## Quantitative Analyses for Valuing Students' Incorrect Responses to Common Assessments

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## Force and Motion Conceptual Evaluation

- 47-item multiple-choice survey ${ }^{1}$
- 40 questions are scored (total of 37 points)

A sled on ice moves in the ways described in questions 1-7 below. Friction is so small that it can be ignored. A person wearing spiked shoes standing on the ice can apply a force to the sled


1. Thorton \& Sokoloff, Am. J. Phys. (1998)

## Typical Analyses

- Give FMCE at beginning and end of course
- Score each response as correct or incorrect
- Count the number of questions answered correctly
- Calculate average normalized gain <g>
- Because $g$ is biased², maybe calculate Cohen's $d$ or another measure of effect size

error bar = pooled standard error of pre/post-test scores
- Compare data sets


## What's wrong with typical analyses?

- Many students answer most questions incorrectly (either before or after instruction)
- We focus on what they DON'T know rather than valuing what they DO know.
- Many students change their answer from one incorrect to another

Are students learning even if they don't choose the correct answer?

Data from 7,288 students
Pretest Score Distribution


Post-test Score Distribution


## Ranking incorrect responses

- Are some incorrect responses better than others?
- What makes one response better than another?
- What productive ideas are students expressing by choosing a particular incorrect answer?


## Assumption 1

Students who have a higher understanding of physics (as measured by the FMCE) are more likely to choose better responses than students who have a lower understanding of physics.

## Item Response Theory (IRT)

- Two-parameter-logistic (2PL) nested-logit model ${ }^{3}$
- Estimates students’ overall knowledge4
- Probability of answering correctly:

$$
P(\theta)=\frac{1}{1+e^{-a(\theta-b)}}
$$

- Probability of choosing the $k^{\text {th }}$ incorrect answer is the product of being incorrect and Bock's nominal response model5:

$$
P_{k}(\theta)=\left(1-\frac{1}{1+e^{-a(\theta-b)}}\right) \frac{e^{a_{k}\left(\theta-b_{k}\right)}}{\sum_{i} e^{a_{i}\left(\theta-b_{i}\right)}}
$$

- Incorrect responses ranked by value of ak

3. Suh and Bolt, Psychometrika (2010)
4. Bock, Psychometrika (1972)

AAPT, Washington, DC
4. Baker, The Basics of Item Response Theory (2001)

## Question 14

A toy car ... can move to the right or left along a horizontal line (the positive part of the distance axis). Assume that friction is so small that it can be
 ignored. Choose the one force graph ... which could allow the described motion of the car to continue. If you think that none is correct, answer choice J.
14. The car moves toward the right (away from the origin) with a steady (constant) velocity.

(H)

(C)


## IRT Plot



## IRT Results

Trace lines for Question 14


## Assumption 2

Students are more likely to choose better responses after instruction than before instruction.

## McNemar-Bowker (MB) chi-square test

- Considers the number of students who give each pre/post response pair (each transition) ${ }^{6,7}$
- Uses chi-square to look for asymmetries

|  |  | Post-test |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A | B | C | D | E | F | G | H |
| $\begin{aligned} & \overleftarrow{\$} \\ & \frac{\$}{\mathbf{o}} \\ & \hline \end{aligned}$ | A | 2881 | 73 | 309 | 42 | 1918 | 22 | 10 | 118 |
|  | B | 55 | 8 | 13 | 1 | 35 | 1 | 1 | 2 |
|  | C | 236 | 2 | 76 | 6 | 104 | 2 | 2 | 7 |
|  | D | 24 | 1 | 9 | 2 | 12 | 0 | 0 | 0 |
|  | E | 101 | 9 | 14 | 1 | 753 | 8 | 0 | 10 |
|  | F | 8 | 0 | 1 | 0 | 7 | 0 | 1 | 0 |
|  | G | 6 | 0 | 1 | 1 | 7 | 0 | 0 | 0 |
|  | H | 32 | 0 | 3 | 0 | 114 | 0 | 1 | 11 |

## MB Results

- Pairwise comparisons show asymmetries
- p-values adjusted using false discovery rate (FDR) method

| Comparison | Adjusted <br> p-value | Percent of <br> Population |
| :---: | :---: | :---: |
| E > A | 0 | $28.3 \%$ |
| C > A | 0.00861 | $7.7 \%$ |
| H $>$ A | $6.63 \mathrm{E}-12$ | $2.1 \%$ |
| E $>$ H | $2.52 \mathrm{E}-22$ | $1.7 \%$ |
| E $>$ C | $3.62 \mathrm{E}-17$ | $1.7 \%$ |
| E > B | 0.00053 | $0.6 \%$ |
| F $>$ A | 0.0402 | $0.5 \%$ |
| C > B | 0.0222 | $0.4 \%$ |
| E > D | 0.0114 | $0.3 \%$ |
| E $>$ G | 0.0402 | $0.2 \%$ |

## Unified Ranking

## E > H > A

- Supported by both methods
- The relationship with $C$ is unclear
- IRT says C < A
- MB says C > A
- More analysis is needed (and maybe more data)
- Response A overwhelms pretest data and may skew MB results
- Future work will involve student interviews
- Ultimate goal: New metric for measuring student understanding and learning. (Down with <g>!)


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## Future work: Developing a new metric for student learning

## Repeat for all Questions



