Individual: 1. The decimal representation is $0.\overline{142857}$. 2006 modulo the period of 6 is is 2. So the 2006th digit is $\boxed{4}$.

2. Let $1 \leq n \leq 1000$ and $n = x^2 - y^2 = (x - y)(x + y)$. Since $x - y, x + y$ are both odd or both even, n is the difference of two squares if and only if n can be factored as the product of two integers with the same parity. The only n that cannot be factored in this manner are integers of the form $2k$, where k is odd. Since $1 \leq k \leq 500$, there are 250 such k. Hence there are $1000 - 250 = \boxed{750}$ integers n with the requested property.

3. There are 10 equiprobable ways to choose 3 people from a group of 5 people. 3 of these involve choosing both women. So the probability is $\boxed{3/10}$.

4. Detailed proof $\boxed{24 = 2 \times 3 \times 4}$.

5. Let x and y be the respective two-digit and three-digit number. We are given the equation $1000x + y = 9xy$. Now $y(9x - 1) = 1000x$, so x divides $y(9x - 1)$. Since x and $9x - 1$ have no factors in common, x divides y. Writing $y = xk$, the equation becomes $1000 = k(9x - 1)$. Hence k and $9x - 1$ are factors of 1000. Since x is a two-digit number, $98 \leq 9x - 1 \leq 999$, and $9x - 1$ must then equal $100, 125, 200, 250, 500$. Hence $9x - 1 = 125$, $x = 14$, $k = 8$, and $y = 112$. Then $x + y = \boxed{126}$.

6. Assume that the statement is false. Let n be the smallest natural number such that L_n and L_{n+2} have a nontrivial factor s. Since $L_2 = 2 + 7 = 9$, $n > 0$. $L_{n+2} = L_n + L_{n+1}$ implies that s divides L_{n+1}. $L_{n+1} = L_{n-1} + L_n$ implies that s divides L_{n-1}. This contradicts the minimality of n.

7. Let S be the set of all positive integers n such that the equation $7a + 11b = n$ does not have a solution with nonnegative integers a, b. We are looking for the largest element of S. Let r be the remainder of n when divided by 7. If $r = 0$, then $n = 7a$, so n is not an element of S. If $r = 1$, then $1, 8, 15 \in S$. Now $22 = 2(11)$, so every other element n with $r = 2$ is not in S as $22 + 7a = 7a + 11(2)$. Similarly, if $r = 2$, then $2, 9, 16, 23, 30, 37 \in S$, but $44 + 7a = 7a + 11(4)$, so no 37 is the largest element of S with $r = 2$. When $r = 3$, the largest $n \in S$ is 59; when $r = 4$, the largest $n \in S$ is 4; when $r = 5$, the largest $n \in S$ is 26; and when $r = 6$, the largest $n \in S$ is 48. Hence the largest element in S is $\boxed{59}$.

8. $\boxed{(1/3)^{5/16}}$

9. Factor 10^5 into primes, $10^5 = 2^5 \cdot 5^5$. Hence 10^5 has 36 distinct positive divisors, each of the form

$$2^i \cdot 5^j \qquad 0 \leq i, j \leq 5$$

Fix an i, we get divisors 2^i5^0, 2^i5^1, 2^i5^2, 2^i5^3, 2^i5^4, 2^i5^5. The product of the 6 divisors above is $2^{6i}5^{15}$. Let $i = 0, 1, ..., 5$, we get

$$N = 2^{6\times15} \cdot 5^{6\times15} = 10^{90} \quad \Rightarrow \quad log(N) = log(10^{90}) = \boxed{90}$$

10. From $f(f(x)) = x$ for $x \neq -d/c$, one can deduce that f is one-to-one as $f(a) = f(b)$ implies $a = f(f(a)) = f(f(b)) = b$. Inspection suggests that

f sends $\mathbf{R}-\{-d/c\}$ to $\mathbf{R}-\{a/c\}$ as $\lim_{x\to+\infty} f(x) = \lim_{x\to-\infty} f(x) = a/c$. By examination, the equation $f(x) = a/c$ has either no solution or infinitely many solutions. As f is one-to-one, a/c cannot be in the range. Let $y = a/c$.

We now show that $y = -d/c$. Assume that $y \neq -d/c$. Then y is in the domain of f and $z = f(y)$ is defined. If $z \neq -d/c$, then $f(z) = f(f(y)) = y$ and y would be in the range of f, which gives a contradiction. If $z = -d/c$, then the expression $f(f(y))$ is not defined, contradicting the hypothesis in the problem. Hence $y = -d/c$ and $a/c = -d/c$.

From $f(19) = 19$, one obtains the equation $19^2c + 19(d-a) = b$. From $f(97) = 97$, one obtains $97^2c + 97(d-a) = b$. Equating these, one obtains $a/c - d/c = 116$. Since $a/c = -d/c$, we have $y = a/c = \boxed{58}$.

