Using Your TI-83/84 Calculator for Hypothesis Testing:
The Chi-Square Goodness-of-Fit Test
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If births were uniformly distributed across the week, we would expect that about 1/7 of all births occur during each day of the week. How closely do the observed number of births fit this expected distribution? The chi-square goodness-of-fit test is used to determine whether an observed frequency distribution is significantly different from the expected distribution, or how "good" (sic) the two distributions fit each other. If we were only interested in one day of the week, we could conduct a 1-proportion z test. However, because we have seven hypothesized proportions, we need to conduct a test that considers all of them together and gives an overall indication of whether the observed distribution differs from the expected one. The chi-square goodness-of-fit test is just what we need. Let's consider the frequency distribution of all 2003 New Jersey births by day of the week.

Day of Week	Number of Births
Sunday	11,302
Monday	16,779
Tuesday	19,252
Wednesday	19,147
Thursday	18,455
Friday	18,732
Saturday	13,156
TOTAL	**116,823**

If you have a TI-84 Plus calculator, there is a built-in chi-square goodness-of-fit (GOF) test. If not, you will need to follow a somewhat more complicated procedure. I will provide instructions for both calculator models; use whichever method applies to your calculator.

χ^2 Goodness-of-Fit Test for the TI-83 Calculator

1. Start by clearing L_1, L_2, and L_3. We will need all three lists in order to compute the χ^2 test statistic.

2. Enter the observed number of births for each day of the week into L_1. (The general procedure is to put observed frequencies in L_1.)

3. Now we need to compute the expected frequency for each day of the week. Because we are hypothesizing that births are uniformly distributed, we calculate the expected frequency as $E = n/k$, where n is the total number of trials (births, in this case) and k is the number of different categories (days of the week, in this case). For this example, $E = 116823/7 = 16689$. We will use this expected frequency for each day of the week. (If we had been hypothesizing unequal frequencies, you would compute a separate expected frequency for each category using $E = np$.) Go ahead and enter 16689 seven times in L_2. (The general procedure is to put expected frequencies in L_2.)

4. The next step is to compute $\frac{(O-E)^2}{E}$ for each category. We'll do that by highlighting L_3 and then typing in the following keystrokes: $($ 2nd 1 $-$ 2nd 2 $)$ x^2 $\div$ 2nd 2 ENTER. These keystrokes will enter the formula shown in the above calculator display. If you have entered the formula correctly, L_3 will fill up with the values shown in the display to the right.

5. Press $\boxed{\text{2nd}}\boxed{\text{MODE}}$ to quit the stat editor and return to the home screen. Once we add up all of the entries in L3, we will have found the value of our chi-square test statistic:

$$\chi^2 = \sum \frac{(O-E)^2}{E}$$

We will use the sum function to do so. Press $\boxed{\text{2nd}}\boxed{\text{STAT}}$ and scroll right to the MATH menu. Then, scroll down to 5:sum(and press $\boxed{\text{ENTER}}$. Type $\boxed{\text{2nd}}\boxed{3}\boxed{)}$ to indicate that you want to sum the contents of L3 and then press $\boxed{\text{ENTER}}$.

6. Your calculator will now display the value of χ^2, which is 3679.866739 for this example.
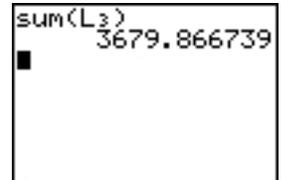
7. Next, we need to find the *P*-value associated with our χ^2 test statistic. To do so, press $\boxed{\text{2nd}}\boxed{\text{VARS}}$ to get the DISTR menu. Scroll down to 8:χ^2cdf(and press $\boxed{\text{ENTER}}$. We want to find the probability of obtaining a χ^2 test statistic of 3679.866739 or higher, so type 3679.866739,1$\boxed{\text{2nd}}\boxed{,}$99,6$\boxed{)}\boxed{\text{ENTER}}$ to tell your calculator to find the probability of obtaining a value between 3679.866739 and 10^{99} (an approximation for ∞) in a χ^2 distribution with 6 degrees of freedom. (Find the number of degrees of freedom by subtracting one from the number of categories, or *k* - 1. For this example, there are seven categories and, hence, six degrees of freedom.)

