

Using Your TI-NSpire Calculator: Binomial Probability Distributions

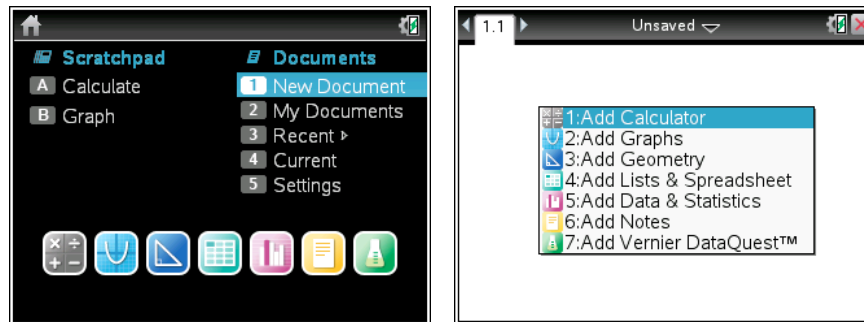
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Statistics I

This handout describes how to use the `binomPdf` and `binomCdf` commands to work with binomial probability distributions. It also describes how to find the mean and standard deviation for any discrete probability distribution and how to plot a probability histogram. Consider the following:

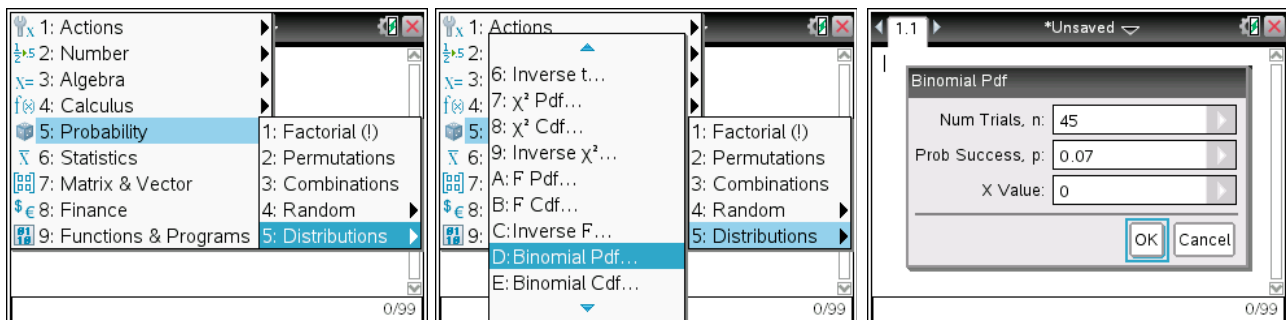
People with type O-negative blood are said to be “universal donors.” About 7% of the U.S. population has this blood type. Suppose that 45 people show up at a blood drive. Let x = the number of universal donors among a random group of 45 people

Let’s begin by finding the probability that none of the 45 donors have type O-negative blood.

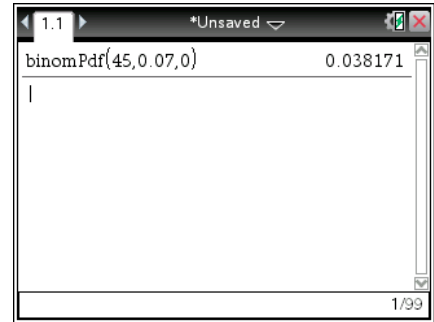
1. From the home screen, indicate that you want to start a **New Document** and select **1: Add Calculator**.



2. Press the `menu` key and then select **5: Probability** followed by **5: Distributions**. We’ll be using **D: Binomial Pdf...** for this calculation.
3. The syntax for the binomial probability density function command is `binomPdf(n, p, x)`. If you prefer, you can type in this command directly instead of navigating through menus.
 - **n** : This is the number of trials. For this example, $n = 45$ (the number of blood donors).
 - **p** : This is the “success” probability. For this example, $p = 0.07$ (the probability that a randomly selected American has type O-negative blood). Note that p must be in decimal form.
 - **x** : This is the number of “successes.” For this example, we want to know the probability that none of the 45 donors have type-0 negative blood, so $x = 0$. Note that x must be a whole number.



