Level Set Segmentation using Non-Negative Matrix Factorization of Brain MRI Images

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***Abstract*—This paper presents a new level set method for image segmentation by integrating the level set formulation and the non-negative matrix factorization (NMF). The proposed model characterizes the histogram of the image by dividing the image into blocks and computing the histograms of the blocks as nonnegative combinations of basic histograms. This is achieved by using the NMF algorithm. The basic histograms form a clustering of the image into distinct regions. Our model also takes into account the intensity inhomogeneity or the bias field that usually corrupts medical images. In a level set formulation, this clustering criterion defines an energy in terms of the level set functions that represent a partition of the image domain. The image segmentation is achieved by minimizing this energy with respect to the level set functions and the bias field. Our method is compared, using synthetic and real images, to other state-of-the-art level set approaches that are based on localized clustering and local Gaussian distribution fitting. It is shown that the proposed approach is more robust to noise in the image and intensity inhomogeneity. These advantages stem from the fact that the proposed model i) depends on the distribution of pixels intensities (the histogram) rather than the direct intensity values and ii) does not introduce additional model parameters to be simultaneously estimated with the bias field and the level set functions.**

I. Introduction

Segmentation of medical images is playing an important role in various medical applications, including quantification of tissue volumes, localization of pathology and computer-integrated surgery. It remains, however, that medical image segmentation is a challenging problem due to the complexity of the anatomical structures, noise from image acquisition and sampling artifacts and intensity inhomogeneity or bias field. These challenges make the classical segmentation techniques, such as thresholding [1], edge detection [2] and region growing [3], ineffective at accurate delineation of complex boundaries.

Active contours, also known as snakes [4], are curves defined within the image that can evolve and deform within the image until they lock onto the objects boundaries. The active contour is represented in a parameterized form by a set of contour points that are propagated under the influence of an internal and an external energy. The internal energy maintains the smoothness of the contour by imposing relevant smoothness and geometrical constraints. The external energy is computed from the image domain and attracts the contour towards the regions boundaries in the image. However, the major drawback of the active contour model lies in its parametric representation. In particular, the contour cannot handle topological changes in the image. For instance, when the contour merges and splits to fit the boundaries in the image, we need to track the contour points and check their orders.

The level set method (LSM) proposes a geometric representation of the contour [5]. Specifically, the contour is represented by defining a higher-dimensional function, referred to as the *level set function*, and then representing the contour as the zero level of this function. Instead of tracking the contour points through time, the level set method evolves the contour by updating the level set function through time. In particular, since the level set does not have any contour points, the merging and splitting of the curve are done automatically and there is no need to track the contour points and their orders during the propagation. The internal and external energies, in the level set approach, are defined in a similar manner as in the active contour method. The advantage of the level set and active contour methods, known as *deformable models*, is that their continuous formulation can achieve pixel-level accuracy, a highly desirable property in the segmentation of medical images.

Recent work on the level set approach took into account the intensity inhomogeneity by defining a local clustering criterion for the image intensities in a neighborhood of each pixel [6]. This local clustering is then integrated with respect to the neighborhood center to give a global criterion of image segmentation, and serve as the external energy term of the level set formulation. Using a similar approach, which clusters the image pixels in a smaller neighborhood, Chen *et al.*, adopted a statistical approach, where the local intensity variations are described by Gaussian distributions with different means and variances [7]. The localized level set method (LLSM) in [6] and the local Gaussian distribution fitting (LGDF-LSM) in [7] have additional nuisance parameters that must be iteratively estimated along with the level set function and the bias field, which are the parameters of interest to the segmentation problem. In the localized LSM approach, the local means of the intensity values in every region have to be estimated along the bias field and the level set function. Similarly, the LGDF-LSM involves simultaneous estimation of the means and standard deviations of every local neighborhood. Given the high-dimensionality and non-convexity of the variational optimization problem, these additional parameters are estimated in an iterative procedure that does not guarantee convergence or optimality of the results [6], [7]. Hence, an imminent drawback of these two state-of-the-art approaches is the number of nuisance parameters that are introduced and must be simultaneously estimated, which decreases the estimation accuracy of the main segmentation parameter: the level set functions.

