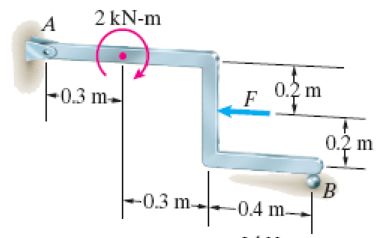
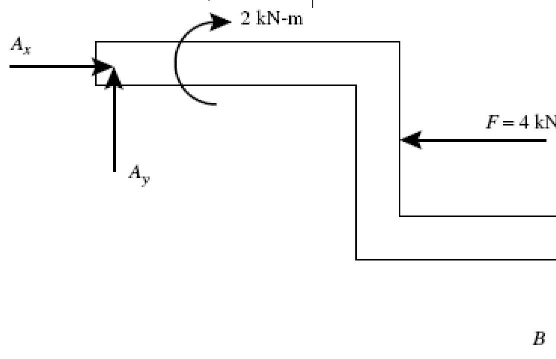
Problem 5.14 (a) Draw the free-body diagram of the beam.

(b) If $F = 4$ kN, what are the reactions at A and B?



Solution:

(a) The free-body diagram



(b) The equilibrium equations

$$\sum M_A : -2 \text{ kN-m} - 4 \text{ kN}(0.2 \text{ m}) + B(1.0 \text{ m}) = 0$$

$$\sum F_x : A_x - 4 \text{ kN} = 0$$

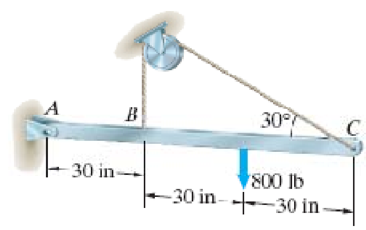
$$\sum F_y : A_y + B = 0$$

Solving:

$$\boxed{A_x = 4 \text{ kN},\ A_y = -2.8 \text{ kN},\ B = 2.8 \text{ kN}}$$

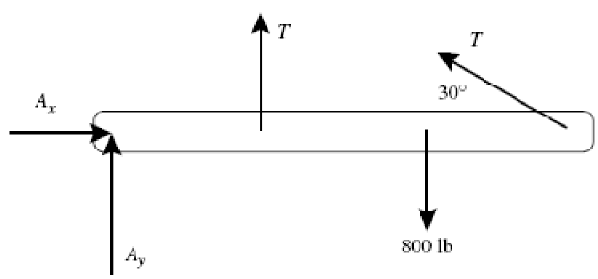
Problem 5.19 (a) Draw the free-body diagram of the beam.

(b) Determine the tension in the cable and the reactions at A.



Solution:

(a) The FBD



(b) The equilibrium equations

$$\sum M_A : \quad (800 \text{ lb})(60 \text{ in}) + T(30 \text{ in})$$

$$+ T \sin 30^\circ (90 \text{ in}) = 0$$

$$\sum F_x : A_x - T \cos 30^\circ = 0$$

$$\sum F_y : A_y + T + T \sin 30^\circ - 800 \text{ lb} = 0$$

Solving:

$$\boxed{A_x = 554 \text{ lb},\ A_y = -160 \text{ lb},\ T = 640 \text{ lb}}$$

Problem 5.47 The suspended weights in Problem 5.46 are each of mass m. The supports at A and E will each safely support a force of 6 kN magnitude. Based on this criterion, what is the largest safe value of m?