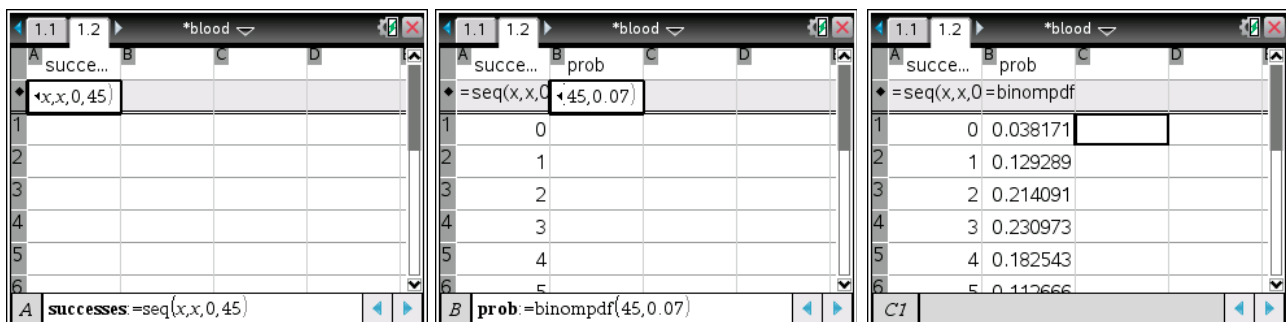
- Your calculator will return the probability that exactly 0 out of the 45 donors have type O-negative blood. You will find that $P(x = 0) = 0.0382$. In other words, there is a 3.82% chance that none of the 45 people are universal donors. *Remember to round all probability values to 3 significant figures.*
- Note that you can type the command directly into your calculator instead of navigating through menus. Give it a try. Type `binomPdf(45, 0.07, 0)` and press `enter`. (You'll end up with the same result more quickly if you learn the syntax for the various calculator commands we'll be using this semester.)
- If you haven't already done so, press `⌘+shift` `⌘+s` and save your file. (I named my file "blood.")



Next, let's generate the binomial probability distribution for $n = 45$ and $p = 0.07$.

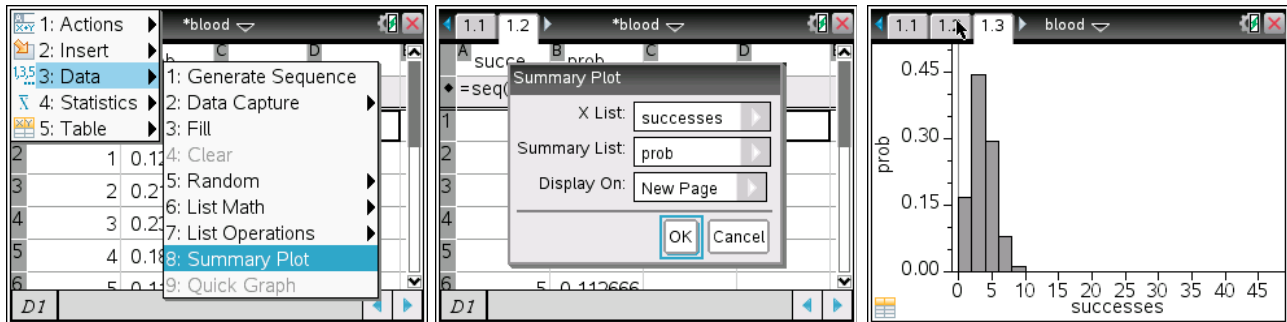
To generate a binomial probability distribution, we simply use the binomial probability density function command without specifying an x value. In other words, the syntax is `binomPdf(n, p)`. Your calculator will output the binomial probability associated with each possible x value between 0 and n , inclusive. The trick is to save all these values. The most efficient way to do so is to work from within a spreadsheet.

- Press `⌘+o` and select **4: Insert** followed by **6: Lists & Spreadsheet** to add a spreadsheet to your document.
- Name column A "successes." We want to fill this list with all of the possible values of x between 0 and 45. Move down to the next box in the column and type `seq(x, x, 0, 45)`. Press the `enter` key and the list will fill with the integers between 0 and 45 inclusive. Note that the general syntax for this command is `seq(x, x, 0, n)`.
- Move over and name column B "prob." We want to fill this list with the probability of obtaining each of the values of x contained in column A. Move down to the second box in column B and type `binomPdf(45, 0.07)`. Press `enter`, and Column B will fill with the probabilities for $0 \leq x \leq n$ successes. Scroll down the spreadsheet to view the complete binomial probability distribution for $n = 45$ and $p = 0.07$.