In this paper, we propose a new region-based level set method that is also able to take into account the intensity inhomogeneity, but does not require the estimation of nuisance parameters in addition to the bias field and the level set function. Moreover, the proposed level set approach uses pixel intensities distribution (histogram) clustering, and also provides a local statistical characterization of the image. This is achieved by integrating the probabilistic non-negative matrix factorization (PNMF) framework [8] into the level set formulation. Specifically, the image is divided into m blocks, and the histogram of every block is computed. An $n \times m$ data matrix V is then created whose columns are the histograms of the image blocks, and n is the number of intensity bins. We show that the PNMF decomposes the histogram matrix V into a mixture of $k \leq m$ basic histograms using the factorization $V \approx WH$. The columns of the factor matrix W consist of the basic histograms of the k regions (to be segmented) in the image. We subsequently model the pixel intensities in every image region using a Gaussian distribution whose mean and variance are given by the mean and variance of the corresponding basic histogram in the factor matrix W. In addition, the bias field is added as a multiplicative factor for each pixel to handle intensity inhomogeneity. The maximum a posteriori probability of the image regions given the pixels intensity values defines the external energy functional of the level set formulation.

This paper is organized as follows: Section II reviews the framework of the level set formulation. Section III introduces the proposed PNMF-based level set approach. In Section IV, we provide and discuss the simulation results based on synthetic and real brain MRI images. Then the main contributions of this paper are summarized in Section V.

II. The Level Set Framework

A. Mumford-Shah Model [9]

Let Ω be the image domain, and $I : \Omega \rightarrow \mathbb{R}$ be a gray-value image. The goal of the segmentation is to find a contour C, which separates the image domain Ω into disjoint regions $\Omega_1, \cdots, \Omega_k$, and a piecewise smooth function u that approximates the image I and is smooth inside each region Ω_i. This is formulated as the minimization of the following Mumford-Shah (MS) functional:

$$\mathcal{F}^{MS}(u, C) = \int_{\Omega} (I-u)^2 d\boldsymbol{x} + \mu \int_{\Omega \setminus C} |\nabla u|^2 d\boldsymbol{x} + \nu |C|, \quad (1)$$

where $|C|$ is the length of the contour C. In the right hand side, the first term is the external energy term, which drives u to be close to the image I, and the second term is the internal energy, which imposes smoothness on u within the regions separated by the contour C. The third term regularizes the contour. The MS model is very general and does not assume a specific form for approximating the function u. It also assumes that the objects to be segmented are homogeneous.

B. Chan and Vese Model [10]

Chan and Vese simplified the Mumford-Shah model by assuming that the approximating function u is piecewise constant:

$$\mathcal{F}^{CV}(\phi, c_1, c_2) = \int_{\Omega} |I(\boldsymbol{x}) - c_1|^2 H(\phi) d\boldsymbol{x} + \int_{\Omega} |I(\boldsymbol{x}) - c_2|^2 (1 - H(\phi)) d\boldsymbol{x} + \nu \int_{\Omega} |\nabla H(\phi)| d\boldsymbol{x}, \quad (2)$$

where H is the Heaviside function, and ϕ is a level set function, whose zero level contour C partitions the image domain Ω into two disjoint regions $\Omega_1 = \{\boldsymbol{x} : \phi(\boldsymbol{x}) > 0\}$ and $\Omega_2 = \{\boldsymbol{x} : \phi(\boldsymbol{x}) < 0\}$. Equation (2) is a piecewise constant model, as it assumes that the image I can be approximated by constant c_i in region Ω_i. In the case of more than two regions, two or more level set functions can be used to represent the regions $\Omega_1, \cdots \Omega_k$.

