The Memristor

Introduction to NanoElectronics

Jessica Dennis
# Table of Contents

Introduction ............................................................................................................................................ 3
What is Hysteresis? ................................................................................................................................. 3
Prediction of the Memristor .................................................................................................................... 4
What is a Memristor? .............................................................................................................................. 6
Physical Model of a Memristor ................................................................................................................ 7
The creation of the Crossbar Array ........................................................................................................... 9
   HP Labs Research ................................................................................................................................. 9
   Crossbar Array .................................................................................................................................. 10
   Connection to Chua’s Memristor ....................................................................................................... 11
Applications .......................................................................................................................................... 12
   Instant-On Computers ...................................................................................................................... 12
   Brain circuits ..................................................................................................................................... 13
The Future of the Memristor ................................................................................................................... 13
Work Cited ............................................................................................................................................ 14
Introduction
For over 150 years, Resistors, Capacitors, and Inductors were believed to be the only three fundamental passive circuit elements. However, in 1971 Leon Chua reasoned through arguments of symmetry that there should be a fourth fundamental element, which he called a memristor (short for memory resistor). Although he showed that such an element has many interesting and valuable circuit properties, until now no one has presented either a useful physical model or an example of a memristor.

Memristors are passive two-terminal circuit elements. The functional relationship between charge and magnetic flux, called memristance, is similar to variable resistance. Memristors can be used to provide controllable resistance. There is no such thing as a generic memristor. Instead, each device implements a particular function. A linear time-invariant memristor is simply a conventional resistor.

Current researchers at HP labs have created the first working prototype of a Memristor, which is the missing fourth fundamental circuit element which creates a relationship between charge and magnetic flux. Memristance arises naturally in most nanoscale systems. The results from measuring memristors serve as the foundation for understanding a wide range of hysteretic current–voltage behavior observed in many nanoscale electronic devices.

What is Hysteresis?
Hysteresis is a very unique circuit phenomenon which describes the history dependence of physical systems. If someone were to apply pressure by pushing on a spring, it is expected that the spring would yield to the pressure applied. When pressure is released, would the spring completely return back to its original shape, or would there be slight deformation? If the spring system allows for deformation, then the system is exhibiting hysteresis, in a broad sense.

A system which exhibits hysteresis can be described as having the potential to be in a number of states which are independent of the system inputs. A system with hysteresis has path-dependence, or "rate-independent memory". When a system has no hysteresis, someone can predict the output of the system at any instant in time, by using only the input to the system at that time. When a system has hysteresis, then this form of prediction is not possible. In order to predict the output of the system, the history of the input (the state of the system for a given input) needs to be considered. In order to correctly predict the output of a system with hysteresis, one must look at the path that the output followed before it reached the current value.

Many physical systems naturally exhibit hysteresis. Hysteresis is commonly applied to magnetic materials. For example, consider a piece of iron that is brought into a magnetic field. Once exposed to the magnetic field, the iron will stay magnetized indefinitely, even after the external magnetic field is removed (unless a magnetic field is applied in the opposite direction). This effect is used commercially to provide memory in a hard disk drive.
The above figure provides an example of a hysteresis loop. Hysteresis loops happen when you repeatedly alter the system back and forth (cycle the field up and down). This process will be discussed later in the report when hysteresis is used to describe how a memristor reacts to a sinusoidal input (AC).

It is interesting to note that there seems to be no etymological link between the term hysteresis and the terms hysterical or history. This is interesting through linguistic comparison because it would be possible to argue the scientific connections to both of these words. The term is actually derived from the ancient Greek word meaning “deficiency” or “lagging behind”. The term hysteresis was coined by Sir James Alfred Ewing.

