CALCULATION PROCEDURE FOR RESPIRATION

During respiration, there is exchange of oxygen, carbon dioxide and water with the body. In this example we will consider a respiration test on our friend Charlie, and we wish to determine Charlie’s rate of oxygen consumption, carbon dioxide production, and water elimination through respiration.

Charlie breathes in air from the room. We obtain the following information about the room air from the weather station hanging on the wall:

<table>
<thead>
<tr>
<th>Measured room conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>P (mm Hg)</td>
</tr>
<tr>
<td>T (°C)</td>
</tr>
<tr>
<td>Relative humidity (%)</td>
</tr>
</tbody>
</table>

We also know that air is approximately 79% nitrogen and 21% oxygen. This is the composition of dry air. Note that the composition of dry air does not consider the water content (humidity), which can vary. The composition of dry air is always approximately 79% nitrogen and 21% oxygen:

<table>
<thead>
<tr>
<th>Composition of Dry Air</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component</td>
</tr>
<tr>
<td>Nitrogen, $y_{N_2}^{in,dry}$</td>
</tr>
<tr>
<td>Oxygen, $y_{O_2}^{in,dry}$</td>
</tr>
</tbody>
</table>

When this air reaches Charlie’s lungs, some of the oxygen is taken away by the blood to be carried throughout his body. The air in the lungs also picks up some carbon dioxide from the blood, and some water from the moist lung tissues. The result is that Charlie exhales air that has less oxygen, but more carbon dioxide and more water in it. In fact, the air we breathe out is always saturated with water (it holds as much water as it can).

The instrumentation we have in the laboratory measures the composition and the flow rate of the air Charlie breathes out. We know that the exhaled air will be at body temperature (37 °C) and barometric pressure (the same as the barometer reading, 755 mm Hg in this example). It is also saturated with water, so the partial pressure of water is 47 mm Hg. The following data are obtained:
Measured data obtained for Charlie's exhaled air
Saturated, $T=37^\circ C$ and $P_b=755$ mm Hg (BTPS)

<table>
<thead>
<tr>
<th>Flow rate $V_{out,BTPS}$</th>
<th>11.94 L/min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fraction of oxygen, $\gamma^{\text{dry,}out}_{O_2}$</td>
<td>0.1708</td>
</tr>
<tr>
<td>Fraction of carbon dioxide, $\gamma^{\text{dry,}out}_{CO_2}$</td>
<td>0.0325</td>
</tr>
</tbody>
</table>

Other information about Charlie’s exhaled breath

<table>
<thead>
<tr>
<th>Body Temperature</th>
<th>$37^\circ C$ (Constant)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ambient Pressure</td>
<td>755 mm Hg (Measured pressure in room)</td>
</tr>
<tr>
<td>Saturated with water (100% relative humidity)</td>
<td>$P_w = 47$ mm Hg (constant)</td>
</tr>
</tbody>
</table>

This information can be summarized in the following flow diagram

You are asked to calculate the molar flow rate of oxygen, carbon dioxide, and nitrogen in all three streams (the inhaled air, exhaled air, and to the body). You must also find the total molar flowrate of dry gas that is inhaled.

This seems complicated! The procedure outlined below takes you through all the calculations step-by-step. You can follow this same procedure when you perform your respiration experiment.
**Calculation Procedure**

First, make a table to keep track of all of our calculations

<table>
<thead>
<tr>
<th>Flowrate (mol/min) on a dry basis:</th>
<th>$\dot{n}_{O_2}$</th>
<th>$\dot{n}_{CO_2}$</th>
<th>$\dot{n}_{N_2}$</th>
<th>$\dot{n}^{\text{dry}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inhaled</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exhaled</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Exchanged with Body</td>
<td>0 (inert)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Step 1**

We are given the total flow rate of exhaled saturated air in L/min. It is more convenient to do all our calculations in moles/minute, so we convert this to a molar flow rate using the Ideal Gas Law (equation 5):

$$P\dot{V} = \dot{n}^{\text{out, BTPS}}RT_{\text{body}}$$

$$(755 \text{ mm Hg})(11.94 \frac{\text{L}}{\text{min}}) = (\dot{n})\left(\frac{62.36 \text{ L mm Hg}}{\text{mol K}}\right)(310 \text{ K})$$

$$\dot{n}^{\text{out, BTPS}} = 0.4664 \frac{\text{mol}}{\text{min}}$$

**Step 2**

It is also more convenient to perform our calculations based on dry air. We must “remove” the water from the saturated air through a calculation. The total flow rate of saturated air at body temperature and barometric pressure can be converted to the flow rate of dry air (at the same temperature and pressure) by Equation 8.

$$\dot{n}^{\text{dry, out}} = \dot{n}^{\text{BTPS, out}} \left(1 - \frac{47 \text{ mm Hg}}{P_b}\right)$$

$$\dot{n}^{\text{dry, out}} = 0.4664 \frac{\text{mol}}{\text{min}} \left(1 - \frac{47 \text{ mm Hg}}{755 \text{ mm Hg}}\right) = 0.4374 \frac{\text{mol}}{\text{min}}$$

This can be entered in the results table.