III. The Proposed PNMF-Level Set Segmentation Approach

A. PNMF-based Clustering Framework

We divide the image into m blocks, and compute the histogram of every block. An $n \times m$ data matrix V is then created, where n is the number of intensity bins in the image. As shown in Fig. 1, the columns of V are the histograms of the blocks in the image. Specifically, the $(i,j)^{th}$ entry, v_{ij}, is the number of pixels in the block j having intensity range in bin i. The rows of V describe the spatial distribution of intensity bin i in the image blocks. The goal is to find $k \leq m$ "basic histograms" that correspond to the distinct regions in the image. This can be achieved using non-negative matrix factorization (NMF). The NMF seeks to factor the data matrix V into two matrices with positive entries $V \approx WH$, where W has size $n \times k$, with each of the k columns defining a metablock, or a region in the image; the entry w_{ij} corresponds to the number of pixels within the intensity bin i in metablock j. The matrix H has size $k \times m$, with each of the m columns representing the metabin region representations of the corresponding block; the entry h_{ij} represents the number of pixels in region i and block j.

Nonnegative matrix factorization involves the non-negativity constraints on the matrix entries which is appropriate for clustering the histogram data matrix. Other matrix decomposition techniques, such as principal component analysis (PCA) or singular value decomposition (SVD) do not guarantee the nonnegativity constraint, and hence loose the physical interpretation of the factorization. However, this non-negativity requirement makes the factorization problem more challenging. We use the PNMF algorithm in [8], which also takes into account the noise in the data matrix. The algorithm starts by randomly initializing matrices W and H with non-negative entries, which are iteratively updated to maximize the maximum a posteriori probability (MAP) criterion assuming the data matrix is corrupted by additive white Gaussian noise. The update equations are given by:

$$\begin{cases} H_{ij} \leftarrow H_{ij} \frac{(W^T V)_{ij}}{(W^T W H + H)_{ij}} \\ W_{ij} \leftarrow W_{ij} \frac{(V H^T)_{ij}}{(W H H^T + W)_{ij}}, \end{cases} \quad (3)$$

It was shown in [8] that the update equations in (3) minimize the following weighted regularized optimization problem:

$$f(W,H) = \|V - WH\|_F^2 + \|W\|_F^2 + \|H\|_F^2. \qquad (4)$$

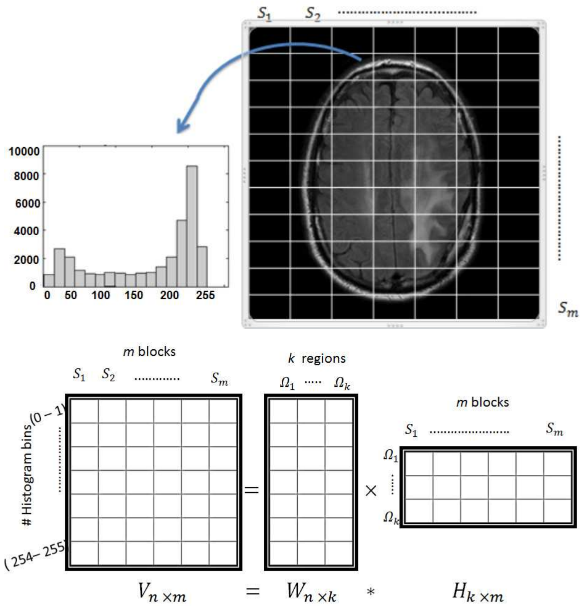

Fig. 1: Building the data matrix and histogram factorization using PNMF.

The PNMF factorization $V \approx WH$ induces a clustering of the histogram data matrix into k basic histograms which correspond to k regions $\Omega_i, i = 1, \cdots, k$. Thus, the PNMF can detect the regions in the image automatically. In the sequel, we will investigate how the non-negative matrix W provides statistical information about the clustered regions in the image. We first consider the synthetic binary image in Fig. 2. The PNMF of the data matrix of this image with $k = 2$ and a block size of 16×16 results in the W matrix shown in Fig. 2. Plotting the entries of each column of W, we obtain two sharp peaks: one peak at the $(0-1)$ range of intensity value, corresponding to the black region, and a second peak at the $(254-255)$ range, corresponding to the white region. Hence, the matrix W seems to provide the distribution of the pixel intensity values in each region, and from this distribution, we can obtain the statistical mean and standard deviation of every region in the image. The same interpretation can be reached on a synthetic gray-scale image (though not shown here for space considerations).