**Prediction of the Memristor**

As discussed earlier in this report, anyone who has taken a circuit analysis class would be able to say that there are three fundamental two-terminal elements used for building circuits. These famous fundamental circuit components would be resistors, capacitors and inductors. These three components are passive elements which are capable of dissipating or storing energy. Unlike active elements, resistors, capacitors and inductors are not capable of generating any form of energy. The behavior of each of these circuit elements can be described by a linear relationship between two of the four basic variables describing a circuit: current (i), voltage (v), charge (q) and magnetic flux (φ).

\[
\text{Resistor: } \quad dv = Rdi \\
\text{Capacitor: } \quad dq = Cdv \\
\text{Inductor: } \quad dφ = Ldφ
\]
In 1971, Leon Chua, an electrical engineer professor at UC Berkley, arranged the linear relationships between each of the four basic variables describing a circuit relationships in the way depicted in the above figure. Each of the four circuit criteria are depicted (current, voltage, charge, and magnetic flux) and the linear relationships described earlier relating each criteria to another create a box containing the fundamental circuit element expressing that relationship.

Through this arrangement of the devices, Chau was able to predict the existence of a fourth passive element. He named this hypothetical element that would create a relationship between magnetic flux and charge the memristor. The figure below shows where the memristor would fit into the above expression. It took nearly 40 years before a model system in which memristance could be expressed was able to take physical form.
Conceptually, it was easy for engineers and scientists to visualize how electric charge could relate to magnetic flux, but there was no obvious physical interpretation between charge and the integral over the voltage.

**What is a Memristor?**

As described in the previous section of this lab report, memristance (M) is the functional relationship between charge and magnetic flux. When used in DC, memristors will behave the same as a regular resistor, meaning that the memristance is constant. When given an alternating input in an AC circuit the memristor behaves very differently. If $M$ is a function of $q$, and $q$ fluctuates with time, then the situation becomes more interesting.

A memristor under AC conditions can switch reversibly between a less conductive OFF state and a more conductive ON state as the polarity of this voltage changes on the device. What is amazing about memristors is that the measure of its resistance does not retrace the exact same path through every lap on its i-v hysteresis loop. Because of this hysteresis effect, the memristor acts as a nonlinear resistor, where the resistance depends on the history of the voltage across it. The name of the memristor is a contraction of ‘memory resistor’, which reflects this property.

When devices are on the nanoscale, memristance becomes the dominant effect on circuit components. In fact, on this scale memristance becomes one million times more important than any other circuit effect, yet on the millimeter scale, memristance is almost unnoticeable.

This unique relationship between $q$ and $\phi$ for a sinusoidal input cannot be expressed through a combination of nonlinear resistive, capacitive and inductive components. The fabrication of such a device would render a nanoscale device whose resistance would depend on the magnitude and polarity of the voltage applied. Through the use of this unique relationship, the memristor would also have the
potentially valuable ability to ‘remember’ its most recent resistance when the applied voltage is turned off.

The most basic mathematical definition of a current-controlled memristor is as follows:

\[ v = \mathcal{R}(w)i \]

\[ \frac{dw}{dt} = i \]

In this mathematical definition, \( w \) is a variable which expresses the state of the device and \( \mathcal{R} \) is the generalized resistance that is dependent on the internal state of the device. In general terms, this expression describes how a memristor has a different resistance-like value which depends on where the state of the device currently lies. For the purposes of this section of the lab report, the state variable is equivalent to the charge \( q \).

In 1976 Chau further generalized the concept of the memristor to to be described in a class of nonlinear systems called memristive systems. This change in definition led to the following equations:

\[ v = \mathcal{R}(w,i)i \]

\[ \frac{dw}{dt} = f(w,i) \]

In the above equations, \( w \) is now able to express a set of state variables and \( \mathcal{R} \) and \( f \) can in general be explicit functions of time. At this point in my research, most papers restricted their discussions to current-controlled time-invariant devices.

It is interesting to note that Chua demonstrated that the \( i-v \) characteristics of some devices and systems, such as thermistors, can be modeled using these memristive equations. Obviously there was no direct connection between these mathematic equations and the physical properties of these systems.

**Physical Model of a Memristor**

In this section of the lab report a physical model of a two-terminal electrical device that behaves like a perfect memristor will be discussed. This model produces hysteretic behavior controlled by \( M \) and the boundary conditions on the state variable \( w \).