**Step 3**

Find the molar composition of the dry outlet gases. We are given the compositions of oxygen and carbon dioxide. The remaining component in the dry gas mixture is nitrogen. We can determine the fraction of nitrogen using equation 2.
\[
\begin{align*}
y_{O_2}^{\text{dry}} + y_{CO_2}^{\text{dry}} + y_{N_2}^{\text{dry}} &= 1 \\
y_{N_2}^{\text{dry}} &= 1 - 0.1708 - 0.0325 = 0.7967
\end{align*}
\]

**Step 4**

Find the flow rate of oxygen, carbon dioxide, and nitrogen in the exhaled breath. We know the dry gas flow rate \( \dot{n}_{\text{dry}} \), and we know the dry gas composition from Step 3, so we can calculate the flow rate of each component using equation 4:

\[
\begin{align*}
\dot{n}_{O_2}^{\text{out, dry}} &= y_{O_2}^{\text{out, dry}} \dot{n}_{\text{dry}} = 0.1708 \left( \frac{0.4374 \text{ mol}}{\text{min}} \right) = 0.0747 \text{ mol/min} \\
\dot{n}_{CO_2}^{\text{out, dry}} &= y_{CO_2}^{\text{out, dry}} \dot{n}_{\text{dry}} = 0.0325 \left( \frac{0.4374 \text{ mol}}{\text{min}} \right) = 0.0142 \text{ mol/min} \\
\dot{n}_{N_2}^{\text{out, dry}} &= y_{N_2}^{\text{out, dry}} \dot{n}_{\text{dry}} = 0.7967 \left( \frac{0.4374 \text{ mol}}{\text{min}} \right) = 0.3485 \text{ mol/min}
\end{align*}
\]

These values represent the flow rates of oxygen, carbon dioxide, and nitrogen in the exhaled air. These can be entered in the results table.

**Step 5**

Nitrogen is inert (all the nitrogen inhaled is exhaled. There is no net transfer of nitrogen to the body). Therefore, the flow rate of nitrogen in is equal to its flowrate out. We can find the inhaled nitrogen flow rate using a mass balance on nitrogen:

\[
\dot{n}_{N_2}^{\text{in}} = \dot{n}_{N_2}^{\text{out}} = 0.3485 \frac{\text{mol}}{\text{min}}
\]

We can enter this in the results table.

**Step 6**

Calculate the total flowrate of dry air inhaled, \( \dot{n}_{\text{in, dry}} \). We know the composition of dry air is 79% nitrogen and 21% oxygen, or \( y_{N_2}^{\text{in, dry}} = 0.79 \) and \( y_{O_2}^{\text{in, dry}} = 0.21 \). We also know the total flow of nitrogen in the inhaled air, \( \dot{n}_{N_2}^{\text{in, dry}} = 0.3485 \text{ mol/min} \), from Step 5. Using Equation 4 we can find the total flowrate of dry air inhaled.

\[
\begin{align*}
\dot{n}_{N_2}^{\text{in}} &= y_{N_2}^{\text{in, dry}} \dot{n}_{\text{dry}} \\
0.3485 \frac{\text{mol N}_2}{\text{min}} &= 0.79 \dot{n}_{\text{in, dry}} \\
\dot{n}_{\text{in, dry}} &= 0.4411 \frac{\text{mol}}{\text{min}}
\end{align*}
\]

**Step 7**
Find the molar flowrate of oxygen in the inhaled air. Now that we know the total flow rate of dry air from Step 6, \( \dot{n}_{in,dry} = 0.4411 \text{ mol/min} \), we can inhaled flowrate of oxygen from equation 4:

\[
\dot{n}_{O_2}^{in} = \gamma_{O_2}^{in,dry} \dot{n}_{in,dry}^{in,dry}
\]

\[
\dot{n}_{O_2}^{in} = 0.021 \left( 0.4411 \frac{\text{mol}}{\text{min}} \right)
\]

\[
\dot{n}_{O_2}^{in} = 0.0926 \frac{\text{mol}}{\text{min}}
\]

This value can be entered in the results table.