B. Proposed Variational Framework

We consider the matrix W in the PNMF factorization $V \approx WH$, which induces a clustering of the image blocks into k regions $\Omega_i, i = 1, \cdots, k$ defined by their basic histogram characteristics given in the matrix W. This clustering model will form the basis of the external energy functional in the level set formulation. Let us assume that the i^{th} column of the matrix W, $\boldsymbol{w}_i$, is the histogram of the i^{th} image region Ω_i. Let μ_i and σ_i be the mean and standard deviation, respectively, of $\boldsymbol{w}_i$. We model the intensity value of a pixel $\boldsymbol{x} \in \Omega_i$ as normally distributed with mean $\mu_i b(\boldsymbol{x})$ and standard deviation σ_i, where $b(\boldsymbol{x})$ is the bias field at pixel $\boldsymbol{x}$. The problem of image segmentation is then formulated as computing the maximum a posteriori (MAP) $p(\{\Omega\}|I)$ of the image regions $\{\Omega\} = \{\Omega_1, \Omega_2, \cdots, \Omega_k\}$ for the image intensity values $I(\boldsymbol{x})$. According to the Bayes' rule $p(\{\Omega\}|I) \propto p(I|\{\Omega\})\ p(\{\Omega\})$. Assuming that the prior probabilities of all partitions $p(\{\Omega\})$ are equal, and the pixels within each region are independent, the MAP estimate reduces to finding the maximum of $\prod_{i=1}^{k} \prod_{\boldsymbol{x}\in\Omega_i} p_i(I(\boldsymbol{x}))$, where $p_i(I(\boldsymbol{x})) = p(I(\boldsymbol{x})|\Omega_i)$, $i = 1, 2, ..., k$. By taking the logarithm, the maximization can be converted to the minimization of the following energy function:

$$E = \sum_{i=1}^{k} \int_{\Omega_i} -\log p_i(I(\boldsymbol{x}))d\boldsymbol{x}, \qquad (5)$$

where $p_i(I(\boldsymbol{x}))$ is given by

$$p_i(I(\boldsymbol{x})) = \frac{1}{\sqrt{2\pi}\sigma_i} \exp\left(-\frac{(I(\boldsymbol{x}) - \mu_i b(\boldsymbol{x}))^2}{2(\sigma_i)^2}\right). \qquad (6)$$

The energy function E is then combined in the level set formulation by representing the disjoint regions with a number of level set functions. We first start with the two-phase formulation, then the regions Ω_1, Ω_2 can be represented with their membership functions defined by $M_1(\phi) = H(\phi)$ and $M_2(\phi) = 1 - H(\phi)$ respectively, where H is the Heaviside function. For more than two regions, two or more level set function are then defined. The minimization problem in Eq. (5) can be equivalently expressed as the following level set energy functional:

$$E(\phi, \mathbf{b}) = \sum_{i=1}^{k} \int_{\Omega} \left(\log(\sqrt{2\pi}\sigma_i) + \frac{(I(\boldsymbol{x}) - \mu_i b(\boldsymbol{x}))^2}{2\sigma_i^2} \right) M_i(\phi) d\boldsymbol{x}. \qquad (7)$$

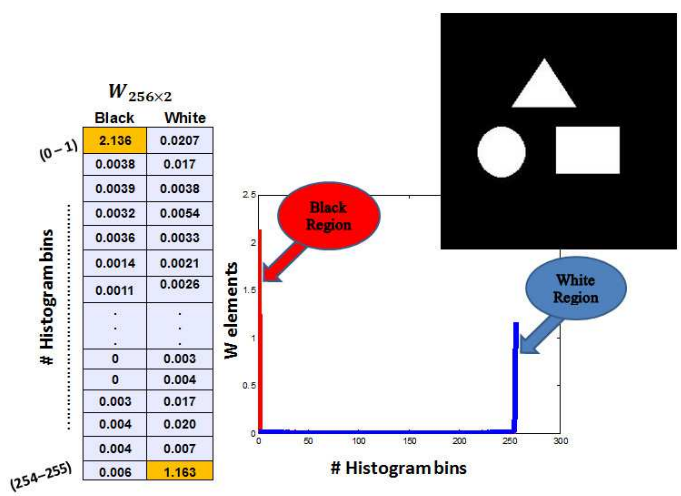

Fig. 2: Non-negative Matrix Factorization $V \approx WH$: The factor "W" of this synthetic binary image provides the *basic histograms* of the two (white and black) regions in the image.

