INFLUENCE OF INHERENT PARTICLE CHARACTERISTICS ON THE FLOW BEHAVIOR AND STRENGTH PROPERTIES OF PARTICULATE MATERIALS

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ABSTRACT: The inherent factors influencing the stress-strain behavior include particle size, particle size distribution, particle shape, angularity, and surface roughness. The influence of the inherent factors on the shear strength properties of particulate materials was studied. The studies indicate that a good correlation between inherent particle characteristics and drained shear strength properties of a wide range of particulate materials exists. In addition, an index test was developed to predict the shear strength properties of particulate materials. The results show that the flow cone (hopper) test data, namely the flow rate, are also influenced by the inherent particle characteristics. A good correlation between drained shear strength properties and the flow rates measured in the cone was found to exist. Therefore, index tests such as flow rate through a flow cone (hopper) can be used to estimate the drained monotonic strength of particulate materials. This proves to be a great advantage because the index test can be conducted rapidly by personnel who do not have extensive experience in soils testing to obtain an estimate of drained shear strength properties. Numerical simulation of the index test using a two-dimensional distinct element program has also been conducted. The simulation studies model the flow and interaction of hundreds of thousands of discrete particles and were used to study particulate behavior at a micro-level.

Keyword: Shear strength, flow behavior, shape, angularity
INFLUENCE OF INHERENT PARTICLE CHARACTERISTICS ON SHEAR STRENGTH

Factors influencing the stress-strain behavior of cohesionless soils

Factors that influence the stress-strain behavior of soils can be classified as inherent factors, intermediate or link factors, and external factors. These factors are shown in Figure 1. Inherent factors affecting stress-strain behavior include particle size, particle size distribution, shape, angularity, and surface roughness. Other inherent factors influencing stress-strain behavior are hardness of the particles mainly influenced by the mineral content, and specific gravity distribution.

Geologic factors influencing stress-strain behavior are relative density, degree of saturation, the initial stress tensor, age, stress history and the initial soil fabric. Soil fabric relates to the arrangement and orientation of particles and voids within a deposit. This can be quantified by measures such as number of particle contacts and orientation of particles. The importance of initial soil fabric on the stress-strain response has long been recognized (Ladd 1974, Lambrechts and Leonards 1978, Mulilis et al. 1977 and Oda 1972). The method of sample preparation influences the initial fabric obtained, and as a result the deformation characteristics. Even minor differences in the fabric obtained as a result of the sample preparation method used can cause significant differences in the stress-strain behavior of sands even at the same relative density (Ladd 1974, Lambrechts and Leonards 1978, Mulilis et al. 1977 and Tatsuoka et al. 1986). Natural sand deposition process usually produces an anisotropic fabric, which is responsible for anisotropic deformation-strength properties of natural deposits of sands (Miura and Toki 1982, Miura et al. 1984, Oda 1972 and Symes et al. 1984). The initial stress tensor includes the applied mean stress level and shear stress existing on the sample before the load is applied. Stress histories of soils determine the prestressing load already applied to the soil, and influences the
stress level at which the reloading mode ends and virgin loading begins. The effect of stress-strain history is partially reflected in the soil fabric as it changes the orientation of the particles and interparticle contact stresses (DeAlba et al. 1976, Ladd et al. 1977, Lambrechts and Leonards 1978 and Seed, 1979). Prestressing reduces the potential for development of irrecoverable strains. The actual effect of prestressing on the subsequent stress-strain behavior is dependent on the stress path the soil was subjected to, and the level of shear strains the soil was subjected to (Lambrechts and Leonards 1978). The fabric produced as a result of prestressing will be stiffer along the direction of prestressing. The fabric will be able to resist any further development of shear strains or stresses below some threshold shear strain if the sample is sheared along the direction of prestressing (Ladd et al. 1977, Lambrechts and Leonards 1978). However, for other loading directions, the fabric might be even softer than the one existing prior to prestressing (Ladd et al. 1977). Aging of the soil may cause changes to the soil fabric produced during deposition, increasing its strength and reducing deformations produced when the soil is loaded. Schmertmann (1991) demonstrated that aging reduced settlement in sands. Daramola (1980) demonstrated that large strain shear modulus increases with aging. The liquefaction potential of the soil was also found to decrease with aging due to the increase in strength of the deposit (Seed 1979).