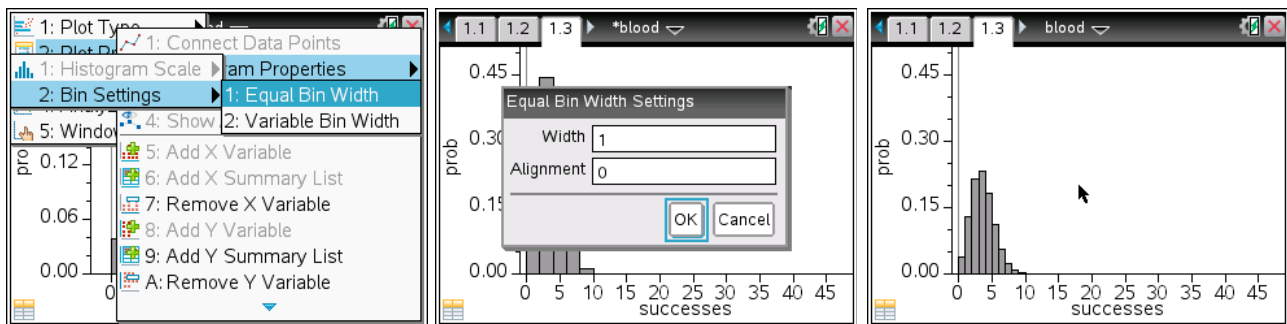


How to construct a probability histogram

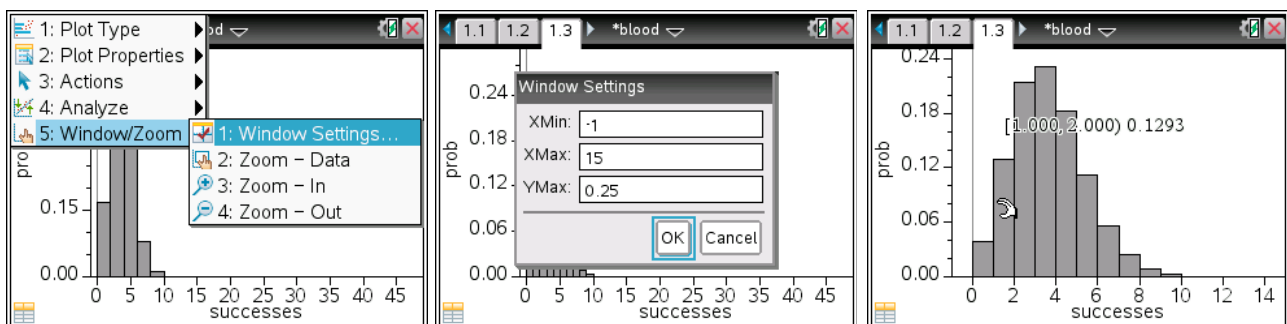
- Let's plot a histogram for this probability distribution. Press the **menu** key and then select **3: Data** followed by **8: Summary Plot**.
- Specify that the x list is *successes* and the summary list is *prob*. I prefer to produce the plot on a new page, but you could plot it within the spreadsheet by splitting the display.



- Note that the histogram that is produced isn't quite what we want. Press the **menu** key and then select **2: Plot Properties** followed by **2: Histogram Properties**. Click on **2: Bin Settings** followed by **1: Equal Bin Width**. Specify a width of 1 and an alignment of 0, and click on "OK."



- Adjust the window settings to make the plot easier to read. Press the **menu** key and then select **5: Window/Zoom** followed by **1: Window Settings**. Experiment with the settings until you end up with a plot that captures the shape of the probability distribution, yet is still readable.



- Use the touch pad to move across the bars, and you'll find the probability associated with a given number of successes. For instance, we find that $P(x = 1) = 0.1293$. Given that x is a discrete random variable, the notation $[1.000, 2.000)$ is equivalent to saying that $x = 1$. You could force the x -axis to be categorical (and separate the bars), but the resulting graph isn't as pretty.

Working with a binomial probability distribution

Use the touch pad to navigate back to your calculator screen (1.1). Next, we'll practice finding some binomial probabilities.

- Let's find the probability that **at least one** of the 45 donors has type O-negative blood. The tedious approach would be to add up $P(x = 1) + P(x = 2) + P(x = 3) + \dots + P(x = 45)$. Don't! Instead, take advantage of the fact that the complement of "at least one" is "none." Thus, $P(1 \leq x \leq 45) = 1 - P(x = 0) = 0.962$. The screen shot to the right shows how you can use the `binomPdf` command to find this probability. *Remember to round probability values to 3 significant figures.*

The screenshot shows a TI-84 Plus calculator screen with the following data:

Command	Result
<code>binomPdf(45,0.07,0)</code>	0.038171
<code>1-binomPdf(45,0.07,0)</code>	0.961829

- It is often helpful to view the spreadsheet at the same time as the calculator using a split screen. Press the `docv` key and select **5: Page Layout** followed by **2: Select Layout**. I chose to use **Layout 2**. Then, I pressed the `menu` key and added a calculator to the right side of the screen.
- Next, let's find the probability that **no more than three** of the donors have type O-negative blood. To do so, we need to find $P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$. You could write down the corresponding probabilities from the `prob` list in your spreadsheet and then add them up.

