Stability Analysis and Breast Tumor Classification from 2D ARMA Models of Ultrasound Images

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***Abstract*— Two-dimensional (2D) autoregressive moving average (ARMA) random fields have been proven to be accurate models of ultrasound breast images. However, the stability properties of these models have not been examined. In this paper, we investigate the stability of 2D ARMA models in ultrasound breast images, and use the estimated 2D ARMA coefficients as a basis for statistical inference using artificial neural networks. Specifically, we use the estimated 2D ARMA coefficients as inputs to a Multi layer perceptron (MLP) neural network to classify the ultrasound breast image into three regions: healthy tissue, benign tumor, and cancerous tumor. Our simulation results on various cancerous and benign ultrasound breast images illustrate the power of the proposed algorithm as attested by different learning algorithms and classification accuracy measures.**

I. INTRODUCTION

Using ultrasound in addition to mammography has helped doctors spot significantly more breast cancers in high-risk women compared with mammograms alone, but it also resulted in four times as many false alarms [6]. These false alarms lead to unnecessary biopsies, which are expensive and unpleasant procedures for the patients. The purpose of our study is to prevent unnecessary biopsies resulting in benign diagnoses by designing a computer-aided diagnosis (CAD) system to improve the accuracy of ultrasound diagnoses. The proposed CAD system uses an artificial neural network (ANN) to select ARMA features for breast cancer diagnosis.

In general, a CAD system for breast tumor detection and classification consists of two stages: (i) feature extraction and (ii) feature classification. Incorporating different features into classification tools results in different CAD systems with varying performances. This is in particular true with CAD systems based on neural networks, whose performance heavily depends not only on the structure of the network but also on the feature vectors used. Neural networks have been used in various ways for the detection and classification of tumors and microcalcifications in ultrasound breast images. In [8], a method for the automatic classification of lesions in ultrasound images by artificial neural networks has been presented. The parameters used for the training of the network are texture and shape related indicators. This choice of training parameters relies on the assumption that the inner part of the lesion and its shape are homogeneous. In [2], a learning vector quantization neural network was used as a classifier to differentiate between malignant and benign solid breast nodules. An autocorrelation feature vector was extracted from a region of interest located by a physician. Woo et al. [11] extracted five shape feature parameters, and fed them to an artificial neural network classifier. They achieved an accuracy of 0.914 on 584 cases. In [7], a hybrid neural network, a combination of an unsupervised self-organizing mapping network and a multilayer perception network with the error backpropagation algorithm, was studied to classify breast tumors using the contour complexity feature estimated by the divider-step method. Their results, on a limited data set of images, achieved an accuracy of 0.93.

The majority of the previous studies relied on raw (grey-level, shape or texture) feature vectors, which are directly extracted from the ultrasound image. However, the underlying physical principles of the imaging modality makes the ultrasound image (and thus all her raw attributes) highly noisy due to speckle, depth dependency and artifacts (e.g., shadowing) which affect the image quality [3]. One way to obtain more reliable image attributes is to replace the noisy image by a stable and accurate model of it. Two-dimensional auto-regressive moving average (ARMA) random fields have been proven to be an accurate model for ultrasound breast images [1]. However, the stability of these models has not been investigated. In this paper, we address the stability issue of 2D ARMA models for ultrasound breast images, and use the estimated 2D ARMA coefficients as feature vectors for a multi-layer perceptron neural network, which classifies the ultrasound image into: (i) normal, (ii) benign tumor, or (iii) cancerous tumor. Furthermore, we test our CAD system using different learning algorithms for the ANN, and compare their respective performances in tumor detection and classification in ultrasound breast images.

II. TWO-DIMENSIONAL ARMA MODELING

We represent the breast image as a 2D random field $\{x[n,m],(n,m)\in\mathbb{Z}^2\}$ [1]. We define a total order on the discrete lattice as follows [1]

$$(i,j)\le(s,t)\Longleftrightarrow i\le s \quad\text{and}\quad j\le t. \tag{1}$$

The 2D ARMA(p_1,p_2,q_1,q_2) model is defined for the $N_1\times N_2$ image $I=\{x[n,m]:0\le n\le N_1-1,0\le m\le N_2-1\}$ by the following difference equation

$$x[n,m]+\sum_{\substack{i=0\\(i,j)\neq(0,0)}}^{p_1}\sum_{j=0}^{p_2} a_{ij}x[n-i,m-j]= w[n,m]+\sum_{\substack{i=0\\(i,j)\neq(0,0)}}^{q_1}\sum_{j=0}^{q_2} b_{ij}w[n-i,m-j], \tag{2}$$

