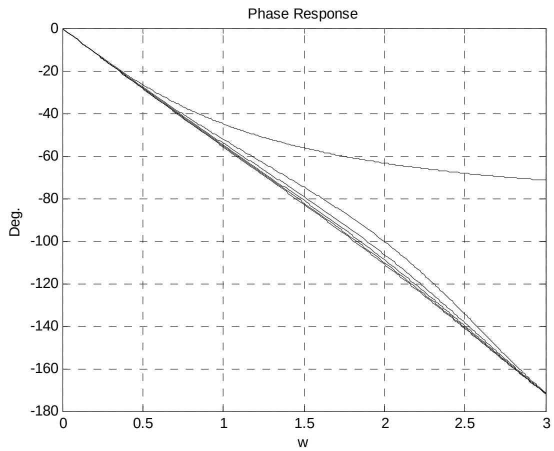

Fig.1(a): The various phase responses

A close examination of these responses clearly shows that the departure from linear phase for various orders can be considered minimal. From Eq.(2), it is seen the phase will be the same as that of the linear response whenever $\omega = n\pi$.

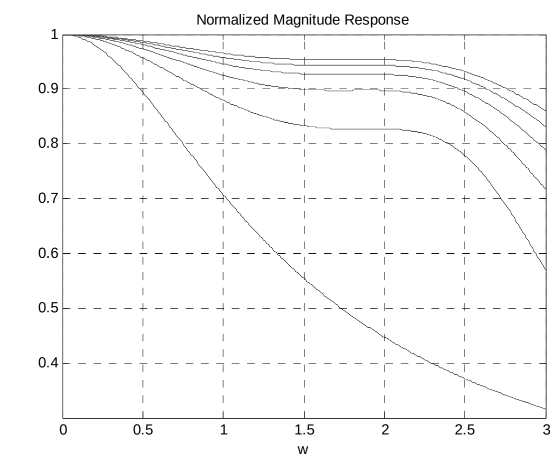

Fig.1(b): The magnitude responses

Fig.1(b) shows the magnitude responses of all-pole transfer functions of various orders up to n =11. The numerator constant is adjusted such that at zero frequency, the response is unity. It is seen that the pass-band of the magnitude response is always less than $\omega = \pi$. This means that within the pass-band, the phase approximation should be considered as good. It is further noted that the magnitude response exhibits a different type of behavior in that there exist some flatness over a range of frequencies and hence is quite different from the well-known Butterworth and/or Chebyshev responses.

Summary and Conclusions

A new type of approximation for linear phase has been introduced, starting from the rational fraction expansion of the hyperbolic cotangent function and truncating it appropriately. It is shown that the approximation for linear phase is quite good particularly in the pass-band. In addition, the magnitude response is not exactly monotonic and exhibits some degree of flatness during a portion of the frequency and hence is different from the familiar Butterworth or Chebyshev response. It has to be pointed out that only odd order approximations are possible by this approach, in that the resulting transfer functions are of odd order.

References

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