**Step 8**

Find the rates of oxygen, carbon dioxide and water transfer to the body. Using a material balance on each component,

\[
\dot{n}_{O_2}^{in} - \dot{n}_{O_2}^{out} + \dot{n}_{O_2}^{body} = 0 \quad \dot{n}_{O_2}^{body} = 0.0747 \frac{\text{mol}}{\text{min}} - 0.0926 \frac{\text{mol}}{\text{min}} = -0.0179 \frac{\text{mol}}{\text{min}}
\]

\[
\dot{n}_{CO_2}^{in} - \dot{n}_{CO_2}^{out} + \dot{n}_{CO_2}^{body} = 0 \quad \dot{n}_{CO_2}^{body} = 0.0142 \frac{\text{mol}}{\text{min}} - 0.0 \frac{\text{mol}}{\text{min}} = 0.0142 \frac{\text{mol}}{\text{min}}
\]

Note that the negative sign for the oxygen flowrate means that oxygen is transferred out of the lungs to the body. Carbon dioxide and water are transferred from the body to the lungs. These values should be entered in the results table. The table is now complete.

Note: Oxygen is transferred to the body so that it can be used by cells for energy. Carbon dioxide is transferred from the body to the lungs because it is a waste product produced by cells. Excess water is transferred from the body to the lungs, and this plays an important role in cooling the body. We will investigate this in more depth in the metabolism and energy lab.
ENGINEERING PRINCIPLES

Mass Balance

We can write a balance on any conserved quantity such as mass, energy, or momentum in the following way.

\[ \text{input} - \text{output} + \text{generation} = \text{accumulation} \]  

Note that quantities being introduced into the system are positive and quantities disappearing from the system are negative. A positive generation quantity indicates that something is being produced in the system, while a negative generation quantity indicates that something is being consumed within the system. Accumulation means that something is building up (+) or disappearing (-) with time.

How does generation occur? One common example is by reaction. Material (or heat energy) could be consumed or produced by a reaction.

Example: Each month you deposit $1,000 into the bank (input) and you withdraw $500 from your account (output). The account also generates $10 interest each month (generation). What is the rate of accumulation of your bank account?

The situation can be expressed using a flow diagram:

The situation can be expressed quantitatively by a balance on the money in the bank account:

\[ \frac{\$1000}{\text{mo}} - \frac{\$500}{\text{mo}} + \frac{\$10}{\text{mo}} = \text{accumulation} = \frac{\$510}{\text{mo}} \]

Example: Charlie breathes in oxygen at a rate of 0.009 mole/min and breathes out oxygen at a rate of 0.07 mole/min. No oxygen builds up within his lungs (accumulation = 0 mole/min). No oxygen is produced in the lungs (generation = 0 mole/min). What is the rate of oxygen exchanged with Charlie’s body?

There is no generation or accumulation of oxygen in Charlie’s lungs. Yet by inspection, we realize that the input by breathing is not equal to the output by breathing. This is because some
of the oxygen in the lungs goes out another way: it is taken away by the blood to be transported through his body. This is just another output!

We can draw the following flow diagram:

We can write the material balance for oxygen in Charlie’s lungs, and solve for the rate at which oxygen is transported to the body:

\[
\dot{n}_{O_2}^{\text{in}} = 0.09 \text{ mol/min} \quad \dot{n}_{O_2}^{\text{out}} = 0.07 \text{ mol/min}
\]

\[
\dot{n}_{O_2}^{\text{body}} = \text{?} \text{ mol/min}
\]

Oxygen is removed from Charile’s lungs at a rate of 0.02 mol O₂/min. Notice that when we say \( \dot{n}_{O_2}^{\text{body}} = -0.02 \text{ mol/min} \), the negative sign indicates removal from the lungs.

**Composition**

Air has a composition of approximately 79% Nitrogen and 21% Oxygen. The fraction of Nitrogen in air is 0.79, and the fraction of Oxygen is 0.21. Notice that the percentages sum up to 100, while fraction of each component sum to 1.0.

We can generalize this by the following equation

\[
\sum_{\text{all components}} y_i = 1 \quad (2)
\]

Where \( y_i \) represents the mole fraction of each component. In the example above, the components are nitrogen and oxygen, and \( y_{O_2}=0.21 \) and \( y_{N_2}=0.79 \). If we have more than two components in the mixture, according to Equation 2 we write \( y_1 + y_2 + y_3 + \ldots = 1 \).
Now suppose we have 10 moles of air ($y_{O_2}=0.21$ and $y_{N_2}=0.79$). How many of each component, oxygen and nitrogen, are there? Intuitively, you might realize that there are 2.1 moles of oxygen and 7.9 moles of nitrogen. To express this in general form,

$$n_i = y_in$$

In Equation 3, $n$ is the total number of moles, and $n_i$ is number of moles of the individual component such as oxygen or nitrogen.