11. Note that $x^2+y^2-2xy = (x-y)^2 \geq 0$, *and* $x^2+y^2+2xy = (x+y)^2 \geq 0$. Thus $\frac{1}{2}(x^2+y^2) \geq xy \geq \frac{-1}{2}(x^2+y^2)$.

$$\frac{a}{2}(x^2+y^2) \geq axy \geq (\frac{-a}{2})(x^2+y^2) \quad if \quad a \geq 0$$

$$(\frac{-a}{2})(x^2+y^2) \geq axy \geq \frac{a}{2}(x^2+y^2) \quad if \quad a < 0$$

For any solution (x, y) of $1 = 2x^2 + axy + 2y^2 = 2(x^2+y^2) + axy$, we have

$$1 \geq 2(x^2+y^2) - (\tfrac{a}{2})(x^2+y^2) = (\tfrac{4-a}{2})(x^2+y^2) \quad \text{if} \quad a \geq 0$$
$$1 \geq 2(x^2+y^2) + (\tfrac{a}{2})(x^2+y^2) = (\tfrac{4+a}{2})(x^2+y^2) \quad \text{if} \quad a < 0$$

In the first case, in order for $x^2+y^2 \leq 1$, $4 \geq a \geq 2$.

Similarly, when $a < 0$, we need $-2 \leq a < 0$ (so that $x^2+y^2 \leq 1$). Thus the minimum value of a is $\boxed{-2}$.

12. Rewriting the limit we get $\lim_{x\to\infty} \frac{\int_0^\infty e^{t^2}\,dt}{e^{x^2}/x}$. By l'Hopital's rule this is equal to $\lim_{x\to\infty} \frac{e^{x^2}}{(2x^2e^{x^2}-e^{x^2})/x^2}$. Simplifying we get $\lim_{x\to\infty} \frac{x^2}{2x^2-1} = \boxed{\frac{1}{2}}$.

Team:

1. The angle $\angle A_nA_1A_2$ has measure $180 - 360/n$ degrees. Hence the angle $\angle A_nA_1B$ will have measure $120+360/n$ degrees. If A_n, A_1, B are three consecutive vertices of a regular m-gon, then $120+360/n = 180-360/m$ for some m. Then $mn-6n-6m = 0$. Adding 36 to both sides, $(m-6)(n-6) = 36$ and $n-6$ is a divisor of 36. The largest possible value for n is $\boxed{42}$, with a corresponding value of $m = 7$.

2. Detailed proof.

3. $\boxed{0}$

4. According to the assumption, write $\begin{pmatrix} a_{n+1} \\ b_{n+1} \end{pmatrix} = 2A \cdot \begin{pmatrix} a_n \\ b_n \end{pmatrix}$ where $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$

Thus

$$\begin{pmatrix} a_{n+3} \\ b_{n+3} \end{pmatrix} = 2A\cdot\begin{pmatrix} a_{n+2} \\ b_{n+2} \end{pmatrix} = 4A^2\cdot\begin{pmatrix} a_{n+1} \\ b_{n+1} \end{pmatrix} = 8A^3\cdot\begin{pmatrix} a_n \\ b_n \end{pmatrix} = \begin{pmatrix} 8 & 24 \\ 0 & 8 \end{pmatrix}\cdot\begin{pmatrix} a_n \\ b_n \end{pmatrix}$$

Therefore

$$\boxed{x = 8,\ y = 24,\ z = 0,\ w = 8}$$

5. Consider the point $P = (n, \sqrt{n^2+1})$ on the curve $y^2 - x^2 = 1$. The vertical line $x = n$ and $y = x$ intersect at the point $Q = (n, n)$.

By similarity $\triangle ONQ \sim \triangle PMQ$, we have

$$\frac{d_n}{n} = \frac{\sqrt{1+n^2} - n}{\sqrt{2}n} \quad \Rightarrow \quad d_n = \frac{(\sqrt{1+n^2} - n)(\sqrt{1+n^2} + n)}{\sqrt{2}(\sqrt{1+n^2} + n)}$$

$$\textit{Hence} \qquad n \cdot d_n = \frac{n}{\sqrt{2}(\sqrt{1+n^2} + n)} = \frac{1}{\sqrt{2}(\sqrt{\frac{1}{n^2} + 1} + 1)}$$

$$\textit{So,} \qquad \lim_{n\to\infty} nd_n = \lim_{n\to\infty} \frac{1}{\sqrt{2}(\sqrt{\frac{1}{n^2} + 1} + 1)} = \frac{1}{2\sqrt{2}} = \boxed{\frac{\sqrt{2}}{4}}$$

6. Let $\alpha_n = (2+\sqrt{3})^n$ and $\beta_n = (2-\sqrt{3})^n$. By Binomial Theorem (or by Mathematical Induction), $\alpha + \beta \in \mathbb{N}$. Since $0 < \beta_n < 1$, we have

$$\lfloor \alpha_n \rfloor = \alpha_n + \beta_n - 1,$$

therefore

$$\alpha_n - \lfloor \alpha_n \rfloor = 1 - \beta_n.$$

Since $\lim_{n\to\infty} \beta_n = 0$,

$$\lim_{n\to\infty} \Big(\alpha_n - \lfloor \alpha_n \rfloor\Big) = \lim_{n\to\infty} (1 - \beta_n) = 1.$$