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where $\{w[n,m]\}$ is a white noise field with variance σ^2, and the coefficients $\{a_{ij}\}, \{b_{ij}\}$ are the parameters of the model. From Eq. (2), the image $\{x[n,m]\}$ can be viewed as the output of the linear time-invariant causal system $H(z_1, z_2)$ excited by a white noise input, where

$$H(z_1^{-1}, z_2^{-1}) = \frac{B(z_1^{-1}, z_2^{-1})}{A(z_1^{-1}, z_2^{-1})} = \frac{\sum_{i=0}^{q_1}\sum_{j=0}^{q_2} b_{ij}\ z_1^{-i} z_2^{-j}}{\sum_{i=0}^{p_1}\sum_{j=0}^{p_2} a_{ij}\ z_1^{-i} z_2^{-j}}, \tag{3}$$

with $a_{00} = 1$. A Two-stage Yule-Walker Least Squares parameter estimation method was proposed in [1]. First, the noise sequence $\{w[n,m]\}$ is assumed to be known. The ARMA parameter estimation problem is then reduced to a simple input-output system identification problem, which is solved by a least-squares (LS) method. The final estimate is then obtained by estimating the noise, using a truncated autoregressive (AR) model, and plugging it back in the Least Squares solution [1].

III. Stability of Two-Dimensional ARMA Models

A 2D-ARMA model can be considered as a discrete system with transfer function $H(z_1^{-1}, z_2^{-1})$, given by Eq. (3), where $B(z_1^{-1}, z_2^{-1})$ and $A(z_1^{-1}, z_2^{-1})$ are co-prime polynomials in the independent complex variables $u_1 = z_1^{-1}$ and $u_2 = z_2^{-1}$, and there are no nonessential singularities of the second kind, i.e., there are no points (u_1^*, u_2^*) such that $A(u_1^*, u_2^*) = B(u_1^*, u_2^*) = 0$. It is well-known that the system is bounded-input bounded-output (BIBO) stable if and only if [5]:

$$A(z_1^{-1}, \infty) \neq 0, \qquad |z_1| \geq 1, \quad \text{and} \tag{4}$$

$$A(z_1^{-1}, z_2^{-1}) \neq 0, \qquad |z_1| = 1, |z_2| \geq 1. \tag{5}$$

Jury's test for one-dimensional stability problems [4], summarized in the below proposition, can be easily adapted to apply to (4).

Proposition 1 (Jury's test [4]): Let $A(z)$ be represented as

$$\begin{aligned} A(z) &= a_n z^n + a_{n-1} z^{n-1} + \cdots a_1 z + a_0 \qquad (6) \\ &= z^{-n}\{a_0 z^{-n} + a_1 z^{-(n-1)} + \cdots a_{n-1} z^{-1} + a_n\}. \end{aligned}$$

Then, a necessary and sufficient condition for the roots $A(z) = 0$, with complex coefficients a_k, to lie inside the unit circle is that the following Δ_{2n} matrix be positive innerwise (PI).

$$\Delta_{2n} = \begin{pmatrix} a_n & a_{n-1} & \cdots & a_1 & 0 & 0 & \cdots & a_0 \\ \ddots & a_n & a_{n-1} & a_{n-2} & 0 & 0 & a_0 & \iddots \\ & 0 & a_n & a_{n-1} & 0 & a_0 & a_1 & \\ & 0 & 0 & a_n & a_0 & a_1 & a_2 & \\ & 0 & 0 & \bar{a}_0 & \bar{a}_n & \bar{a}_{n-1} & \bar{a}_{n-2} & \\ & 0 & \bar{a}_0 & \bar{a}_1 & 0 & \bar{a}_n & \bar{a}_{n-1} & \\ \iddots & \bar{a}_0 & \bar{a}_1 & \bar{a}_2 & 0 & 0 & \bar{a}_n & \ddots \\ \bar{a}_0 & \bar{a}_1 & \cdots & \cdots & \bar{a}_{n-1} & 0 & \cdots & \bar{a}_n \end{pmatrix},$$

where the over bar indicates complex conjugates.
Now, to check Eq. (5), let us write $A(z_1^{-1}, z_2^{-1})$ as a polynomial in z_2^{-1} with coefficients, which are polynomials in z_1^{-1}:

$$A(z_1^{-1}, z_2^{-1}) = \sum_{k=0}^{p_2} a_{p_2-k}(z_1^{-1}) z_2^{-k}. \tag{7}$$