Ambient factors influencing the stress-strain behavior include the stress path the soil is subjected to include factors such as drainage conditions as well as rate and type of loading (cyclic and monotonic). In addition, the temperature has also a considerable influence on the stress-strain behavior.

To study the influence of inherent particle characteristics on the shear strength and flow behavior of particulate materials, particle size, particle size distribution, particle shape, angularity and surface texture has to be determined. Particle size and particle size distribution...
can be obtained from a sieve analysis. It is more difficult to determine accurately particle shape, angularity, and surface texture (Oda 1972, Jensen 1974, Poulos et al. 1985). Unambiguous interpretation of particle characteristics will always be difficult, due to the large number of natural factors that must be accounted for.

**Materials selected for experimental research**

The various materials selected for this study will be briefly described here. The materials used consist of both naturally occurring as well as manufactured materials. The manufactured materials available for testing are angular Ottawa sand of three size ranges, $^{1}$#15, #45, and #90, regular Ottawa sand of two size ranges, 0.2 mm and #20/70, and glass ballotini of two size ranges, 0.1 m and #10/20. The naturally occurring materials that were studied are Daytona Beach sand, and Syncrude tailings sand. In addition to the above materials, medium grained and long grained rice was also studied. Index properties of the materials are listed in Table 1.

**Methodology used to determine the shape parameters of particulate materials**

The variables involved in the description of particle shape are numerous, and for a given natural deposit the shape of soil particles can vary greatly. Since shape of particles is an important factor influencing the behavioral characteristics of the material, it is desirable to obtain a quantitative measure. The image analyzer proved to be a useful tool to define morphological characteristics of particulate materials. In addition to providing quantitative indices of the shape of the particle, the use of the image analyzer also provides the investigator with a better insight into the nature of the material. The proposed new Shape and Angularity Factors have advantages over other proposed methods in that they provide a good description of the particle shape in terms of two parameters, thus making it practical to correlate the parameters with engineering

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$^{1}$ # indicates ASTM standard sieve sizes.
properties. They share a common problem associated with treating particles having edges that are intersected twice by a radial segment. Fortunately, most of the materials found in nature have very few such edges.

The outline of a two-dimensional projection of a particle can be quantified numerically by discretizing the perimeter. Thus, the "true" shape of the particle is approximated by an equivalent polygon. The Shape Factor is related to the deviation of global particle outline from a standard outline or datum. The Angularity Factor is a measure of the number and sharpness of the corners.

In this study, a new set of parameters, the deviation from a circular shape, is introduced. Consider a two-dimensional projection of a particle that has been discretized by means of equal sampling intervals (Figure 2). A circle can also be discretized in the same manner, with the same sampling interval as the particle under consideration (Figure 2). Considering the "chords" of the circle as a datum, and using the same radial segments for both the circle and the particle, the angle between corresponding chords can be obtained. Depending on the direction of the particle chord vector relative to that of the circle, these angles may assume positive or negative values. These angles are denoted \( \alpha_i \) where \( i = 1 \) to \( N \), and are called Distortion Angles.

For a specific particle, distortion angles can be plotted as a function of the cumulative sampling interval to form a Distortion Diagram. It is noted that the zero bearing can be selected at an arbitrary radial segment because rotation of the particle does not result in a change in shape. The distortion diagram will be shifted along the abscissa depending on where the zero bearing is, but the order of the distortion angles \( \alpha_i \) relative to each other will not be affected.