Anomalies caused by electrical switching in thin-film devices have been discussed through literature for the past 50 years. In general terms, the smaller devices get, the larger role memristance serves in the overall performance of the device. In extremely small devices, magnetic flux and charge work together to cause hysteresis loops in the device’s performance plot. This means that memristance needs to be clearly understood in order to obtain the desired effect from a given device.
For the purpose of this device description, consider a thin semiconductor film of thickness $D$ sandwiched between two metal contacts, as shown in the following figure.

The total resistance of the device is defined by two variable resistors that are connected in series with each other. Specifically, the semiconductor film has a region with a high concentration of dopants (positive ions) having low resistance $R_{ON}$, and the remainder has a dopant concentration that is essentially zero, giving a much higher resistance $R_{OFF}$.

The application of a $v(t)$ across the device will cause the charged dopants to drift. This dopant drift will move the boundary between the two regions. For the simplest case of ohmic electronic conduction and linear ionic drift in a uniform field with average ion mobility $\mu \nu$, the following is obtained:

$$v(t) = \left( R_{on} \frac{w(t)}{D} + R_{off} \left( 1 - \frac{w(t)}{D} \right) \right) i(t)$$

$$\frac{dw(t)}{dt} = \mu \nu \frac{R_{on}}{D} i(t)$$

Which yields the following formula for $w(t)$:
\[ w(t) = \mu v \frac{R_{on}}{D} q(t) \]

By inserting the above equation into the previous \( v(t) \) equation the memristance of this system is obtained, which is for \( R_{on} \ll R_{off} \)

\[ M(q) = R_{off} \left( 1 - \frac{(\mu v)(R_{on})}{D^2} q(t) \right) \]

**The creation of the Crossbar Array**

**HP Labs Research**

Even though the concept of the memristor was predicted in the 70’s, production of such a device was not possible because of the scale of the device. The device ultimately created by HP labs was built on the nanometer scale, composed of a thin (50 nm) titanium dioxide film between two 5 nm thick electrodes.

In 1995, HP Labs created a research group aimed at tackling Moore’s Law. At the time, the existing semiconductor road map did not extend past 2010. The research team took advice from Phil Kuekes, the creative force behind the Teramac (tera-operation-per-second multiarchitecture computer). Kuekes gave the advice to build an architecture that would work even if a substantial amount of the individual devices in the circuit were dead on arrival. This approach would ultimately provide for electronics that would have the potential to keep improving even after the devices got so small that defects would become common.

It was decided that the simplest interpretation of the Teramac architecture was the crossbar, which has since become the standard for nanoscale circuits because of its simplicity and redundancy.

*Figure 5 - Crossbar Architecture shown here in a scanning tunneling microscope image*
Crossbar Array
The Crossbar Array is an array of perpendicular wires. Anywhere two perpendicular wires cross, there is a switch connecting them. In order to connect any horizontal wire to a vertical wire at any orientation on the grid, the switch sandwiched between those two wires must be closed. The general idea was to enable communication by opening and closing these switches by applying voltages to the ends of the wires. By this description, it is fair to describe a crossbar array as basically a storage system, with an open switch representing a zero and a closed switch representing a one.

Crossbar Arrays use a wealth of redundancy, ensuring that any defect present in the device can be avoided. Also, because of their simplicity, crossbar arrays have a much higher density of switches than a comparable integrated circuit based on transistors.

A switch used in a crossbar array is a 40-nanometer cube of titanium dioxide (TiO₂) which is expressed in two separate layers: The lower layer of TiO₂ has a perfect 2:1 oxygen-to-titanium ratio, making it an exceptional insulator. In contrast, the upper layer of TiO₂ is missing 0.5 percent of its oxygen (TiO₂₋₀.₅), causing vacancies make the TiO₂₋₀.₅ a conductive metallic material.
The engineered oxygen deficiencies in the TiO$_{2-x}$ manifest as oxygen vacancies scattered throughout the upper layer. A positive voltage applied to the switch repels the positive oxygen deficiencies in the metallic upper TiO$_{2-x}$ layer, sending them into the insulating TiO$_2$ layer below. That causes the boundary between the two materials to move down, increasing the conductivity of the entire switch. The more positive voltage is applied, the more conductive the switch becomes.