Equation (7) can be rewritten as:

$$E(\phi, \mathbf{b}) = \sum_{i=1}^{k} \left[\int_{\Omega} e_i(\boldsymbol{x}, \mathbf{b}) M_i(\phi(\boldsymbol{x})) d\boldsymbol{x} \right], \tag{8}$$

where $e_i(\boldsymbol{x}, \mathbf{b}) = \log(\sqrt{2\pi}\sigma_i) + \frac{(I(x)-\mu_i b(x))^2}{2\sigma_i^2}$.

By adding the geometrical constrains to the energy functional $E(\phi, \mathbf{b})$, we obtain the total energy functional in the level set framework,

$$\mathcal{F}(\phi, \mathbf{b}) = \alpha E(\phi, \mathbf{b}) \; + \; \beta R(\phi) \; + \; \gamma Lg(\phi), \tag{9}$$

where $R(\phi)$ and $L_g(\phi)$ are the regularization terms, and α, β and γ are weighting parameters. The first term, $R(\phi)$, is a distance regularization term [11] that is minimized when $|\nabla\phi| = 1$, a property of the signed distance function. This term is defined by

$$R(\phi) = \frac{1}{2} \int_{\Omega} (|\nabla\phi| - 1)^2 d\boldsymbol{x}. \tag{10}$$

The second regularization term is defined by

$$L_g(\phi) = \int_{\Omega} g |\nabla H(\phi(\boldsymbol{x})| d\boldsymbol{x}, \tag{11}$$

which computes the arc length of the zero level set contour, $(\int_{\Omega} |\nabla H(\phi(\boldsymbol{x})| d\boldsymbol{x})$, and therefore maintains the smoothness of the contour. The contour length is weighted by the edge indication function defined by

$$g = \frac{1}{1 + |\nabla(G_\sigma * I)|^2}, \tag{12}$$

where $G_\sigma * I$ is the convolution of the image I with the smoothing Gaussian kernel G_σ. The edge indication function g works to stop the level set evolution near the variational edges because it is close to zero at this place and positive otherwise. Therefore, the length regularization term L_g works to minimize the length of the contour at the boundaries of the objects in the image.

Finally, the total level set energy functional that needs to be minimized in order to achieve segmentation is expressed as:

$$\mathcal{F}(\phi, \mathbf{b}) = \alpha \sum_{i=1}^{k} \left[\int_{\Omega} e_i(\boldsymbol{x}, \mathbf{b}) M_i(\phi(\boldsymbol{x})) dx \right] + \frac{\beta}{2} \int_{\Omega} (|\nabla\phi| - 1)^2 dx + \gamma \int_{\Omega} g |\nabla H(\phi) d\boldsymbol{x}. \tag{13}$$

C. Energy Minimization

Segmentation is achieved by minimizing the energy functional $\mathcal{F}$ with respect to the two variables ϕ and $\mathbf{b}$. The energy minimization is achieved iteratively by minimizing $\mathcal{F}$ with respect to each variable given the other held constant at each value during the previous iteration. We first fix $\mathbf{b}$, then the minimization of the energy functional $\mathcal{F}(\phi, b)$ with respect to ϕ is achieved by using standard gradient descent method, namely solving the gradient flow equation:

$$\frac{\partial\phi}{\partial t} = -\frac{\partial\mathcal{F}}{\partial\phi}. \tag{14}$$

