3. Equivalent Force, Moment Systems

SECTION OBJECTIVES: By the end of this section, you will be able to:

a. Describe what a moment of a force is, and its characteristics;

b. Calculate the moment produced by a force acting on a two- or three-dimensional body;

c. Calculate the resultant forces and moments that are acting on a two- or three-dimensional body.

READING: Chapter 4

The next level of complexity in solving static equilibrium problems is to consider bodies that are 3-D, rather than a mere particle or point.

In this case, all the forces do not necessarily act on a single point. An example is the supported beam shown below:

![Supported Beam Diagram]

To find the reaction forces, $R_A$ and $R_B$, we need more tools than we have now (the only tool we have is $\sum \vec{F} = 0$). While this is still necessary, it is not sufficient.

To solve problems such as this, we need to learn the concept of a moment, which is defined as the tendency of a force to rotate a body about a point or axis.

A football is a good example:
A moment is a vector quantity, so it has both a magnitude and a direction.

The magnitude of a moment about a point O, or $|\vec{M}_O|$ is defined to be:

$$|\vec{M}_O| = |\vec{F}|d$$  \hspace{1cm} (3.1.)

where $|\vec{F}|$ is the magnitude of the force vector $\vec{F}$ and $d$ is the perpendicular distance from point O to the line of action of the force $\vec{F}$.

What are the units (English and SI) for a moment?

Some other characteristics of moments:

- In 2-D problems, we can represent a moment on our FBDs.
- In 2-D problems, a moment that produces counterclockwise rotation (right hand) is defined as “positive”, while a clockwise rotation (left hand) is defined to be “negative”.
- For two forces, $F_A$ and $F_B$, that each produce a moment on a body, the resultant moment can be shown to be

$$\vec{M}_R = \vec{M}_A + \vec{M}_B$$

**Example:** It is known that a force with a moment of 960 N·m about D is required to straighten the fence post CD. If the capacity of the winch puller AB is 2400 N, determine the minimum value of distance $d$ to create the specified moment about point D.
Schematic

Known

Given

Find

Analysis
Example: Find the moment of the force \( F \) about point \( O \)
Vector representation of a MOMENT

For some problems, finding vectors using the scalar method above is not convenient because the perpendicular distance, \( d \), is not always easy to find. One example of this is in 3-D problems. For these cases, it is more convenient to use a vector method to find moments.

In vector form, the moment of a force, \( \vec{F} \), about a point \( O \), is given by

\[
\vec{M}_O = \vec{r} \times \vec{F}
\]  \hspace{1cm} (3.2.)

where

You might remember this as something called a “cross product”, and the magnitude of this cross product is given by

\[
|\vec{M}_O| = |\vec{r}| |\vec{F}| \sin \theta
\]  \hspace{1cm} (3.3.)

where \( \theta \) is the angle formed when \( \vec{r} \) and \( \vec{F} \) are set “tail to tail” and you use the “right-hand rule” for cross products.

notice from the figure that \( \frac{d}{|\vec{F}|} = \sin \theta \) or \( |\vec{r}| \sin \theta = d \)

Finally, notice that which position vector \( \vec{r} \) you choose to use is arbitrary, as long as it starts at the point of interest (the point you’re finding the moment about) and ends somewhere on the “line of action” of the force.
Example (Prob 4.46): Determine the moment of the 80 N force about the origin $O$ letting $\mathbf{r}$ be the vector

a) from $O$ to $A$

b) from $O$ to $B$
Example (Prob 4.48): Determine the moment of the 100-kN force

a) about $A$,

b) about $B$. 
Couples

Couples are a related concept to MOMENTS. Their definition is:

Two equal magnitude, non-collinear, parallel forces of opposite directions

Some characteristics of couples:

a. the sum of the forces in any direction (x, y or z) is zero

b. because the forces sum to zero, the body will not *translate*, only rotate, because of the couple

c. the magnitude of the couple is its moment, and this is a measure of how strongly it wants to rotate a body

d. a couple has a direction or sense (a vector)

e. the plane that is normal to the couple direction cannot be changed, but it can be translated freely (a “free” vector)
Example (Prob 4.115):

Determine the sum of the moments exerted on the plate by the two couples.
Sometimes, it will be convenient or necessary to “change” a problem from its original, by combining all forces into a single force plus a couple. Here’s an example.

**Example (Prob 4.156):** Two forces act on the beam. If you represent them by a force $F$ acting at $C$ and a couple $M$, what are $F$ and $M$?
Here’s a tougher problem...

**Example:** If you represent the two forces and couple acting on the airplane by a force $F$, what is $F$, and where does the line of action intersect the $x$ axis?
Centroids

Related to this concept of equivalent force-couple systems is something called a ‘centroid’, which is a geometric center of a shape.

A centroid may be related to the center of gravity, if the entire body is made up of stuff with a single density. Consider the two cones below.
What is useful for you (in this and in future classes) is to be able to find the centroid of a composite body.

**Example:** Locate the centroid of the shaded area shown.
Example: Locate the centroid of the composite area shown.