The distortion diagram is, in fact, a mapping technique. As the particle shape can be fully reconstructed from the distortion diagram, the diagram will be used as a basis for further
quantification of particle shape. The smaller the sampling interval, the more accurate the particle outline will be represented by the equivalent polygon.

Particle form is defined in terms of the deviation of the particle from that of a circle. As the values of distortion angles ($\alpha_i$) alternate between positive and negative values, the sum of the absolute values is an indication of the particle's deviation from a circular shape. This sum can also be represented as the area under the distortion diagram, divided by the sampling interval. To represent particle form, a non-dimensional Shape Factor can be defined by dividing the sum of the absolute values of $\alpha_i$ by the sum corresponding to a flat particle.

$$\text{Shape Factor, } SF = \frac{\sum_{i=1}^{N} |\alpha_i|_{\text{particle}}}{\sum_{i=1}^{N} |\alpha_i|_{\text{flat sheet}}}$$

where $\sum_{i=1}^{N} |\alpha_i|_{\text{flat sheet}} = N \times 45^\circ$.

For practical particle shapes, values of Shape Factor will lie between zero and one; the former value corresponds to a circle, and the latter value corresponds to a flat plate.

The angularity of a particle can be defined in terms of the number and sharpness of the corners. A measure of angularity can be obtained from the sum of the difference between $180^\circ$ and the internal angles of the particle, $\beta_i$ (Figure 2). Theoretically, the angularity of any smooth shape should be zero. If the sampling intervals were very small, the internal angle of a smooth shape would approach $180^\circ$, and hence its angularity would approach zero. Instead of summing up the absolute differences in $\beta$ angles, the sum of the squares of the differences in internal angles was taken to amplify the influence of sharper corners. Since the shape of a particle is approximated by an N-sided polygon, a small angularity will be obtained even for a smooth particle if its global shape is non-circular. To reduce this discrepancy, an angularity term was
defined as: $\sum_{i=1}^{N}(\beta_{i\text{, particle}} - 180)^2 - \sum_{i=1}^{N}(\beta_{i\text{, circle}} - 180)^2$. A normalized Angularity Factor was then defined as:

$$\text{Angularity Factor, } AF = \frac{\sum_{i=1}^{N}(\beta_{i\text{, particle}} - 180)^2 - \sum_{i=1}^{N}(\beta_{i\text{, circle}} - 180)^2}{3 \times 180^2 - \sum_{i=1}^{N}(\beta_{i\text{, circle}} - 180)^2} \quad (2)$$

Equation (2) will give a value of 0 for the angularity of a sphere, and an angularity of 1 for the sharpest particle analyzed thus far. In this study, it was found that a sampling interval of $9^\circ$ was an optimal compromise between the tediousness of measurement and the accuracy of the results. For particles with aspect ratios greater than 3:1, a sampling interval of $4.5^\circ$ is recommended. SF provides a good measure of the overall form of the particle, and AF gives a reasonable estimate for the angularity of the particle. These two indices can be reported as a range of values or their mean and standard deviation can be reported. For correlation purposes, the latter scheme is more advantageous. More detailed descriptions of the method can be obtained from Sukumaran (1996).

**Relationship between shear strength and shape and angularity factors**

The large strain drained friction angle is influenced by the inherent particle characteristics. The undrained peak friction angle on the other hand, is influenced by the fabric, void ratio and stress path. Therefore, to study the inherent particle characteristics on frictional characteristics, the large strain drained friction angle is plotted against Shape and Angularity Factor in Figure 3 for all the materials included in this study. It can be seen that as the Shape and Angularity Factor increases, the large strain drained friction angle increases in general. For correlation purposes, a single measure would prove more convenient. A new term, Form Factor, FF, was therefore defined as:
The large strain drained friction angle is plotted vs. Form factor in Figure 4. A very strong correlation between Form Factor and drained friction angle is found to exist. This observation can be easily understood because as the SF and AF increases, the inter-particle friction increases.

