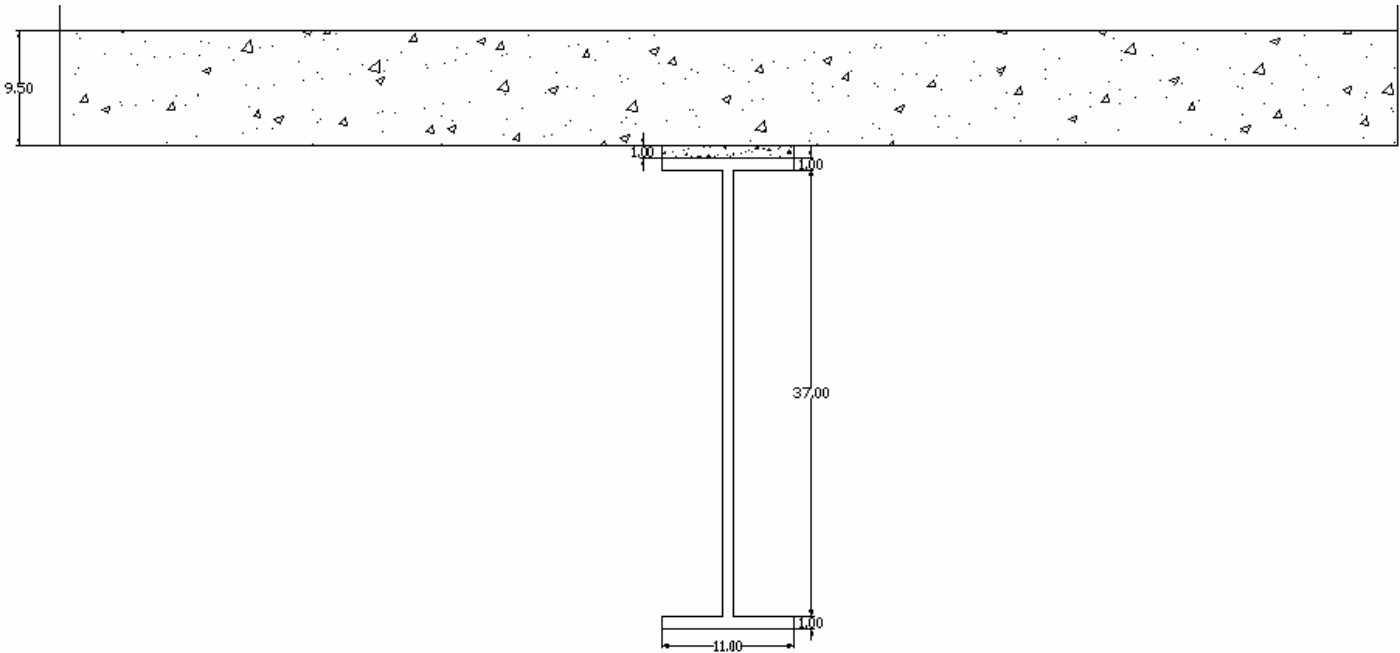


Analysis and Design of Steel Frames
 Composite Beam Homework
 Assigned: Thursday March 27th
 Due: Thursday April 3rd

The beam shown in the diagram below is a plate girder which is fully composite with the concrete slab. Please note that there is a 1 inch thick concrete haunch as commonly used on bridge construction. This is an interior beam, the span is 84 ft and the beams are spaced at 9.25 ft on center. Assume $F_y = 50$ ksi and $f'_c = 4.5$ ksi. The web thickness is 1 inch and the modular ratio is 8.



Problem 1: Determine the effective width of the concrete slab.

Determination of the Effective Width:

b_{eff} is the lesser of:

$$\frac{L}{4} = \frac{84 \text{ ft}}{4} \left(12 \frac{\text{in}}{\text{ft}} \right) = 252 \text{ in}$$

$$\text{or } S = 9.25 \text{ ft} \left(12 \frac{\text{in}}{\text{ft}} \right) = 111 \text{ in} \rightarrow \text{Governs}$$

Problem 2: Determine the moment capacity of the composite beam, being sure to apply an appropriate phi factor.

Determination of Moment Capacity of Composite Beam:

$$a = \frac{A_s F_y}{0.85 f'_c b_e} = \frac{(59 \text{ in}^2)(50 \text{ ksi})}{(0.85)(4.5 \text{ kis})(111 \text{ in})} = 6.95 \text{ in}$$

$$\begin{aligned} \phi M_n &= \phi A_s f_y \left(\frac{d}{2} + t_s - \frac{a}{2} \right) = 0.90 (59 \text{ in}^2) (50 \text{ ksi}) \left(\frac{39}{2} \text{ in} + 1 \text{ in} + 9.5 \text{ in} - \frac{6.95 \text{ in}}{2} \right) = 70,423 \cdot 9 \text{ in} - k \\ &= 5,868 \cdot 7 \text{ ft} - k \end{aligned}$$

Problem 3: Determine the moment of inertia of the composite section

Determination of Beam Deflection:

Component	Width (in)	Length (in)	Area (in ²)	d (in)	Ad (in ³)	y(in)	Ay ² (in ⁴)	I (in ⁴)
Top Flange	1	11	11	38.50	423.50	1.54	26.06	0.92
Web	1	37	37	19.50	721.50	17.46	11280.66	4221.08
Bottom Flange	1	11	11	0.50	5.50	36.46	14623.36	0.92
Concrete Haunch	1	1.375	1.375	39.50	54.31	2.54	8.86	0.11
Concrete Deck	9.5	13.875	131.8125	44.75	5898.61	7.79	7997.09	991.34

$$Moment\ of\ Inertia = Sum(ad^2) + sum(I) = 39,150 \text{ in}^4$$

Problem 4: Determine the live load deflection of the beam assuming a 77.8 kip point load at mid span.

$$\Delta_{Max} = \frac{PL^3}{48 EI} = \frac{(77.8 \text{ k})(84 \text{ ft})^3 \left(12 \frac{\text{in}}{\text{ft}}\right)^3}{48 (29,000 \text{ ksi})(39,150 \text{ in}^4)} = 1.46 \text{ inches}$$

Problem 5: Determine the number of $\frac{3}{4}$ inch shear studs required to obtain full composite action in the beam. Assume a concrete unit weight of 150 pcf.

Determination of Shear Stud Quantity:

$$C_{\max} = 0.85 f'_c b_e t_s = 0.85 (4.5 \text{ ksi}) (111 \text{ in}) (9.5 \text{ in}) = 4,033 \cdot 46 \text{ kips}$$

$$T_{\max} = A_s F_y = (39 \text{ in}^2) (50 \text{ ksi}) = 1,950 \text{ kips} \Rightarrow \text{Controls}$$

$$E_c = w_c^{1.5} \sqrt{f'_c} = (150 \text{ pcf})^{1.5} \sqrt{4.5 \text{ ksi}} = 3,897 \text{ ksi}$$

$$Q_n = 0.5 A_{sc} \sqrt{f'_c E_c} = 0.5 \left(\pi \left(\frac{3}{8} \right)^2 \right) \sqrt{(4.5 \text{ ksi}) (3,897 \text{ ksi})} = 29.25 \text{ kips}$$

$$N = \frac{T_{\max}}{Q_n} = \frac{1,950 \text{ kips}}{29.25 \text{ kips}} = 66.7 \frac{\text{studs}}{\text{half beam}} = \frac{133.4}{\text{beam}} \Rightarrow \text{Use 134}$$