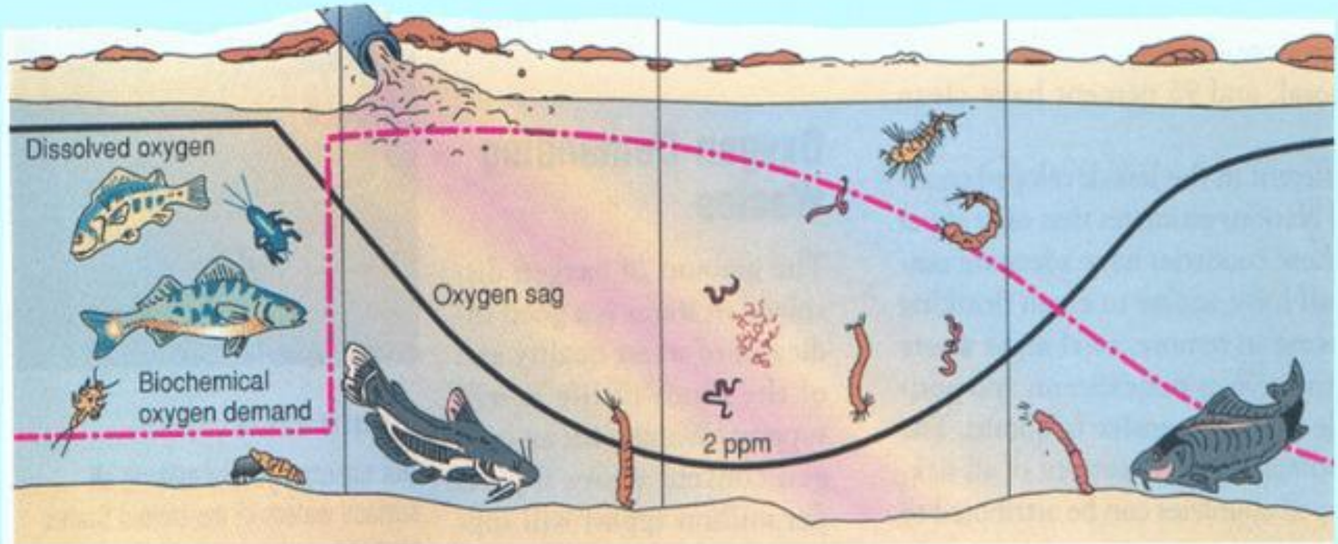
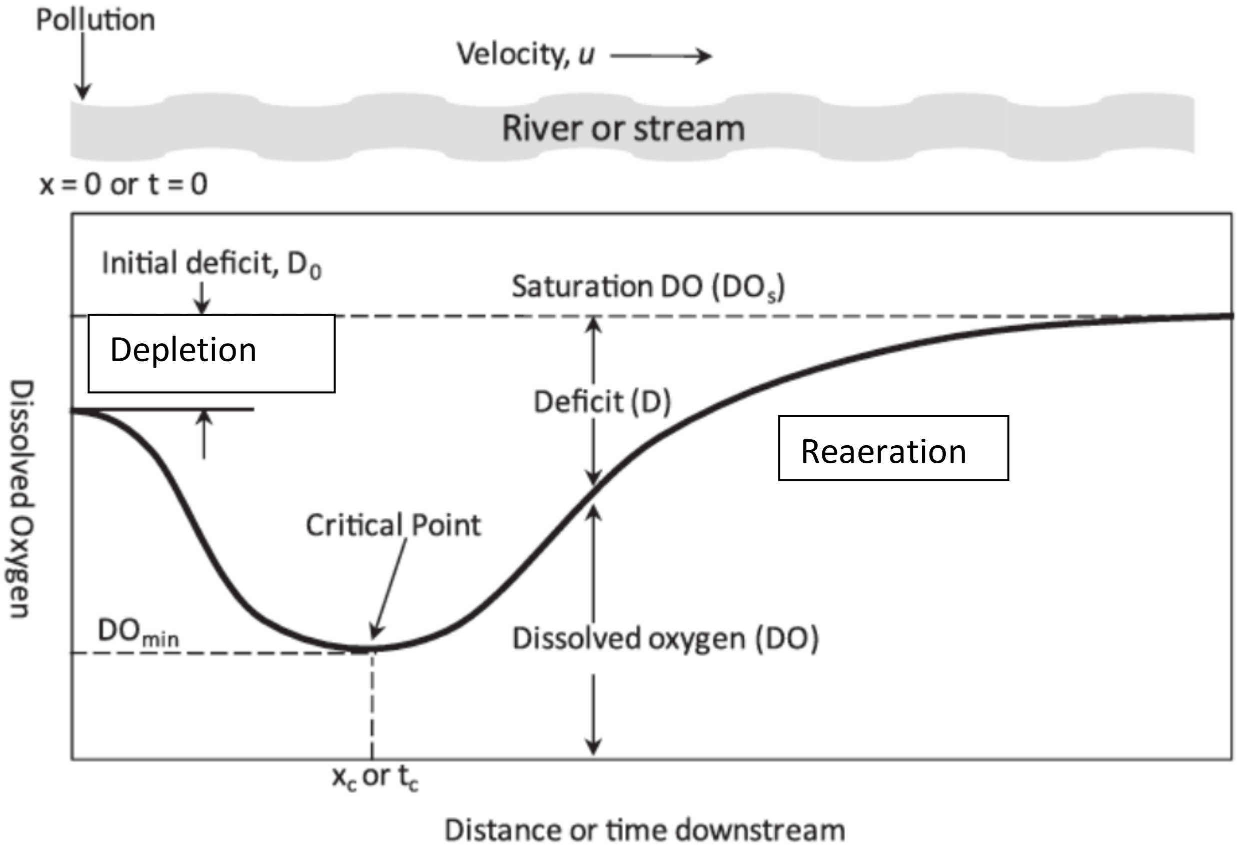
Apply classical engineering river model (Streeter-Phelps DO-sag)