Therefore, to check (5), we require that the matrix Δ_{2p_2} be PI, where $\Delta_{2p_2} =$

$$\begin{pmatrix} a_{p_2}(z_1^{-1}) & a_{p_2-1}(z_1^{-1}) & \cdots & a_1(z_1^{-1}) & 0 & \cdots & a_0(z_1^{-1}) \\ \ddots & a_{p_2}(z_1^{-1}) & a_{p_2-1}(z_1^{-1}) & a_{p_2-2}(z_1^{-1}) & 0 & a_0(z_1^{-1}) & \iddots \\ 0 & a_{p_2}(z_1^{-1}) & a_{p_2-1}(z_1^{-1}) & a_0(z_1^{-1}) & a_1(z_1^{-1}) & & \\ 0 & 0 & a_n(z_1^{-1}) & a_1(z_1^{-1}) & a_2(z_1^{-1}) & & \\ 0 & 0 & \bar{a}_0(z_1^{-1}) & \bar{a}_{n-1}(z_1^{-1}) & \bar{a}_{n-2}(z_1^{-1}) & & \\ 0 & \bar{a}_0(z_1^{-1}) & \bar{a}_1(z_1^{-1}) & \bar{a}_n(z_1^{-1}) & \bar{a}_{n-1}(z_1^{-1}) & & \\ \iddots & \bar{a}_0(z_1^{-1}) & \bar{a}_1(z_1^{-1}) & \bar{a}_2(z_1^{-1}) & 0 & \bar{a}_n(z_1^{-1}) & \ddots \\ \bar{a}_0(z_1^{-1}) & \bar{a}_1(z_1^{-1}) & \cdots & \cdots & \bar{a}_{p_2-1}(z_1^{-1}) & \cdots & \bar{a}_{p_2}(z_1^{-1}) \end{pmatrix}.$$

The entries of this matrix are functions of z_1^{-1} with $|z_1| = 1$. It was pointed out by Siljak in [10] that for positivity checking of such a matrix, one requires the positivity checking of the matrix at one point, say at $z_0 = 1$, and the positivity checking of the determinant for all $|z_1| = 1$.

Theorem 1: [5] $A(z_1^{-1}, z_2^{-1}) \neq 0$ for $|z_1| = 1, |z_2| \geq 1$ if and only if

- $\Delta_{2p_2}(z_0)$ is PI for a $z_0, |z_0| = 1$;
- Det $\{\Delta_{2p_2}(z_1^{-1})\} > 0$, for all $z_1, |z_1| = 1$,

where Det denotes the determinant.

Example: stability of a 2D ARMA(1,1,1,1) model: For a 2D ARMA(1,1,1,1), we have

$$H(z_1^{-1}, z_2^{-1}) = \frac{b_{00} + b_{10} z_1^{-1} + b_{01} z_2^{-1} + b_{11} z_1^{-1} z_2^{-1}}{a_{00} + a_{10} z_1^{-1} + a_{01} z_2^{-1} + a_{11} z_1^{-1} z_2^{-1}}, \tag{8}$$

where we assume that all the coefficients are real, and $a_{00} \neq 0$. The first stability condition, described in Eq. (4), yields

$$a_{00} + a_{10} z_1^{-1} \neq 0 \text{ for all } z_1, |z_1| \geq 1 \text{ if and only if } |\frac{a_{10}}{a_{00}}| < 1.$$

Equation (5) establishes the second stability condition, and can be verified by considering the matrix Δ_2, where

$$\Delta_2 = \begin{pmatrix} a_{00} + a_{10} z_1^{-1} & a_{01} + a_{11} z_1^{-1} \\ a_{01} + a_{11} \bar{z}_1^{-1} & a_{00} + a_{10} \bar{z}_1^{-1} \end{pmatrix}.$$

The determinant of Δ_2 is given by $Det\{\Delta_2\} = |a_{00} + a_{10} z_1^{-1}|^2 - |a_{01} + a_{11} z_1^{-1}|^2$. After some simplifications, we obtain that $Det\{\Delta_2\} > 0$, for all $z_1, |z_1| = 1$ if and only if $(a_{00}^2 - a_{11}^2 + a_{10}^2 - a_{01}^2) + 2(a_{00}a_{10} - a_{01}a_{11})x \geq 0$ for all $-1 \leq x \leq 1$. Finally, the following proposition states the necessary and sufficient conditions for a 2D ARMA(1,1,1,1) to be stable.