As explained in Figure 1, inherent, geologic and ambient factors influence the stress-strain and strength characteristics of granular soils. It is believed that flow cone test results - flow rate and pluviated void ratio are also influenced by inherent particle characteristics. Accordingly, it was decided to investigate the effects of shape and angularity factors on the flow cone test results. A successful correlation would suggest that the flow cone could be correlated to the stress-strain properties.

2. RELATIONSHIP BETWEEN FLOW CONE TEST RESULTS AND SHEAR STRENGTH PROPERTIES

2.1 Description of the flow cone test apparatus

The flow cone test apparatus is comprised of a preparation pipe and a flow cone (Tragesser 1992). The apparatus is as shown in Figure 5. The flow cone resembles a funnel having a right circular cylinder welded to an inverted frustum of a cone. The cone has a volume of 533 in$^3$. The milled channel at the end of the flow cone permits the passage of a flow control slide ($\frac{1}{8}$ in $\times \frac{1}{4}$ in). The flow control slide used in the tests had an opening of 3/4" diameter. When the slide hole is aligned with the flow cone exit, the material flows out by gravity. Orifice openings of 1/2" and 1/4" are also available.

The preparation pipe was used to allow for a more uniform and consistent deposition of the sample material into the flow cone. The preparation pipe has a capacity of 469 in$^3$. For ease
in handling, the cone is filled in two stages. The preparation pipe unit terminates in a
prefabricated pipe drain. This detachable metal plate with 30 holes with an internal diameter of
3/8" is attached at the base. The total area available for pluviation is 21.38 cm². The cable and
pulley attaches to the top of the preparation pipe to maintain a constant height of fall during
pluviation.

Material is dry pluviated into the flow cone using the preparation pipe. The pipe is filled
with material while resting on a flat surface to prevent material from flowing out. Once the
preparation pipe is filled, the pipe is moved using the cable and pulley to a position just vertically
above the flow cone and placed at the desired height of fall. During the pluviation process, the
preparation pipe is raised at a constant rate to maintain a constant height of fall. The preparation
pipe is moved in a circular pattern within the walls of the flow cone to produce a more uniform
deposition. The flow cone is filled to the top using this approach, and the excess material is
trimmed off carefully using a straight-edge. During trimming considerable care is taken not to
densify the material within the cone.

The flow cone is fitted with a 0.75" opening at the bottom that is kept closed during the
filling procedure. After filling and trimming the material level with the top of the flow cone, the
slide is opened and the material flowing out is collected. The solid weight flow rate of the
material \(w\) is measured by weighing the solids \(W_s\) flowing from the cone in a given time, \(t\).

\[
\text{Weight Flow Rate (w)} = \frac{W_s}{t}
\]  

(4)

It has been found that \(w\) is independent of \(t\) as long as the level of the material in the cone is
above the top of the inverted frustrum. All the material used to fill the flow cone is collected and
weighed to obtain the unit weight of the material within the flow cone. The volume flow rate \(v\)
is calculated knowing the weight or mass flow rate \((w)\), the total weight of material that is required to fill the flow cone \((W)\) and the total volume of the flow cone \((V)\) (Figure 6).

\[
\text{Volume Flow rate (VFR or \(v\))} = \frac{w}{W} V = \frac{w}{\gamma_d}
\]

where

\(w\) = weight or “mass” flow rate

\(W\) = total weight of material required to fill the flow cone

\(V\) = total volume of flow cone

\(\gamma_d\) = pluviated unit weight = \(\frac{W}{V}\)

In addition, the solid volume flow rate equals:

\[
\text{Solid Volume Flow Rate (SVFR)} = \frac{V_{\text{solids}}}{t} = \frac{W_s}{G_s \gamma_w t} = \frac{w}{G_s \gamma_w} = \frac{VFR}{(1+e)}
\]

or

\[
\text{VFR} = \frac{1+e}{G_s \gamma_w} w = \frac{w}{\gamma_d}
\]

VFR has an advantage over the other two measures when comparing materials with different inherent particle characteristics because the effect of different specific gravities and pluviated void ratios are directly included in the measurement.