- Alternatively, we can use the `sum` command and specify the range of items in the `prob` list that we wish to add up. The key is to remember that the first item in the list corresponds to $P(x = 0)$; that is, the row number corresponds to $x + 1$. The syntax for the sum command is `sum(list, start, end)`. For this example, type `sum(prob, 1, 4)` and press `enter`. This command tells your calculator to sum the contents of rows 1 through 4 in the `prob` list. We find that $P(x \leq 3) = 0.613$. In other words, there is a 61.3% chance that no more than three of the 45 donors have type O-negative blood.

The screenshot shows a TI-84 Plus calculator in split-screen mode. The left window displays a spreadsheet with the following data:

Row	Successes (x)	Probability (P(x))
1	0	0.03817
2	1	0.12928
3	2	0.21409
4	3	0.23097
5	4	0.18254
6	5	0.11266

The right window shows the calculator screen with the command `sum(prob,1,4)` and the result `0.612523`.

- A third approach is to use the `binomCdf` command. This command finds the cumulative probability of obtaining x or fewer successes. The syntax is `binomCdf(n, p, x)`. This command is a useful whenever you are asked to find the probability of "no more than" x successes. To find the probability that no more than three of the donors have type O-negative blood, use `binomCdf(45, 0.07, 3)`. This command tells your calculator to find $P(x = 0) + P(x = 1) + P(x = 2) + P(x = 3)$, which is exactly what we need to do here. We obtain the same answer as before, $P(x \leq 3) = 0.613$.

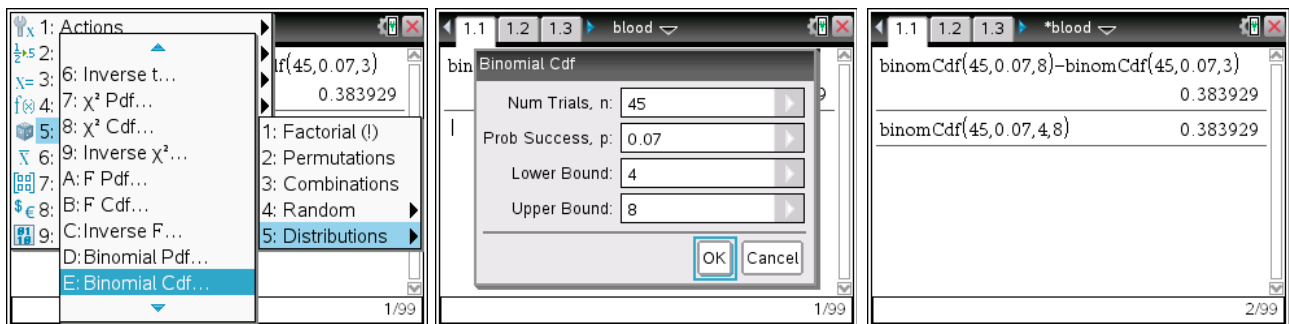
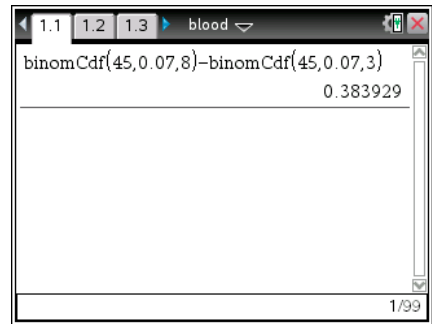
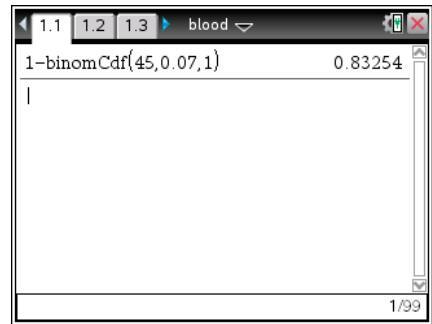
The screenshot shows a TI-84 Plus calculator in split-screen mode. The left window displays the same spreadsheet as in the previous screenshot. The right window shows the calculator screen with the command `binomCdf(45,0.07,3)` and the result `0.612523`.