A negative voltage on the switch attracts the positively charged oxygen particles, pulling them out of the TiO$_2$. This reaction renders the switch as a whole to become more resistive. The more negative voltage is applied, the less conductive the cube becomes.

What makes this switch memristive is that when the voltage is turned off, positive or negative, the oxygen bubbles do not migrate. They stay where they are, which means that the boundary between the two titanium dioxide layers is frozen. That is how the memristor ‘remembers’ how much voltage was last applied.

**Connection to Chua’s Memristor**
Leon Chua’s original predicted model for the hypothetical memristor’s behavior is shown in the figure shown below. The graph produced by my HP Labs’ research team’s experimental results is shown in the following figure. The loops represent and map the switching behavior of the device. Both models begin with a high resistance, and as the voltage increases, the current slowly increases. As charge flows through the device, the resistance drops, and the current increases more rapidly with increasing voltage until the maximum is reached. The results obtained by HP labs researcher’s is clearly suggestive of the ideal memristor proposed by Chua.
Applications
Moore’s Law states that the density of transistors on an integrated circuit doubles about every 18 months. This rule of thumb has held true for more than 40 years, but there is an agreement in the industry that the process of miniaturizing transistors will only be able to continue for about another decade. The emphasis in design will have to change to devices that are not just increasingly infinitesimal but are in fact increasingly capable.

When memristors are combined with transistors in a hybrid chip, memristors would be expected to improve the performance of digital circuits without shrinking transistors from their current day dimensions. The purpose is to use transistors more efficiently, and this process could potentially extend the drop-off of Moore’s Law by another decade.

Instant-On Computers
The key feature of a memristor is that when the voltage is turned off, the device remembers the resistance it was last at until. That resistance-saving property makes memristors very appealing for use in computer memory.

The ability to always remember resistance values means that a memristor could be used as a nonvolatile memory. If a laptop contained memory built using memristors, removing the battery from the computer would not lose any memory. When the battery was returned, the computer would turn on exactly where the user left off. Memristors would potentially be able to remove lengthy reboot or unintentional loss of memory due to a power failure.
Brain circuits
One extremely interesting example of a potential application for memristors would be the creation of a circuit that was able to mimic the processes of the human brain. Within a decade, memristors could potentially allow mankind to properly emulate, instead of simply simulate, networks of neurons and synapses.

Using current day transistors, the system needed to simulate a mouse brain in real time would involve solving an astronomical number of partial differential equations. It is estimated that a digital computer capable of this workload would need to be the size of a small city, and it would need to be powered by several dedicated nuclear power plants.

In contrast, by using memristors instead of transistors, an electronic circuit that is theorized to be small enough to fit in a shoebox, would be able to function according to the same physical principles as a brain. This feat would be possible because memristors behave functionally like synapses, replacing a few transistors in a circuit with memristors could lead to analog circuits that can think like a human brain. A hybrid circuit which contains memristors and transistors could help scientists research actual brain function and disorders.

The Future of the Memristor
For memristor based memory devices to become a reality, the reliable design and manufacture of electrode contacts, interconnects and the active region of the memristor must be possible. Also, because signal gain is not possible with a memristor, research needs to be put into creating a high resistance ratio between the ON and OFF states. Fundamentally, a far deeper understanding of the memristor is extremely important.

Transistors clearly have a steady hold on the current design market, and the idea of proposing an alternative to a currently productive technology will be a steep slope to climb. Most manufacturers will embrace this technology only after a well-functioning, large-scale array is clearly demonstrated. When that demonstration occurs, there will surely be a rejuvenated race towards smaller devices.
Work Cited