By calculus of variations, we compute the derivative $\frac{\partial\mathcal{F}}{\partial\phi}$ and express Eq. (14) as follows:

$$\frac{\partial\phi_l}{\partial t} = -\alpha \sum_{i=1}^{k} \left(\frac{\partial M_i(\phi)}{\partial\phi_l} e_i \right) + \beta \left(\nabla^2\phi_l - \text{div}\left(\frac{\nabla\phi_l}{|\nabla\phi_l|} \right) \right) + \gamma\delta(\phi_l) \; \text{div}\left(g \frac{\nabla\phi_l}{|\nabla\phi_l|} \right), \tag{15}$$

Then, for fixed ϕ the optimal bias field $\mathbf{b}$ that minimizes the energy $\mathcal{F}$ is estimated by:

$$\mathbf{b} = \frac{\sum_{i=1}^{k} \int_{\Omega} \frac{I(\boldsymbol{x})\mu_i}{\sigma_i^2} M_i(\phi_l) d(\boldsymbol{x})}{\sum_{i=1}^{k} \int_{\Omega} \frac{\mu_i^2}{\sigma_i^2} M_i(\phi_l) d(\boldsymbol{x})}. \tag{16}$$

In implementation, the Heaviside function is approximated by [12]

$$H_\epsilon(\boldsymbol{x}) = 0.5 \sin(\arctan(\frac{x}{\epsilon})) + 0.5, \tag{17}$$

and the dirac delta function, $\delta_\epsilon(\boldsymbol{x})$, is estimated by

$$\delta_\epsilon(\boldsymbol{x}) = 0.5 \cos(\arctan(\frac{x}{\epsilon})) \; \frac{\epsilon}{\epsilon^2 + x^2}. \tag{18}$$

IV. Simulation Results and Discussion

We first assess the performance of the proposed PNMF-based level set approach by segmenting synthetic images and then apply it to real brain MRI images with and without the glioblastoma tumor, though we do not show the normal brain segmentation for space consideration. Specifically, we are interested in delineating the gray-matter, white matter, cerebrospinal fluid (CSF) and tumor regions within the brain. We automate the initialization of the level set function by using the fuzzy c-means (FCM) algorithm and initiate the level set function as $\phi_o = -4\epsilon(0.5 - B_k)$, where ϵ is a constant regulating the Dirac function, and B_k is a binary image obtained from the FCM result. We compare the proposed method to the level set formulations of Li *et al* (localized-LSM) [6] and Chen *et al* [7] (LGDF-LSM), and study the robustness to noise. The segmentation accuracy is quantitatively assessed using the root mean square error ($RMSE$). We set the internal and external energy weighting coefficients $\alpha = \beta = \gamma = 1$ so that all energy terms are equally weighted. We found that the segmentation results are robust to these coefficients in the interval $[0.1, 20]$. We also choose the block size to be 8×8 for partitioning the images. The choice of the block size presents a tradeoff between the smallest size of the region that we would like to capture and computational complexity. A small block size is able to capture a similarly small distinct region in the image at the expense of computational power.

A. Robustness to noise

We study the robustness of the proposed PNMF-based level set method to noise and intensity inhomogeneity by comparing it with the localized-LSM model in [6], and the improved LGDF-LSM model in [7] using $RMSE$. Figure 3 shows three synthetic images with the segmentation results of the three models. We can notice that the proposed PNMF-LSM is more robust to noise and intensity inhomogeneity than the other two approaches. Table (I) presents the comparison, using $RMSE$ values, between the three methods on 10 synthetic images.