- Through creative combinations of the `binomPdf` and `binomCdf` commands, you will find it possible to solve any sort of binomial probability problem that comes your way. Suppose that we want to find the probability that **at least two** of the donors have type O-negative blood. Recognize that this is the complement of no more than one of the donors having this blood type,

and you can easily solve this problem. We find that $P(x \geq 2) = 1 - P(x \leq 1) = 0.833$.

7. As a final example, let's find the probability that **between 4 and 8** of the donors have type O-negative blood. That is, find $P(4 \leq x \leq 8)$. You could solve this problem using the `sum` command and your binomial probability distribution. Alternatively, you can strategically use the `binomCdf` command. Note that $P(4 \leq x \leq 8) = P(x \leq 8) - P(x \leq 3)$. By subtracting out $P(x \leq 3)$, we are making sure that $P(x = 4)$ is included in our calculations. Using the `binomCdf` command as shown to the right, we find that $P(4 \leq x \leq 8) = 0.384$. Using this approach saves you from having to generate the binomial probability distribution and figure out the correct rows to add up.

8. The previous example can be solved more easily by navigating through the menus of your Nspire or by using a slightly different command syntax. Press the `menu` key and select **5: Probability** followed by **5: Distributions**. Select **E: Binomial Cdf**. The dialog box that opens up allows you to specify the desired lower and upper bounds directly, in this case, 4 and 8. Note that we end up with the same result.



Summary: binomPdf vs. binomCdf commands

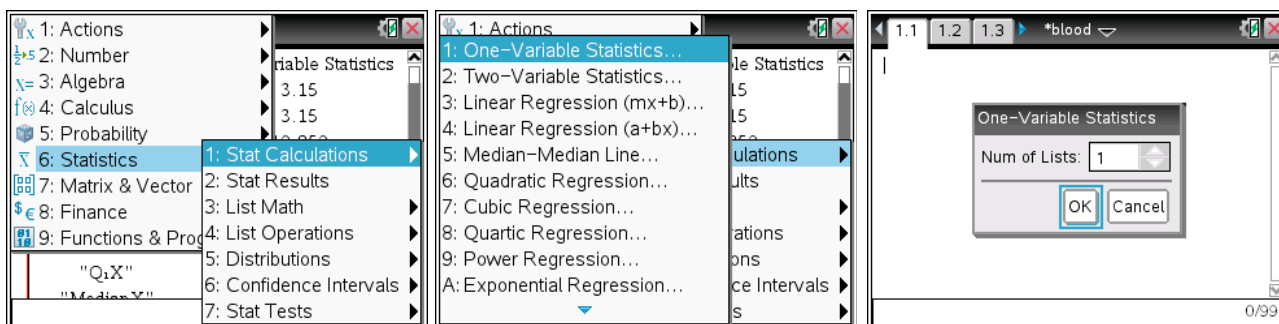
Here are some useful applications of the `binomPdf` and `binomCdf` commands:

- To find $P(x = k)$, use `binomPdf(n, p, k)`
- To find $P(x \leq k)$, use `binomCdf(n, p, k)`
- To find $P(x < k)$, use `binomCdf(n, p, k-1)`
- To find $P(x > k)$, use `1-binomCdf(n, p, k)`
- To find $P(x \geq k)$, use `1-binomCdf(n, p, k-1)`

Note: k refers to some number of successes between 0 and n .

How to find μ_x and σ_x for a discrete probability distribution

1. What is the **expected number** of universal donors out of the 45 people who show up at the blood drive? For all discrete random variables, $E(x) = \mu_x = \sum x \cdot P(x)$. For the special case of a binomial probability distribution, $E(x) = \mu_x = np$. For this example, $E = (45)(0.07) = 3.15$ (round to 3.2). We can find both μ and σ for a binomial probability distribution (or any other type of discrete probability distribution) by using the one-variable statistics command on the TI-Nspire.
2. Press the **menu** key and select **6: Statistics** followed by **1: Stat Calculations**. Then, select **1: One-Variable Statistics**. Somewhat un-intuitively, you need to specify that you will be using one list. The X1 list is the list containing the x values (*successes* for this example), and the frequency list is the one containing the corresponding $P(x)$ values (*prob* for this example).



3. Your calculator will return the output shown below. Note that what it calls \bar{x} is really μ . Rounding to one more decimal place that we started with, we find that the expected number of universal donors out of 45 is 3.2 people, with a standard deviation of 1.7 people.