A similar equation can be written for a mixture that is flowing at a certain flowrate $\dot{n}$ (moles/min)

$$\dot{n}_i = y_\dot{n}$$

**Ideal Gas Law (Review)**

You have used the Ideal Gas Law in your Chemistry class. Recall the Ideal Gas Law provides a relationship between the temperature (T), pressure (P), volume (V) and number of moles (n) of a gas:

$$PV = nRT$$

Recall that R, the universal gas constant, can be expressed in different units. Values of R in different units are provided in many engineering texts, and it is convenient to choose the units of R that are consistent with the pressure, volume, and temperature units that you are working with. In our experiments we will be typically work with volume in Liters, pressure in mm Hg, and Temperature in K. The corresponding value of R is:

$$R = \frac{62.36 \text{ L} \cdot \text{mm Hg}}{\text{mol} \cdot K}$$

Example: Suppose you have 1.0 mole of air at $P=760$ mm Hg and $T=298$ K. What volume does this air occupy?

Solving Equation 5 for $V$, we can calculate the volume occupied by the air:

$$V = \frac{nRT}{P} = \left(1 \text{ mole} \frac{62.36 \text{ L mm Hg}}{\text{mol K}} \right) \frac{298 \text{ K}}{760 \text{ mm Hg}} = 24.45 \text{ L}$$

**PARTIAL PRESSURE**

Suppose we have $n$ moles of a gas mixture. The partial pressure $p_A$ is the pressure that would be exerted by $n_A$ moles of the gas A alone in the same total volume and at the same temperature.
Another way to think of the partial pressure of a gas in a mixture is that each gas will contribute to the total pressure, in proportion to its mole fraction in the mixture. In a mixture of air at atmospheric pressure (760 mm Hg), the partial pressure of oxygen would be $0.21(760 \text{ mm Hg}) = 159.6 \text{ mm Hg}$. The partial pressure of Nitrogen would be $0.79(760 \text{ mm Hg}) = 600.4 \text{ mm Hg}$. Together the partial pressures add up to the total pressure of 760 mm Hg. A general expression of partial pressure is:

$$p_A = y_A P$$  \hspace{1cm} (6)

**CONVERSION FROM BTPS TO STPD**

The partial pressure of water in air varies – on humid days it is higher than on dry days. The amount of water that air can hold before becoming saturated varies with temperature. At body temperature, 37°C, the partial pressure of water is 47 mm Hg. The mole fraction of water in saturated air at body temperature (37°C) and barometric pressure, from equation 6, is:

$$y_w^{BTPS} = \frac{P_w}{P} = \frac{47 \text{ mm Hg}}{P_B}$$  \hspace{1cm} (7)

In this experiment we will need to “remove” the water from a saturated air stream, so we can perform the calculations using dry air. First, we can recognize that the fraction of dry air is $(1-y_w)$, from Equation 2. Then we can substitute this into Equation 3 to find the flow rate of dry air:

$$\dot{n}_{out}^{dry} = \dot{n}_{out}^{BTPS} (1-y_w) = \dot{n}_{out}^{BTPS} \left(1 - \frac{47 \text{ mm Hg}}{P_B} \right)$$  \hspace{1cm} (8)
EXERCISE PHYSIOLOGY TERMS

The computer software used in this laboratory uses a system of nomenclature that is standard to exercise physiology. Common terminology is given below. In Engineering, the standard nomenclature is sometimes different. The engineering terminology and nomenclature is explained after each definition.

**BTPS** – Body Temperature Pressure Saturated. 37°C and barometric pressure, saturated with water. The vapor pressure of water at 37°C is 47 mm Hg. The fraction of water in air at 37°C is 47 mm Hg/P_B.

**STPD** – Standard Temperature Pressure Dry. 0°C and 760 mm Hg, dry.

**VO2** ($\dot{V}_{O2}$) = calculated volumetric rate of oxygen consumption, (L/min or ml/min) – calculated by software. The software presents VO2 values at in L/min at STPD – remember that the volume would be different at a different temperature or pressure.

**VCO2** ($\dot{V}_{CO2}$) = calculated volumetric rate of carbon dioxide production, (L/min or ml/min) – calculated by software. The software presents VO2 values at in L/min at STPD.

$V_E$ = measured volumetric flow rate of air (L/min) moving out of the lungs at the designated conditions, either BTPS or STPD. We have chosen BTPS for the Freshman Clinic report.

$V_I$ = calculated volumetric flow rate of air (L/min) moving into the lungs, at BTPS.

$V_t$ = tidal volume, the measured volume of air exhaled in a single normal breath (L).

$O_2$ % (FEO₂-mix) = percent oxygen expired, on a dry basis – measured. (In engineering we commonly work with fractions instead of percentages. We will use $y_{O2}^{out,dry}$ to denote the mole fraction of oxygen in the exhaled gas on a dry basis.)

$CO_2$ % (FECO₂-mix) = percent carbon dioxide expired, on a dry basis – measured. (In engineering we commonly work with fractions instead of percentages. We will use $y_{CO2}^{out,dry}$ to denote the mole fraction of oxygen in the exhaled gas on a dry basis.)