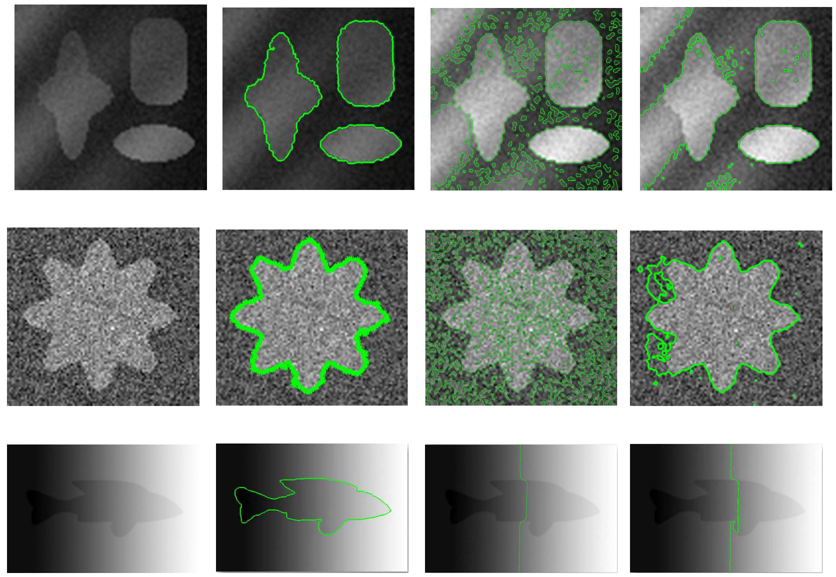

Fig. 3: Performance evaluation of the proposed PNMF-LSM approach, the localized-LSM in [6] and the improved LGDF-LSM in [7] on three synthetic images corrupted with different level of noise and intensity inhomogeneity. The first column represents the original images. The second column shows the segmentation of the proposed PNMF-based level set algorithm. The third and fourth columns show the results of the localized-LSM and the improved LGDF-LSM models, respectively.

TABLE I: Quantitative evaluation based on $RMSE$ of the three segmentation models: The proposed PNMF-LSM, the localized-LSM [6] and the improved LGDF-LSM [7], applied to 10 synthetic images.

Image No.	Proposed PNMF-LSM	Localized-LSM	Improved LGDF-LSM
1	0.2181	0.7205	0.7239
2	0.0851	0.0763	0.8710
3	0.1940	0.5095	0.3854
4	0.2802	0.6103	0.9220
5	0.1426	0.4335	0.2230
6	0.0741	0.0377	0.8624
7	0.0257	0.0452	0.8879
8	0.0687	0.9976	0.0761
9	0.0916	0.3100	0.1292
10	0.2015	0.2358	0.1212

As seen from the table, the proposed PNMF-based level set model provides more stable and lower $RMSE$ values than the localized-LSM and the improved LGDF-LSM models.

B. Application to Real Brain MRI Images

We apply the proposed PNMF-based level set approach on real brain MRI images with a tumor obtained from the University of Alabama at Birmingham School of Medicine. The main goal is to segment the different brain and tumor structures into the following regions: gray matter, white matter, cerebrospinal fluid (CSF), edema, and tumor (if it exists). Figure 4 shows the segmentation result of the proposed approach on a gadolinium-enhanced T1-Weighted MRI scan of a patient with glioblastoma multiform. The green contour in Fig. 4b indicates the growing tumor, the red contour in Fig. 4d indicates the gray matter, the blue contour in Fig. 4e indicates the white matter, and the yellow contour in Fig. 4f indicates the the CSF with the back ground. The pink contour in Fig. 4c indicates the region of the brain that corresponds to necrosis and invasive tumor/edema. The red and blue arrows in its binary representation Fig. 4i point to the area of necrosis and high signal intensity in Fluid-attenuated Inversion Recovery (FLAIR) sequences (not shown), respectively. We notice from Fig. 4 that the PNMF-LSM model is able to separate the different brain structures. The binary representations of each brain structure in Fig. 4 show the exact boundaries of these structures. The proposed PNMF-based level set approach retrieves the histogram of each region (or brain structure, see Fig. 4m obtained from the factor matrix W. We show also the bias field in the MRI image (Fig. 4g), which is estimated through the level set formulation. We also apply PNMF-LSM to a normal brain MRI images though not shown in this paper for space consideration.

