M&Ms Quality Control: A Chi-Square Analysis

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Have you ever wondered why packages of M&Ms never seem to have enough blue M&Ms? Does it seem like you always get a package of mostly brown M&Ms? In this exercise, we are going to conduct a chi-square (χ^2) goodness-of-fit analysis to determine whether the color proportions for a sample of milk chocolate (i.e., "plain") M&Ms are consistent with the expected proportions as reported on the manufacturer's website (http://us.mms.com/us/about/products/milkchocolate/). Our sample will consist of one "fun-size" bag of M&Ms per student attending class tonight; we will analyze the pooled class data. The expected color proportions are as follows:

Color	Proportion (p)
Brown	0.13
Blue	0.24
Orange	0.20
Green	0.16
Red	0.13
Yellow	0.14

The χ^2 goodness-of-fit test allows us to determine if the observed frequency distribution (the color counts from our M&Ms sample) is significantly different from the expected frequency distribution (calculated from the claimed proportions on the M&Ms website). You can read more about this test statistic in section 11-2 of your textbook.

✎ **Before you begin, state and symbolize the null hypothesis that you will be testing.**

To test this hypothesis, you will need to calculate the χ^2 goodness- of-fit test statistic,

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

where O is the observed frequency and E is the expected frequency for each color. You can compute E for each color by multiplying the total number of M&Ms in our sample (n) by the claimed proportion for that color (p).

Count the number of M&Ms of each color in your bag (without eating any M&Ms until after you finish counting). Record the color counts for your bag of M&Ms in Table 1, and add your results to the class data tally. When everyone has reported their data, write down the class color totals in column 1 of Table 2 and then proceed to calculate the goodness-of-fit test statistic. (Refer to the chi-square calculator handout for specific instructions on how to use your TI-83 or TI-84 Plus calculator to do so.)

✎ **Table 1. Individual data**

	Milk Chocolate M&Ms Colors						
Observed Frequency (O)	Brown	Blue	Orange	Green	Red	Yellow	**Total**

✎ **Table 2. Class data**

Color	**Observed Frequency** (O)	**Expected Frequency** $(E = np)$	$\frac{(O-E)^2}{E}$
Brown			
Blue			
Orange			
Green			
Red			
Yellow			
Total:	$\sum O = n =$	$\sum E =$	$\chi^2 = \sum \frac{(O-E)^2}{E} =$

These totals should match.

Now that you have computed the χ^2 GOF test statistic, you'll need to figure out how many degrees of freedom there are by subtracting 1 from the number of color categories (k – 1).

✎ **Degrees of freedom (df) = _____**

You need this information because the chi-square distribution is different for each number of degrees of freedom. You can now use your calculator to compute the P-value corresponding to your χ^2 GOF test statistic. Press 2nd VARS to get the DISTR menu. Scroll down to 8:χ^2cdf(and press ENTER. Next, type in the χ^2 value you computed above , 999999 , the number of degrees of freedom) and press ENTER. This command will give you the probability of obtaining a statistic greater than your computed value in a chi-square distribution with your number of degrees of freedom. (Refer to the chi-square calculator handout for more detailed instructions.)

✎ **What is your P-value? _____ (Round to 3 significant figures)**

✎ Using a significance level of α = .05, should you reject or fail to reject the null hypothesis (H_0)?

✎ Based on the outcome of your hypothesis test, what is your conclusion regarding the quality-control process at the M&Ms manufacturing plant? (Write out the complete formal hypothesis test in your notebook.)

Now, let's revisit the 1-proportion z interval....

✎ Calculate the proportion of M&Ms in the class sample that are blue. That is, find $\hat{p}$.

✎ Using the class sample data, construct a 95% confidence interval estimate of the proportion (p) of all M&Ms that are blue. Report your answer as $\hat{p} - E < p < \hat{p} + E$.

✎ Does this confidence interval contain the proportion of blue M&Ms claimed by the manufacturer ($p = 0.24$)? Assuming that the manufacturer's claimed proportion is the "true" population proportion, what can you say regarding the success of your confidence interval? Did our class sample data yield a confidence interval that was one of the 95% that succeed at capturing p?