Modeling Effect of O_2 demanding waste on rivers

- In the simple DO model, two key processes are considered:
  - Source of DO: Reaeration from atmosphere
  - Sink of DO: Oxidation of organic matter (carbonaceous)
- The key model assumptions are:
  - Continuous discharge of waste at a given location
  - Uniform mixing of river water and wastewater
  - No dispersion of waste in the direction of flow (ie, plug flow assumed)

Streeter-Phelps Oxygen Sag Curve



Model Equations: Streeter-Phelps

- Now, rate of increase of DO deficit (D),

$$\frac{dD}{dt} = r_D - r_R \Rightarrow \frac{dD}{dt} = k_d L_0 e^{-k_d t} - k_r D \qquad \text{------------(3)}$$

Solution of eq 3 is known as the classic Streeter-Phelps Oxygen Sag Equation:

$$D = \frac{k_d L_0}{k_r - k_d}\left(e^{-k_d t} - e^{-k_r t}\right) + D_0 e^{-k_r t} \qquad \text{..........(4)}$$

$$\Rightarrow D = \frac{k_d L_0}{k_r - k_d}\left(e^{-k_d x/u} - e^{-k_r x/u}\right) + D_0 e^{-k_r x/u} \qquad \text{------------(5)}$$

Where, D_o = DO deficit at t = 0; x = distance d/s (=ut) ;

u = stream velocity ; t = time

Streeter-Phelps Oxygen Sag Curve

- It is important to identify critical point where DO is minimum.
- At Critical point, dD/dt = 0

Solving Eq (3) for this condition,

$$t_c = \frac{1}{k_r - k_d} \ln\left\{ \frac{k_r}{k_d} \left[1 - \frac{D_0 (k_r - k_d)}{k_d L_0} \right] \right\} \text{-------------(6)}$$

From eq (3),

$$\frac{dD}{dt} = k_d L_0 e^{-k_d t} - k_r D$$

At critical point,

$$\frac{dD}{dt} = 0 = k_d . L_o e^{-k_d t_c} - k_r D_c$$

$$D_c = \frac{k_d}{k_r} L_o e^{-k_d t_c} \text{-------------------------------------(7)}$$

So, **$DO_{min} = DO_{sat} - D_c$**