**Relation between shear strength properties and flow rate**

As previously stated, this study focuses on the effects of inherent particle characteristics, hence all other contributing factors must be strictly controlled. The initial void ratio and fabric will depend not only on the material characteristics but also on the method of deposition. Dry pluviation was used as the depositional method for triaxial tests because it was the method adopted in the flow cone. The specimen preparation technique used in the flow cone was
replicated in the triaxial tests to correlate the strength properties to the flow rate. The material was sheared in completely drained conditions to large strains.

Only those materials with $D_{50}$ less than 0.6 mm are plotted, to avoid the effects of arching in the flow cone. Figure 7 shows the relation between volume flow rate and the large strain drained friction angles. The dashed line is drawn to show the departure from linearity of the points. The flow rates measured showed a remarkable dependency on the frictional characteristics of the material. This was determined to be due to the fact that the inherent particle characteristics influencing the large strain friction angle of the materials have a similar influence on the flow characteristics of the material. Thus, the flow cone can be used to estimate the large strain friction angles of materials with a wide range in particle characteristics.

**NUMERICAL EXPERIMENTS**

Experimental investigations of particulate flow in a hopper are restricted by the limited quantitative information obtained about what actually happens inside particulate assemblies. To obtain more detailed information about particulate assemblies, some preliminary numerical experiments simulating hopper flow was conducted using PFC$^{2D}$ (Itasca 1997). PFC$^{2D}$ models the movement and interaction of stressed assemblies of rigid circular particles using the distinct element method (DEM). Cundall (1988) and Hart et al. (1988) give a thorough description of the method. The model consists of a two-dimensional collection of discrete, circular particles. However, because it is a two-dimensional program, the results obtained will only give information about the expected trends and not the actual magnitudes.

**Influence of material friction on flow rate**

The modeled hopper has the same dimensions as the one used in the experimental studies referred to in the earlier section. Only half the hopper was modeled. A linear elastic contact
model was used using a constant normal and shear stiffness (Cundall and Strack 1979). To simulate the particle deposition process, 2000 particles of radius 1 to 2 mm (Density = 2.5×10^{-3} g/mm^3) are randomly generated within a prescribed region specified by the walls of the hopper. The particles are then subjected to a gravity field so that they settle within the defined hopper walls. During this process, particles collide with the container walls and with each other. Computations are continued until an equilibrium configuration is obtained. Flow was then initiated by removing the horizontal wall at the orifice and the simulation was continued until all the particles were discharged from the hopper.

The wall friction value was varied between 0.3, 0.4, and 0.5 to study the effect of its variation on rate of discharge of the material at the orifice. Figure 8 shows the total mass of particles discharged plotted against time for the various wall friction values. It can be seen that after an initial acceleration stage, the mass flow rate is constant until the very end of the discharge. The change in wall friction does not appear to influence the mass flow rate measured. The ball friction values were also varied between 25°, 30°, and 35°. Figure 9 shows the total mass of particles discharged against time. It can be seen as the ball friction increases, the mass flow rate decreases. This is more evident from Figure 10 where mass flow rate is plotted versus ball friction. It can be seen that the relationship is bilinear, with the flow rate remaining a constant up to a ball friction value of 30° and thereafter it decreases.

Figures 11 and 12 show typical particle velocity distributions during the constant discharge period. From Figure 11 it can be seen that the flow is radial with all particles flowing towards the virtual apex of the side-walls. The flow pattern does not change much when the wall friction value changes. But from Figure 12 it is evident that a change in ball friction does change the flow pattern. For lower values of ball friction, there is a clear zone of slow moving material adjacent to the wall. Also, there appears to be more mass movement of the material towards the
orifice, resulting in a "mass flow" type of discharge. For larger values of ball friction, a "funnel flow" type of discharge is obtained.