Proposition 2: Consider the 2D ARMA(1,1,1,1) model given by Eq. (8). Let

$$\alpha = a_{00}^2 - a_{11}^2 + a_{10}^2 - a_{01}^2, \tag{9}$$

and

$$\beta = 2(a_{00}a_{10} - a_{01}a_{11}). \tag{10}$$

Then the model is stable if and only if the following two conditions are satisfied:

(i) $|\frac{a_{10}}{a_{00}}| < 1$

(ii) $\begin{cases} \alpha - \beta > 0, & \text{if } \beta > 0; \\ \alpha + \beta > 0, & \text{if } \beta < 0. \end{cases}$

IV. Neural Network for Tumor Detection and Classification

Multi-layer perceptron neural network (MLP) is the most popular and widely used nonlinear network for solving many practical problems in applied science, biology, and engineering. The reason for the popularity of the MLP network is that it is very flexible and can be trained to assume the shape of the patterns in the data, regardless of the complexity of these patterns. The training of the MLP is supervised in that, for each input, the corresponding output is also presented to the network. The initial weights are set at random. The mean square error (MSE), the most commonly used error indicator, of the prediction over all training pattern for a network with one output neuron can be written as [9]:

$$E = \frac{1}{2N}\sum_{i=1}^{N}(t_i - y_i)^2, \tag{11}$$

where E denotes the MSE, t_i and y_i are the target and predicted output for the i^{th} training pattern, respectively, and N is the total number of training patterns. Depending on the error surface equation, the MLP weights are adjusted using three different methods:

a) Back propagation with momentum method:

$$W_{\text{new}} = W_{\text{old}} + \Delta W; \quad \Delta W = -\eta\frac{\partial E}{\partial w} + \alpha \Delta W_{\text{old}}, \tag{12}$$

where η and α are the learning rate and momentum rate, respectively.

b) Delta bar delta learning (adaptive learning rate):

$$W_{\text{new}} = W_{\text{old}} - \eta_{+}\frac{\partial E}{\partial w}, \tag{13}$$

where

$$\eta_{+} = \begin{cases} \eta_{\text{old}} + a, & \text{if } \frac{\partial E}{\partial w} > 0; \\ \eta_{\text{old}} b, & \text{if } \frac{\partial E}{\partial w} \leq 0, \end{cases} \tag{14}$$

where η_{+} is the adaptive learning rate, a and b are constants less than one.

c) The Levenberg-Marquard algorithm:

$$W_{\text{new}} = W_{\text{old}} + \Delta W; \quad \Delta W = -\frac{\frac{\partial E}{\partial w}}{H + e^{\gamma} I}, \tag{15}$$

where H is the Hessian matrix of the MSE with respect to the weights. The term e^{γ} produces a conditioning effect on the second derivative such that the error monotonically decreases.

V. Simulation Results

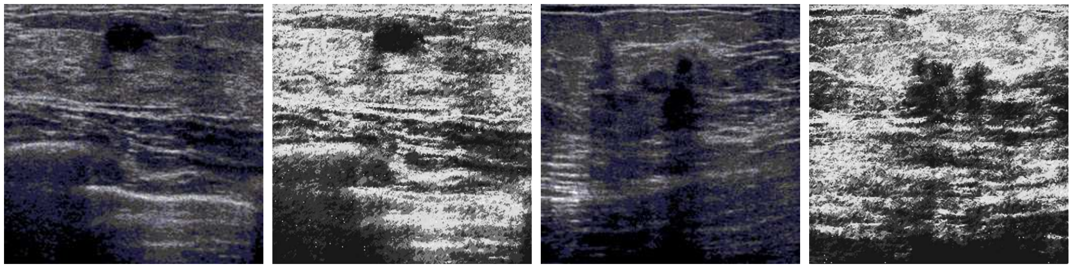
we found that ARMA$(1,1,1,1)$ is a sufficient model order to accurately represent ultrasound breast images [1]. Figure 1 shows two ultrasound images, one with a cancerous tumor and one with a benign tumor, and their respective ARMA$(1,1,1,1)$ models. The ARMA parameters were estimated using a window of size 16×16. The choice of the window size presents an inherent trade-off between the accuracy of the representation and the accuracy of the classification [1]. A large window size would lead a better representation of the ARMA model, but might include pixels from different classes. We found that for 256×256 images, a 16×16 window size leads to good discriminatory ARMA feature vectors. Each image is therefore represented by a number of 1×8 ARMA feature vectors, which contain the 8 parameters $a_{00}, a_{01}, a_{10}, a_{11}, b_{00}, b_{01}, b_{10}, b_{11}$ for each 16×16 sub-block image. Without loss of generality, we chose $a_{00} = b_{00} = 1$. Therefore, the size of the feature vectors reduces to 6 instead of 8. We decide that an image has a cancerous (resp., benign) tumor if at least one of the sub-block images is classified as a cancerous (resp., benign) tumor. Otherwise, we conclude that the image is healthy and contains no tumors.