Solution: Written with the mass value of 80 kg replaced by the symbol m, the equations of equilibrium from Problem 5.46 are

$$\sum F_x = A_X + E_X = 0,$$

$$\sum F_y = A_Y - 2\ \text{m}(9.81) = 0,$$

and $\sum M_A = 0.3E_X - 0.2\ \text{m}(9.81) - 0.4\ \text{m}(9.81) = 0.$

We also need the relation

$$|A| = \sqrt{A_X^2 + A_Y^2} = 6000\ \text{N}.$$

We have four equations in the three components of the support reactions plus the magnitude of A. This is four equations in four unknowns. Solving for the unknowns, we get the values

$$A_X = -4243\ \text{N},$$

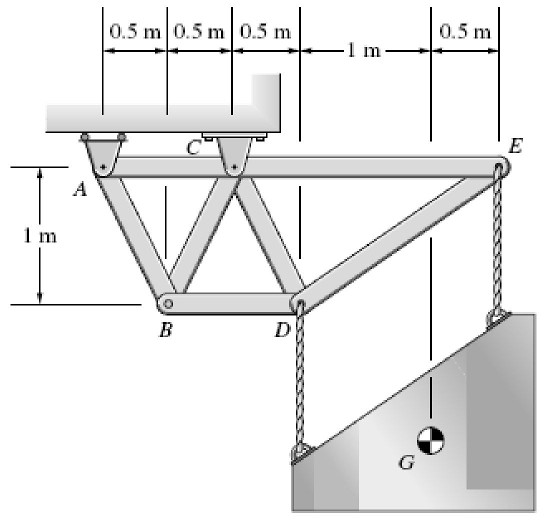
$$A_Y = 4243\ \text{N},$$

$$E_X = 4243\ \text{N},$$

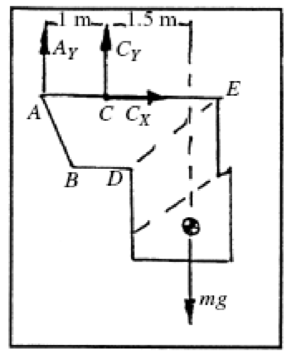
and $m = 216.5$ kg.

Note: We could have gotten this result by a linear scaling of all of the numbers in Problem 5.46.

Problem 5.59 A speaker system is suspended by the cables attached at D and E. The mass of the speaker system is 130 kg, and its weight acts at G. Determine the tensions in the cables and the reactions at A and C.



Solution: The weight of the speaker is $W = mg = 1275$ N. The equations of equilibrium for the entire assembly are



$$\sum F_x = C_X = 0,$$

$$\sum F_y = A_Y + C_Y - mg = 0$$

(where the mass $m = 130$ kg), and

$$\sum M_C = -(1)A_Y - (1.5)mg = 0.$$

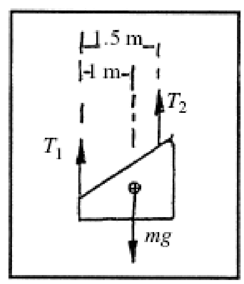
Solving these equations, we get

$C_X = 0$,

$C_Y = 3188$ N,

and $A_Y = -1913$ N.

From the free body diagram of the speaker alone, we get



$$\sum F_y = T_1 + T_2 - mg = 0,$$

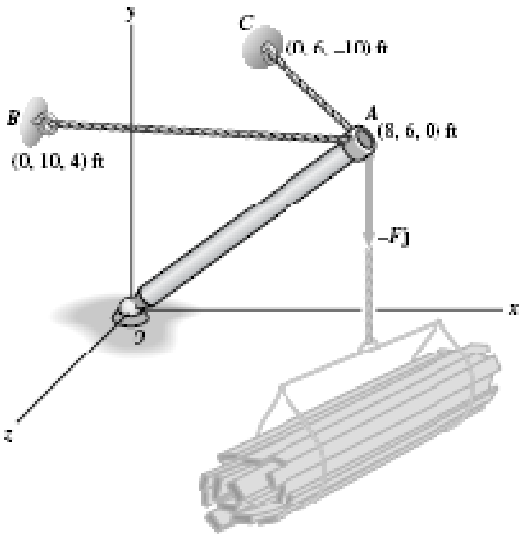
and $\sum M_{\text{left support}} = -(1)mg + (1.5)T_2 = 0.$

Solving these equations, we get

$T_1 = 425.$ N

and $T_2 = 850$ N

Problem 5.89 The suspended load exerts a force $F = 600$ lb at A, and the weight of the bar OA is negligible. Determine the tensions in the cables and the reactions at the ball and socket support O.