V. Conclusion

In this paper, we proposed a new deformable model for image segmentation based on a variational level set formulation and probabilistic non-negative matrix factorization. The PNMF algorithm clusters the histogram data matrix into basic histograms that correspond to the distinct brain regions in the image. The proposed model is also able to account for the intensity inhomogeneity in medical images without a priori knowledge. The segmentation is achieved by minimizing an energy functional with respect to the level set function and

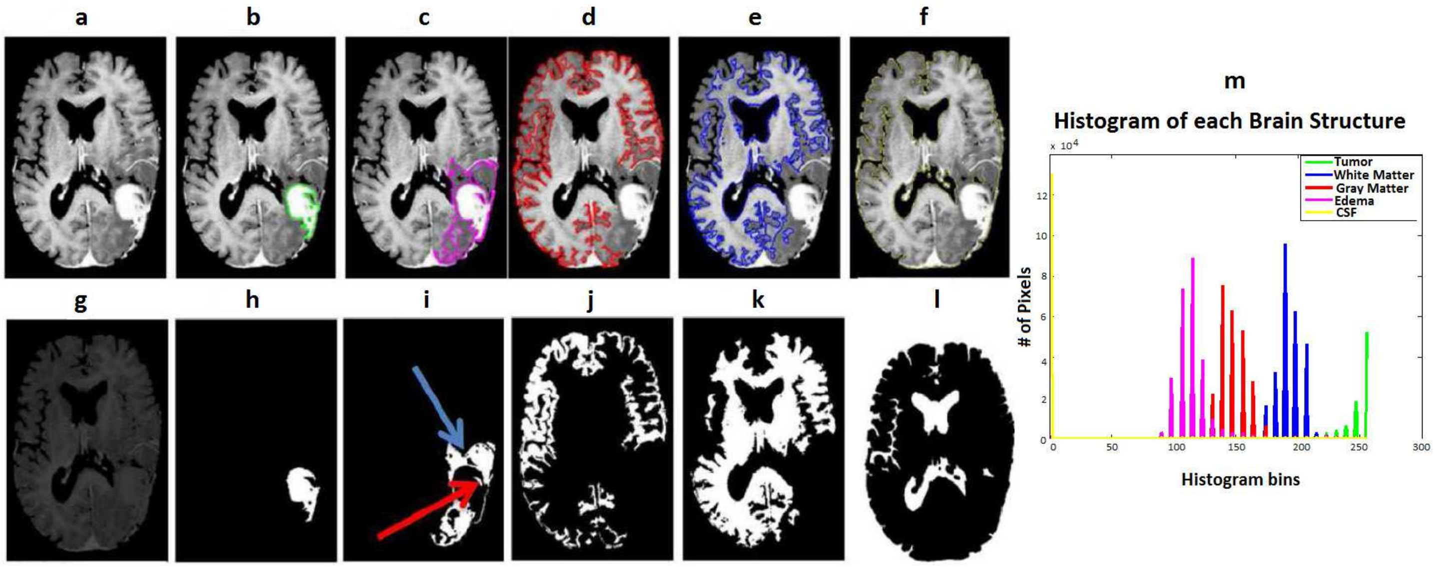

Fig. 4: Segmentation of a gadolinium-enhanced T1-Weighted MRI scan of a patient with glioblastoma multiform using the proposed PNMF-LSM algorithm. The green contour in (b) indicates the growing tumor, the red contour in (d) indicates the gray matter, the blue contour in (e) indicates the white matter, and the yellow contour in (f) indicates the the CSF with the back ground. The proposed PNMF-based level set approach retrieves the histogram of each brain structure shown in (m) obtained from the factor matrix W. The bias field in the MRI image is shown in (g). The pink contour in (c) indicates the region of the brain that corresponds to necrosis and invasive tumor/edema. The red and blue arrows in its binary representation (i) point to the area of necrosis and high signal intensity in Fluid-attenuated Inversion Recovery (FLAIR) sequences (not shown), respectively. (h), (j), (k) and (l) show the binary representations of the growing tumor, the gray matter, the white matter and the CSF with the background, respectively.

the bias field only. Unlike other approaches in the level set framework, no additional parameters need to be estimated. Moreover, the proposed PNMF-LSM is robust to noise compared to other state-of-the-art level set approaches, and it is computationally efficient as well. This can be explained by the fact that the proposed PNMF-LSM: 1) relies on the histogram of the image for clustering rather than the pixel intensity values, and 2) does not estimate additional parameters other than the level set functions and the bias field.

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