We conducted our simulations using 573 ARMA feature vectors of healthy, benign and cancerous ultrasound breast images. The stability analysis, based on proposition 2, revealed that all 573 feature vectors satisfied stability criterion (i) and all but 5 vectors satisfied stability criterion (ii). It would be interesting to investigate the medical implications of the unstable ARMA features. Such an investigation will be the focus of a future work, and will involve a larger data set and a precise clinical evaluation of the images by our medical collaborators.

The ARMA feature vectors were used as the input to an MLP neural network with six input layer nodes, one hidden layer containing four nodes, and three output nodes. The three algorithms, presented in Section IV, were used to learn the neural network: The momentum backpropagation (BP), the delta-bar-delta, and the Levenberg-Marquardt algorithms. The mean square error (MSE) curves of all three algorithms, displayed in Fig. 2, converge in less than 200 epochs. We observed that the performance of the delta-bar-delta algorithm depends heavily on the choice of its parameters a, b (see Eq. 14). For instance, for $a = b = 0.1$, the algorithm fails to converge (Fig. 2(a)), whereas for $a = 0.01, b = 0.1$, the algorithm converges, and thus results in a much better classification accuracy (Fig. 2(b)). Therefore, if the delta-bar-delta learning algorithm is used, one should carefully chose its additive and multiplicative parameters by trial and error. Table V summarizes the performance of the neural network using the three learning algorithms. The steady-state MSE, accuracy, sensitivity and specificity are listed for each case. We observe that the MLP neural network based on the Levenberg-Marquardt algorithm achieves the highest accuracy (95%), sensitivity (98%), and specificity (97%). In particular, the MLP Levenberg-Marquardt outperforms the k-means algorithm presented in [1].

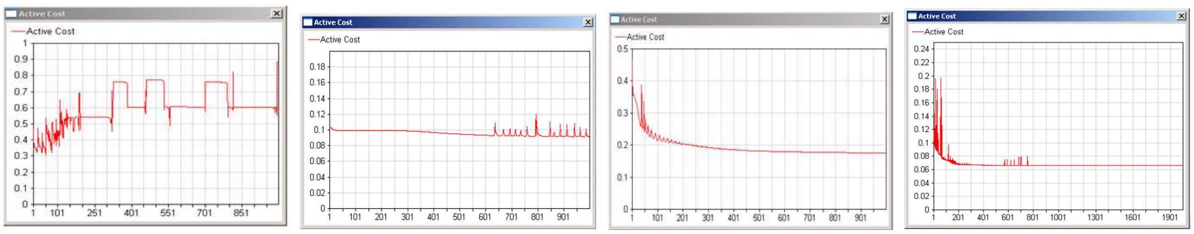
VI. Conclusion

Ultrasound breast images can be accurately represented by 2D ARMA models [1]. We showed that such models are stable under mild conditions, and thus form a stable and more robust (to noise) platform for the analysis of ultrasound breast images. We used the estimated parameters of the 2D ARMA models as feature vectors for tumor detection and classification using an MLP neural network with various learning algorithms. We observed that the performance of the neural network heavily depend on the learning algorithm



(a) (b) (c) (d)

Fig. 1. Ultrasound images and their ARMA$(1,1,1,1)$ models: (a) Cancerous tumor; (b) ARMA$(1,1,1,1)$ model of (a); (c) Benign tumor; (d) ARMA$(1,1,1,1)$ model of (c).



(a) (b) (c) (d)

Fig. 2. Mean square error (MSE) versus number of iterations. The MLP neural network was designed with six input layer nodes, one hidden layer containing four nodes, and three output nodes: (a) MLP Delta Bar Delta with parameters $a = b = 0.1$; (b) MLP Delta Bar Delta with parameters $a = 0.01, b = 0.1$; (c) MLP backpropagation with momentum rate $\alpha = 0.7$; (d) MLP Levenberg-Marquardt algorithm.

TABLE I

MLP CLASSIFICATION ACCURACY OF CANCEROUS AND BENIGN TUMORS

Learning algorithm	steady-state MSE	Accuracy	Sensitivity	Specificity
Momentum BPA	0.18	83 %	96%	94 %
Delta Bar Delta	0.09	89%	94 %	96%
Levenberg-Marquard	0.065	95%	98%	97%

and its parameters. The proposed CAD system provides a promising starting point to test the hypothesis that two-dimensional ARMA models can contribute significantly to the clinical goals of detection, classification, and testing of breast cancer in ultrasound images.

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