8. We obtained a very large χ^2 value, so you shouldn't be surprised that there is a very small *P*-value associated with it. So small, in fact, that your calculator rounds it to Ø. Don't be alarmed when this happens; just indicate that the *P*-value is "near zero." The *P*-value won't be exactly zero, but we are unable to find out just how low it is for extremely large χ^2 values.
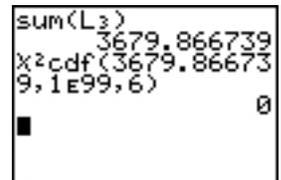
9. <u>Round chi-square values to 3 decimal places and *P*-values to 3 significant figures.</u> You would report the results for this example as $P(\chi^2 > 3679.867) \approx 0$.

10. *What does this mean?* If births were in fact distributed uniformly across the seven days of the week, an observed χ^2 value of 3679.867 would occur about 0% of the time. This result is certainly unusual, so we reject H_0 and conclude that the sample data are consistent with births being non-uniformly distributed across the seven days of the week.

χ^2 **Goodness-of-Fit Test for the TI-84 Plus Calculator**

1. It is relatively easier to conduct a χ^2 goodness-of-fit test if you happen to have a TI-84 Plus calculator. Steps 1-3 are the same as I described previously for the TI-83 calculator (except that you only need to clear L1 and L2).

2. Once you have entered the observed frequencies in L1 and the expected frequencies in L2, press $\boxed{\text{STAT}}$ and scroll right to the TESTS menu. Then, scroll down to D:χ^2GOF-Test and press $\boxed{\text{ENTER}}$. Your calculator will prompt you for the list with the observed frequencies (L1 for this example),

the list with the expected frequencies (L$_2$ for this example), and the number of degrees of freedom. You can find the degrees of freedom by subtracting 1 from the number of categories (k - 1); there are 6 degrees of freedom for this example. Next, highlight `Calculate` and press `ENTER`.

```
X²GOF-Test
 Observed:L₁
 Expected:L₂
 df:6■
 Calculate Draw
```

3. Your calculator will return the output shown to the right. It gives you both the value of the χ^2 test statistic and its associated *P*-value. `CNTRB` provides a list of the CoNTRiButions of each category to the overall χ^2 value. Use ▶ to scroll through these numbers; note that `CNTRB` lists the same values that appear in L$_3$ when you follow the TI-83 calculator procedure above. <u>Round chi-square values to 3 decimal places and *P*-values to 3 significant figures</u>. You would report these results as $P(\chi^2 > 3679.867) \approx 0$.

```
X²GOF-Test
 X²=3679.866739
 P=0
 df=6
 CNTRB={1738.85…
■
```

4. *What does this mean?* If births were in fact distributed uniformly across the seven days of the week, an observed χ^2 value of 3679.867 would occur about 0% of the time. This result is certainly unusual, so we reject H_0 and conclude that the sample data are consistent with births being non-uniformly distributed across the seven days of the week.

Below I have provided the formal hypothesis test for this example. I will expect you to use this same format when reporting the results of a χ^2 goodness-of-fit test. Pay special attention to the wording of the conclusion and how I report the χ^2 value, degrees of freedom, and *P*-value at the end. This was an unusual hypothesis test in that the original claim became the null hypothesis. More typically, the original claim is tested as the alternative hypothesis.

<u>Claim</u>: Births are evenly distributed across the seven days of the week.

Let p_1 = the proportion of births on Sunday, p_2 = the proportion of births on Monday, … , and p_7 = the proportion of births on Saturday.

H_0: $p_0 = p_1 = p_2 = p_3 = p_4 = p_5 = p_6 = p_7 = 1/7$ (Births are evenly distributed across the seven days of the week.)

H_1: At least one $p_i \neq 1/7$ (Births are not evenly distributed across the seven days of the week.)

Conduct a χ^2 goodness-of-fit test with a significance level of α = .05

Reject H_0 because our P-value was near zero, and 0 < .05

<u>Conclusion</u>: There is sufficient evidence to warrant rejection of the claim that births are evenly distributed across the seven days of the week (χ^2 = 3679.867, df = 6, P $\approx$ 0).

The M&Ms exercise that we will conduct during class tonight will provide you with an example of how to conduct a chi-square goodness-of fit-test in situations where the expected frequencies are <u>not</u> equal.