The preliminary numerical simulation results are encouraging. For future research, it is expected that more simulation runs will be undertaken by varying the particle diameter and ball friction values, so as to correspond to the materials tested. The mass flow rates obtained from the numerical simulations will be compared with experimental results. In addition, flow patterns will be studied to obtain a better idea of granular material interaction.

CONCLUSIONS

A good correlation was found to exist between the large strain drained friction angles determined from drained, axial compression tests and the Shape and Angularity Factor. As the Shape and Angularity Factor increases, the large strain drained friction angle increases. A good correlation with Form Factor was also found to exist.

An index test, namely the flow cone was developed to predict the large strain drained friction angle. The volume flow rate measured from the flow cone was found to have a good correlation with the large strain drained friction angle. As the friction angle increases, the volume flow rate decreases.

Numerical simulation studies were conducted to study the effect of wall friction and inter-particle friction on the flow pattern as well as the flow rate. It was found that the wall friction had very little influence on the flow rate as well as the flow pattern observed. The inter-particle friction had the same effect on the mass flow rate as observed in the flow cone. As the inter-particle friction value increases, the mass flow rate decreases. Volume flow rate was not measured in the numerical simulation studies. Also, the flow patterns indicate that as the inter-particle friction value increases, the flow pattern changes from "mass flow" to "funnel flow".
REFERENCES


Figure 1: Factors affecting the behavior of sands (Leonards 1995)
Figure 2: Illustration of the distortion angle used to determine shape and angularity factor
Figure 3: Shape and Angularity Factor vs. $\phi_{\text{drained}}$
Figure 4: $\phi_{\text{drained}}$ vs. Form Factor
Cable and Pulley System
With Handle

Pipe Opening

Specimen Preparation
Pipe

Flow Cone

Support Rods

Flow Control Slide

Receptacle

Table

17.75”

Figure 5: Flow Cone Test Apparatus
\[ V_v = e V_s \]

\[ W_{air} = 0 \]

\[ W_s = G_s V_{solids} \gamma_w \]

Figure 6: Phase Diagram showing the relationship between mass and volume flow rate
Figure 7: Large strain $\phi_{\text{drained}}$ vs. Volume Flow Rate
Figure 8: Mass discharge rates for various values of wall friction
Figure 9: Mass discharge rates for various values of ball friction
Figure 10: Mass Flow Rate vs. Ball Friction
Figure 11: Particle velocity vector plot for different wall friction

Wall Friction = 0.3          Wall Friction = 0.4          Wall Friction = 0.5
Figure 12: Particle velocity vector plot for different ball friction

Friction = 25°  Friction = 30°  Friction = 35°
<table>
<thead>
<tr>
<th>Material</th>
<th>Mean particle size, D50 (mm)</th>
<th>Coefficient of uniformity, Cu</th>
<th>Maximum void ratio, e_{max}</th>
<th>Minimum void ratio, e_{min}</th>
<th>(e_{max} - e_{min})</th>
<th>Specific gravity</th>
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<tr>
<td>Ottawa #20/70</td>
<td>0.53</td>
<td>2.4</td>
<td>0.78</td>
<td>0.47</td>
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<tr>
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<td>0.21</td>
<td>2.4</td>
<td>0.85</td>
<td>0.55</td>
<td>0.30</td>
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<td>1.06</td>
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<tr>
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<td>2.1</td>
<td>1.11</td>
<td>0.75</td>
<td>0.36</td>
<td>2.57</td>
</tr>
<tr>
<td>Ottawa #90 (angular)</td>
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<td>2.2</td>
<td>1.10</td>
<td>0.73</td>
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<tr>
<td>Glass ballotini 0.1 mm</td>
<td>0.17</td>
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<td>0.91</td>
<td>0.34</td>
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<td>2.41</td>
</tr>
<tr>
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<td>0.91</td>
<td>0.34</td>
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<td>1.00</td>
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<td>0.36</td>
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<td>-</td>
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<td>Long Grained Rice</td>
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<td>1.0</td>
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