Solution: From the diagram, the important points in this problem are A (8, 6, 0), B (0, 10, 4), C (0, 6, −10), and the origin O (0, 0, 0) with all dimensions in ft. We need unit vectors in the directions A to B and A to C. Both vectors are of the form

$$\mathbf{e}_{AP} = (x_P - x_A)\mathbf{i} + (y_P - y_A)\mathbf{j} + (z_P - z_A)\mathbf{k},$$

where P can be either A or B. The forces in cables AB and AC are

$$\mathbf{T}_{AB} = T_{AB}\,\mathbf{e}_{AB} = T_{ABX}\mathbf{i} + T_{ABY}\mathbf{j} + T_{ABZ}\mathbf{k},$$

and $\mathbf{T}_{AC} = T_{AC}\,\mathbf{e}_{AB} = T_{ACX}\mathbf{i} + T_{ACY}\mathbf{j} + T_{ACZ}\mathbf{k}$.

The weight force is

$$\mathbf{F} = 0\mathbf{i} - 600\mathbf{j} + 0\mathbf{k},$$

and the support force at the ball joint is

$$\mathbf{S} = S_X\mathbf{i} + S_Y\mathbf{j} + S_Z\mathbf{k}.$$

The vector form of the force equilibrium equation (which gives three scalar equations) for the bar is

$$\mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{F} + \mathbf{S} = 0.$$

Let us take moments about the origin. The moment equation, in vector form, is given by

$$\sum \mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{T}_{AB} + \mathbf{r}_{OA} \times \mathbf{T}_{AC}$$

$$+ \mathbf{r}_{OA} \times \mathbf{F} = \mathbf{0},$$

where $\mathbf{r}_{OA} = 8\mathbf{i} + 6\mathbf{j} + 0\mathbf{k}$.

The cross products are evaluated using the form

$$\mathbf{M} = \mathbf{r} \times \mathbf{H} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 6 & 0 \\ H_X & H_Y & H_Z \end{vmatrix},$$

where $\mathbf{H}$ can be any of the three forces acting at point A. The vector moment equation provides another three equations of equilibrium. Once we have evaluated and applied the unit vectors, we have six vector equations of equilibrium in the five unknowns T_{AB}, T_{AC}, S_X, S_Y, and S_Z (there is one redundant equation since all forces pass through the line OA). Solving these equations yields the required values for the support reactions at the origin.

If we carry through these operations in the sequence described, we get the following vectors:

$$\mathbf{e}_{AB} = -0.816\mathbf{i} + 0.408\mathbf{j} + 0.408\mathbf{k},$$

$$\mathbf{e}_{AC} = -0.625\mathbf{i} + 0\mathbf{j} - 0.781\mathbf{k},$$

$$\mathbf{T}_{AB} = -387.1\mathbf{i} + 193.5\mathbf{j} + 193.5\mathbf{k} \text{ lb},$$

$$|\mathbf{T}_{AB}| = 474.1 \text{ lb},$$

$$\mathbf{T}_{AC} = -154.8\mathbf{i} + 0\mathbf{j} - 193.5\mathbf{k} \text{ lb},$$

$$|\mathbf{T}_{AC}| = 247.9 \text{ lb},$$

$$\mathbf{M}_{AB} = \mathbf{r}_{OA} \times \mathbf{T}_{AB} = 1161\mathbf{i} - 1548\mathbf{j} + 3871\mathbf{k} \text{ ft-lb},$$

$$\mathbf{M}_{AC} = \mathbf{r}_{OA} \times \mathbf{T}_{AC} = -1161\mathbf{i} + 1548\mathbf{j} + 929\mathbf{k} \text{ ft-lb},$$

and $\mathbf{S} = 541.9\mathbf{i} + 406.5\mathbf{j} + 0\mathbf{k}$ lb